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Distillation column, Model reduction, PI controller, Fuzzy Inference System, MATLAB tool

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# FUZZY CONTROLLER OF MODEL REDUCTION DISTILLATION COLUMN WITH MINIMAL RULES

#### Abstract

In this paper the control of a binary distillation column is described. This control is done with fuzzy logic, one with PI- like fuzzy controller and the other with modified PI fuzzy controller, using the minimal rules for fuzzy processing. This work is focused on model reduction of Wood and Berry binary distillation column to get the best performance. It is desired to minimize the rules in order to reduce the computation time to make a faster decision. Comparisons will be made between two versions of fuzzy controllers utilizing reduced rules to verify the outputs. The controlled variables are top composition with high concentration and bottom composition with low. To demonstrate the performance of the fuzzy PI control schemes, results are compared with a classical PI controller and optimal methods, like Differential Evolution (DE), Invasive Weed Optimization (IWO). The proposed structure is able to quickly track the parameter variation and perform better in load disturbances and also for set point changes. Then all the processes of the distillation column with it's fuzzy controllers are simulated in MATLAB software as the results are shown.

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## **1. INTRODUCTON**

Distillation is one of the most common and best understood separation methods, widely used in the process industries, e.g. in partial fractionation of crude oil, separation of noble gases, and production of distilled alcoholic beverages, etc. Hence design and operation of the distillation columns has been studied in many textbooks e.g. (Perry & Green, 2008; Navef Ghasem, 2014; Gorak & Schoenmakers, 2014). There are many approaches in the controller design for distillation columns. A survey has been published in (Skogestad, 1997) where the focus is on the construction of the SISO loops governed by PID controllers. Design of the stabilizing controllers for unstable distillation columns was given in (Jacobsen & Skogestad, 1991). Robust H∞ control of the distillation process was described in Lundstrom, Skogestad & Doyle (1999). Control of the distillation processes based on Model Predictive Control (MPC) was introduced in Cutler & Ramaker (1979). In recent years, multiple fuzzy control setups were independently investigated and verified for different distillation columns mainly on simulation scenarios (Miccio & Cosenza, 2014; Vasickaninová, Bakošová & Mészáros, 2016; Drgona, Takác, Hornák, Valo & Kvasnica, 2017). Fuzzy control provides a formal methodology for implementing a knowledge of human about a system. Since it gives a convenient method for constructing nonlinear controllers via the use of heuristic information, it is a practical alternative for a variety of control applications (Reznik, 1997). Direct fuzzy control of the distillation columns was studied in (Aaron, Antony & Kumaravel, 2018; Fileti, Antunes, Silva & Pereira, 2007), while in (Glankwamdee, Tarathammatikorn & Chattana-anan, 1999) the supervisory fuzzy system for adjusting the parameters of the classical PI controllers is proposed for a binary distillation column.

In this paper a fuzzy logic based control (PI) schemes have been proposed for distillation column. Fuzzy Inference Systems (FIS) are proposed to adjust the manipulated variables (reflux flow rate L) and (steam flow rate V) to get the desired composition of products (top XD) and (bottom XB) for a binary distillation column. To control the top (desired value = 0.98) and bottom (desired value = 0.02) product composition two separate fuzzy inference systems has been designed. The scheme uses fuzzy rules and reasoning to determine the desired outputs based on the error signal and integral of it (version 1), while (version 2) is modified depends on the error and derivative of it.

The paper is organized as follows. Section II presents a detailed description of the distillation column. Section III is devoted to fuzzy controller synthesis. Experimental results and discussion are presented in Section IV. Robustness analysis is presented in section V and Conclusions are drawn in Section VI.

# 2. DISTILLATION COLUMN

The distillation column feed tank is carrying methanol and water mixtures. During the process the methanol water mixture can be heated. The light weight molecules rise to the top of the column and weighty components moves downstairs to the column. The separation takes place in a vertical column where heat is added to a reboiler at the bottom and removed from condenser at the top. Figure 1 shows distillation column.



