Applied Computer Science, vol. 15, no. 1, pp. 18–35
doi:10.23743/acs-2019-02

Submitted: 2018-12-06
Revised: 2019-01-01
Accepted: 2019-03-20

resource-constrained project scheduling problem, discounted cash flows, payment project scheduling, multi-stage project, milestones

Marcin KLIMEK [0000-0002-5912-197X]*

TECHNIQUES OF GENERATING SCHEDULES FOR THE PROBLEM OF FINANCIAL OPTIMIZATION OF MULTI-STAGE PROJECT

Abstract

The article presents the problem of scheduling a resource-constrained project with discounted cash flow maximization from the perspective of a contractor. The contractor's expenses (cash outflows for the contractor) are associated with the execution of activities. The client's payments (cash inflows for the contractor) are performed after fulfilling the agreed project stages. The following techniques are suggested for solving the problem: the activity right-shift procedure, the backward scheduling with the optimization of completion dates for the agreed project stages and the modified triple justification technique. The effect of these techniques of generating schedules is illustrated for an exemplary project. Finally, an experimental analysis of the proposed procedures is presented.

1. INTRODUCTION

One of the most often raised problems related to operational research in recent years is the Resource-Constrained Project Scheduling Problem (RCPSP). For the RCPSP can be used different optimization criteria: time criteria, namely the minimization of makespan, or financial criteria, namely the maximization of discounted cash flows etc. The discussion of the applied models may be found in the study (Hartmann & Briskorn, 2012). The analysis of cash flows related to the implemented project is particularly significant when planning the project. These cash flows are discounted in the majority of studies, their NPV (Net Present

* State School of Higher Education, Department of Computer Science, Sidorska 95–97, 21-500 Biala Podlaska, Poland, m.klimek@dydaktyka.pswbp.pl

Value) is calculated with the assumed discount rate. The maximization of the sum of discounted cash flows is the most frequent objective function in research taking into account the economic aspects in scheduling a project (Hartmann & Briskorn, 2012). The first optimization model with discounting cash flows Max-NPV for a project without resource constraints was proposed by Russell (1970). Numerous optimization models RCPSP-DC (RCPSP with Discounted Cash Flows) are analyzed for a project with limited resources (Hartmann & Briskorn, 2012; Herroelen, Reyck & Demeulemeester, 1997).

One of the analyzed problems of the RCPSP-DC is Payment Project Scheduling (PPS) (client (Dayanand & Padman, 1997; Herroelen, Reyck & Demeulemeester, 1997; Leyman & Vanhoucke, 2016). Research for the PPS problem include the determination of, among others, the number of tranches, the due dates and the amounts in particular payment tranches etc. A payment schedule, optimized from the point of view of the contractor (Klimek & Łebkowski, 2015, 2017; Mika, Waligóra & Węglarz, 2005; Ulusoy, Sivrikaya-Serifoglu & Sahin 2001), less frequently from the perspective of the client (Dayanand & Padman, 2001), is searched for in PPS problems. There are also attempts to find a compromise in the form of a payment implementation plan satisfactory both for the client and for the contractor (Bahrami & Moslehi, 2013; Ulusoy & Cebelli, 2000). The principles of settling works between the client and the contractor are determined and they may be the effect, for instance, of joint negotiations. The need for negotiations results from the fact that the client's and the contractor's interests with regard to payments for the project are usually divergent. The most favorable situation for the client is when the client pays the contractor only once after the end of the entire project, as in the Lump-Sum Payment model (LSP). The contractor prefers receiving the client's highest payments as soon as possible which may be allocated for funding the contractor's project works. The client's payments before the end of the project (Vanhoucke, Demeulemeester & Herroelen, 2003) are performed in the cash flow model assigned to events – Payment at Event Occurrences model (PEO). Events activating a payment are most often related to stages of works or activities, for instance payments are made after the end of selected activities or after the end of each activity in the Payments at Activities' Completion Times model (PAC). Payments spread evenly throughout the project, made in equal time intervals, are made in the Equal Time Intervals model (ETI) with a specific number of payments determined before building the schedule and the Progress Payments model (PP) with an unknown number of payments, depending on the duration of the project in the planned schedule. The last payment in each of the LSP, PEO, PAC, ETI, PP models is made upon the end of the project.