Fig. 1. Shows distillation column

In the present work, Wood and Berry distillation column model is taken for case study. The 2 x 2 MIMO process is presented by Wood and Berry (1973), Hamdy, Ramadan & Abozalam (2018). The process transfer function matrix of the distillation process is given by:

$$\begin{bmatrix} x_D \\ x_B \end{bmatrix} = G(s) \begin{bmatrix} L(s) \\ V(s) \end{bmatrix}$$
 (1)

where  $G(s) = \begin{vmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{vmatrix}$  is the system matrix For Wood and Berry is:

$$G(s) = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-s}}{21s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix}$$
(2)

Decoupling is used to reduce the control loop interactions. The theory of decoupling control for MIMO processes has been well-established in many textbooks and papers (Luyben, 1970; Liu, Wang, Mei & Ding, 2013). In this work, and due to the advantages of simplified decoupling, so that can be used. The decoupler is:

$$D(s) = \begin{bmatrix} 1 & \frac{-g_{12}}{g_{11}} \\ \frac{-g_{21}}{g_{22}} & 1 \end{bmatrix}$$
(3)

The resulting transfer matrix decoupler T(s) is (Luyben, 1970):

$$T(s) = \begin{bmatrix} g_{11}(s) - \frac{g_{12}(s)g_{21}(s)}{g_{22}(s)} & 0\\ 0 & g_{22}(s) - \frac{g_{12}(s)g_{21}(s)}{g_{11}(s)} \end{bmatrix}$$
(4)

Then, according to equation (2) and (4), the diagonal matrix of WB column is given by:

$$\overline{g_{11}}(s) = \frac{12.8e^{-s}}{16.7s+1} - \frac{6.237(14.36s+1)e^{-7s}}{228.69s^2+31.89s+1}$$
(5)

$$\overline{g_{22}}(s) = \frac{-19.4e^{-3s}}{14.4s+1} + \frac{9.745(16.7s+1)e^{-9s}}{228.69s^2+31.89s+1}$$
(6)

To simplify the equations (5) and (6), pade approximation (Kalpana, Harikumar, Senthilkumar, Balasubramanian & Abhay, 2017) is used to remove the nonlinear term in equations and after applying some mathematical arrangements, it gets:

$$G_{11WB} = \frac{-1432 \, s^4 + 7368 \, s^3 + 1843 \, s^2 + 155.2 \, s + 3.75}{3819 \, s^5 + 9491 \, s^4 + 3971 s^3 + 547.1 s^2 + 30.05 \, s + 0.5714} \tag{7}$$

$$G_{22WB} = \frac{2093 \, s^4 - 2698 \, s^3 - 710.1 \, s^2 - 59.71 \, s - 1.43}{3293 \, s^5 + 3615 \, s^4 + 1146 \, s^3 + 144.1 \, s^2 + 7.747 \, s + 0.1481} \tag{8}$$

The diagonal transfer matrix T(s) obtained in equation (7) and (8) are complex since its high order transfer functions. Controller tuning can therefore be difficult. It is then often suggested to approximate them by a simpler transfer functions to facilitate controller tuning, so model reduction techniques is used to reduce the order of these equations. Model reduction is a technique widely used in part of dynamic analysis and design of systems, in this paper, optimization technique, which is Particle Swarm Optimization (PSO) algorithm, can be selected. This algorithm is a biologically inspired algorithm and it is a population based stochastic nature. After applying the steps algorithm for PSO to equ(7) and (8), and select generation count limit = 200, population size = 50, problem dimension = 5, mutation probability = 0.06, number of elites = 2, after exploitation improvement program, the reduced transfer function for applying PSO algorithm to equation (7) is:

$$G_{11rWB} = \frac{0.8498s + 0.5051}{s^2 + 0.9989s + 0.077} \tag{9}$$



The step responses of the reduced order model and the original system are compared in Figure 2.

Fig. 2. Step Response Comparison of Original and Reduced Order System G11

For transient specifications, the original and reduced systems are compared as shown in Table 1.

Characteristics	$G_{11WB}$	$G_{11rWB}$
Ts	48.1	45.8
Tr	27.8	25.8
Мр	0.287%	0%
Ess	6.58	6.56

Tab. 1. Transient specification comparison between G<sub>11WB</sub> and G<sub>11rWB</sub>

And for equation (8) is:

$$G_{22rWB} = \frac{0.7324s - 1}{0.7811s^2 + 1.047s + 0.1049} \tag{10}$$

The step responses of the reduced order model and the original system are compared in Figure 3.



Fig. 3. Step Response Comparison of Original and Reduced Order System G22

For transient specifications, the original and reduced systems are compared as shown in Table 2.

Characteristics	G22WB	G22rWB
Ts	39.8	37.2
Tr	22.3	20.2
Мр	0.675%	0%
Ess	-9.72	-9.53

Tab. 2. Transient specification comparison between G22WB and G22rWB

As seen from Figures (2, 3) and Tables (1,2), the reduction models by the proposed method are very close to the original model especially in settling and rise time.

#### 3. DISTILLATION COLUMN CONTROLLERS

There are many control strategies applied to distillation column multivariable system (Jin, Wang & Liu, 2016), (Prodanović, Nedić & Filipović & Dubonjić, 2017), in this work using decentralized decoupling structure control strategy, where the proposed method of control design involves combination of simplified decoupler, and decentralized controller (PI-like fuzzy control and modified PI fuzzy) for each loop. Figure 4 Shows controlled structure block-diagram.



Fig. 4. Block-diagram of the proposed control system

There are many methods for desiging distillation column using fuzzy controller depends on rules minimization procedures (Hung & Benito Ferndndee, 1993; Margaglio, Lamanna & Glorennec, 1997; Farzin & Mirshekari, 2014). In this work using (9 rules) and two versions:

### 1. Version 1 (PI-like fuzzy controller PILFC).

The equation giving a conventional PI-controller is (Reznik, 1997; Avatefipour, Piltan, Reza & Nasrabad, 2014; Javadi & Hosseini, 2009):

$$u(t) = Kp \times e(t) + Ki \times \int e(t)dt$$
(11)

where *Kp* and *Ki* are the proportional and the integral gain coefficients. A block diagram for a fuzzy control system looks like Figure 5.



Fig. 5. A block diagram of a PI fuzzy control system (version 1)

#### 2. Version 2 (Modified PI- like fuzzy controller MPILFC).

Now the fuzzy controller and the rules table have other inputs. It means that the rules themselves should be reformulated. Sometimes it is difficult to formulate rules depending on an integral error as in Figure 5. Because it may have the very wide universe of discourse, so that this version 2 has the error and the change-of-error inputs and one needs just to integrate the output of a controller. One may consider the controller output not as a control signal, but as a change in the control signal. The block diagram for this system is given in Figure 6. It is clear that the gain factor Ki is used with the error input and Kp with the change-of-error. The change-of-control output  $\Delta u(t)$  is added to u(t-1). It is necessary to stress here that this takes place outside the PI-like fuzzy controller, and is not reflected in the rules themselves.



Fig. 6. A block diagram of a modified PI fuzzy control system (version 2)

The structure of FLC contains four main parts (Avatefipour, Piltan, Reza & Nasrabad, 2014) as shown in Figure 7. Fuzzification, inference mechanism, rule base and defuzzification, where fuzzification part is used for converting real input to fuzzy input. The rule-base part contains the expert knowledge in the form of a set of rules.



Fig. 7. Shows the fuzzy logic structure

## 4. RESULT AND DISCUSSION

For selection the optimum values of (Kp, Ki) in fuzzy control for both versions, using the best values (Alawad & Jebar, 2020), which is using optimization methods.

# 4.1. Case study 1 (*G*<sub>11*rWB*</sub>)

By applying the version 1 and version 2 for  $G_{11rWB}$  and use the following gains in (Alawad & Jebar, 2020): Kp = 24.501, Ki = 24.601. Figure 8 shows the membership functions of version 1 for two inputs, while Figure 9 shows the output and Figure 10 shows the membership functions of version 2 for two inputs. Figure 11 shows the output.



Fig. 8. Membership functions for two inputs (version 1) of  $G_{11rWB}$ 



Fig. 9. Membership function for output (version 1) of G<sub>11rWB</sub>



Fig. 10. Membership functions for two inputs (version 2) of G<sub>11rWB</sub>



Fig. 11. Membership function for output (version 2) of G<sub>11rWB</sub>

The fuzzy controllers rule base composed of 9  $(3\times3)$  rules as shown in Table 3. Also we use the Mamdani inference system as inference engine and centroid method for defuzzification in the fuzzy controllers.

e de	Ν	Z	Р
Ν	N	N	Z
Z	N	Z	Р
Р	Z	Р	Р

Tab. 3. Rules for fuzzy control

The following results are obtained and compared with others controllers using (PI) with optimization techniques (Alawad & Jebar, 2020) and (PI) with fuzzy as shown in Table 4. Figure 12 Shows the step response comparison between all controllers for  $G_{11rWB}$ . In the simulation results reported in this work as shown in Figure 12, the controlled variable XD was always considered to have a set-point equal to 0.98.

Tab. 4. Transient response parameters of  $G_{11rWB}$  for different controllers

Controller types	Mp%	Ts (minute)	Tr (minute)	Ess
Without controller	0.13	7.72	3.67	0.13
PI-MATLAB	12.4074	46.6524	13.8450	0
PIFC (Alwadie, Ying & Shah, 2003)	0	16.66	9.352	0
PI-DE (Alawad & Jebar, 2020)	2.3725	0.6299	0.0977	0
PI-IWO (Alawad & Jebar, 2020)	7.9654	3.1801	0.3889	0
PILEFC(version1)	2.9279	0.4802	0.0548	0
MPILFC (version2)	0.0922	2.5613	1.2387	0



Fig. 12. Step Response comparison between all controllers for G<sub>11rWB</sub>

As seen from Table 4 and Figure 12. version 1 is better than version 2 in speed response (less Ts and Tr) but with an over shoot (Mp=2.9279%), also version 1 gives a small improvement when compared with optimization method (PI-DE).

#### 4.2. Case study 2 ( $G_{22rWB}$ )

By applying the version 1 for  $G_{22rWB}$  and use the following gains (Alawad & Jebar, 2020) Kp = -0.2717, Kp = -0.028. Figure 13 shows the membership functions of version 1 for two inputs, while Figure 14 shows the output.



Fig. 13. Membership functions for two inputs (version 1) of G<sub>22rWB</sub>



Fig. 14. Membership function for output (version 1) of G<sub>22rWB</sub>

And by applying the version 2 for  $G_{22rWB}$  and use the following gains (Alawad & Jebar, 2020) Kp = -0.2717/2 = -0.13585,  $Ki = -0.028 \times 1.5 = -0.042$ . Figure 15 shows the membership functions of version 2 for two inputs, while Figure 16 shows the output.



Fig. 15. Membership functions for two inputs (version 2) of G<sub>22rWB</sub>



Fig. 16. Membership function for output (version 2) of  $G_{22rWB}$ 

The following results are obtained and compared with others controllers (Alawad & Jebar, 2020; Alwadie, Ying & Shah, 2003) as shown in Table 5. Figure 17 shows the step response comparison between all controllers for  $G_{22rWB}$ . In the simulation results reported in this work as shown in Figure 17, the controlled variable XB was always considered to have a set-point equal to 0.02.

Controller types	Mp%	Ts (minute)	Tr (minute)	Under shoot	Ess
Without controller	N/A	N/A	N/A	N/A	N/A
PI-MATLAB	9.4941	26.3414	6.0747	3.7694	0
PIFC (Alwadie, Ying & Shah, 2003)	0.5	25.732	15.83	0	0
PI-DE (Alawad & Jebar, 2020)	0	7.7918	4.3258	6.2333	0
PI-IWO (Alawad & Jebar, 2020)	17.9261	11.3792	1.7423	12.1463	0
PILEFC (version1)	7.2023	7.3392	1.9995	10.3027	0
MPILFC (version2)	4.0792	10.1798	5.5103	0	0

Tab.5. Transient response parameters of G<sub>22rWB</sub> for different controllers



Fig. 17. Step Response comparison between all controllers for G<sub>22rWB</sub>

As seen from Table 5 and Figure 17 version 1 is better than version 2 in speed response (less Ts and Tr), but over shoot (7.2037%) and under shoot (10.3027%) is not best when compared with PI-DE. The system G22rWb is not appear in Figure 16, because the system is unstable.

## **5. CONCLUSIONS**

Two advanced controllers were developed in this paper, i.e., version1 and a version 2 fuzzy logic controller and their performances compared in simulation for a case study, i.e., a Wood and Berry binary distillation column, which is characterized by high nonlinearities and parameter uncertainties in the underlying mathematical model. Triangular membership functions are used to represent the input and output variables

The performance and the control synthesis of the fuzzy control approaches are moreover compared with classical PI controller, FPI, (DE) and (IWO) optimization methods. All the simulation results confirmed the robustness and the effectiveness of the fuzzy control action, with evident advantages for the (version 1) fuzzy controller, but the main disadvantage it may have the very wide universe of discourse. In distillation columns, tight composition control of products with 98% purity level is not achievable with classical PID controllers only due to sensitivity to disturbance. This work focuses on one of the most extended forms of conventional decoupling called simplified decoupling. The simplified decoupling technique has the simple decoupler form, but controller cannot be designed directly from the decoupled process model without using the model reduction technique.

Generally, (PILFC) is better than (MPILFC), in transient response specifications (Mp%, Ts, Tr), when compared with (MPILFC), this is clear for minimum-phase system  $G_{11rWB}$ , but with The non-minimum phase system  $G_{22rWB}$ , the proposed (MPILFC) is better in average values of transient response, when compared with (PILFC).

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