Models which include the use of gradual project settlement, with specified milestones, determined before project is planned by the client and the contractor (Klimek & Łebkowski, 2015; He, Wang, Jia & Xu, 2009; 2012; He & Xu, 2008) taking into account, for instance, the project progress, the costs of the execution of activities, the costs of resource involvement etc. are developed. Earlier payments, before the project completion, for instance in stages, are unfavorable for the client. A bonus-penalty system is introduced during settlements to compensate the client's need to make earlier payments (He & Xu, 2008). Penalties are used for exceeding the agreed due dates for the project or its stages, while the bonuses are used for the completion of the project or its stages earlier than the agreed due date. Accurate due dates or time windows are defined in which the completion of an activity (stage) is neither awarded nor punished. The penalties and bonuses are to mobilize the contractor of works to execute the project as quick as possible. Without additional stimuli, the contractor prefers to delay the execution of particular activities (stages) with which costs are associated (subsequent expenses have a lower discounted value) The bonus-penalty system is effective from the perspective of the contractor if the benefits from the bonuses are higher than the contractor's costs related to the acceleration of completing an activity (stage), and the penalties for the lack of punctuality are higher than the contractor's benefits from a later completion of an activity (stage) The bonus-penalty system is effective from the perspective of the contractor if the bonuses for the contractor for an accelerated execution of activities (stages) are not higher than the client's benefits from an earlier execution of activities (stages), and the penalties for the lack of punctuality are higher than the client's losses related to delays in the execution of activities (stages).

Apart from settlements determined between the client and the contractor, the project's financial optimization from the perspective of the contractor takes into account other cash flows, for instance the contractor's expenses borne in connection with the execution of activities, the costs of resource involvement, transport and purchase of materials etc. The contractor's expenses are usually more frequent than receipts and their value depends on the incurred costs of works. Cash flows assigned to activities may be implemented at various moments of the activity's execution. However, it is most often assumed that expenses (cash outflows) are borne when the activity commences, while receipts (cash inflows) are acquired upon finishing the activity. Receipts/expenses related to the activity are also converted into a single cash flow performed directly before or directly after the end of the activity.

Various optimization models for projects with single-mode or multi-mode for executing the activities, performed with the use of renewable, double constrained, or non-renewable resources etc. are examined for RCPSP-DC (Leyman & Vanhoucke, 2016). Capital in Capital Constrained Project Scheduling Problem models (CCPSP) (Leyman & Vanhoucke, 2017; Smith-Daniels, Padman & Smith-Daniels, 1996) is one of the limited, non-renewable resources taken into account when building the schedule – the expenses and receipts need to balance one another at any moment of the project's duration, for instance activities may be executed only when financial funds for their execution obtained from the completion of previous works are available.

This article analyzes the single-mode RCPSP from the perspective of the contractor of the project, with expenses borne on account of the execution of activities and with receipts acquired from the client for the completed agreed stages of works. The problem with stage settlement of project works was not examined in this form in research related to RCPSP, except for the author's studies. Stage cash flows are examined for multi-mode RCPSP (He et al., 2009; He, Liu & Jia, 2012; He & Xu, 2008). The proposed optimization model for the purposes of the client's settlements with the contractor defines agreed stages (milestones) (Klimek, 2017; Klimek & Łebkowski, 2013, 2015, 2017): group of activities to be executed, due dates, the amount of payments for the execution of works as well as the amount of penalties decreasing the stage payments charged for a delayed completion of works. The proposed model with defined milestones may be useful in practice since it enables the settlement of the project depending on the degree of its completion. The milestone technique is used in practical projects to determine particularly important events on the way to achieving the project's objectives. It facilitates project management, increases the possibility to control its execution and the punctuality of works. According to the author, it may also be used for financial settlements between the contractor and the client.

It is recommended to schedule works for the financial optimization model of a multi-stage project so that the activities are started as late as possible, and the agreed project stages are completed as early as possible. The purpose of the study is to analyze the techniques of generating solutions, prepared by the author, dedicated for the examined model, taking into account the specific nature of stage settlements, namely the right shift of activities for schedule with a fixed resource allocation, backward scheduling with the optimization of completion times for project stages, or modified justification techniques. The effect of these techniques is illustrated for an exemplary project. The simple experimental analysis of the techniques of generating solutions is conducted for test instances from the PSPLIB library (Project Scheduling Problem LIBrary) (Kolisch & Sprecher, 1997) with additionally defined financial settlements of the project.

2. PROBLEM FORMULATION

A nonpreemptive single-mode RCPSP is analyzed in which the project is presented in the AON representation (Activity-On-Node) as a directed graph $G(V, E)$ in which V is the set of nodes corresponding to activities, and E is the set of arcs presenting precedence relations (Eq. 1) between activities (finish-start zero-lag precedence).

$$ST_i + d_i \le ST_j, \qquad \forall (i, j) \in E \tag{1}$$

where: i – index of activity, i = 1, ..., N_A (N_A – number of activities),
ST_i – starting time of activity i,
d_i – duration of activity i.

Activities are executed with the use of constrained, renewable resources the number of which is constant in time (Eq. 2). The number of used resources cannot exceed a_k at any time t, throughout the schedule's execution time.

$$\sum_{i \in J(t)} r_{ik} \leq a_k, \qquad \forall t : t = 1, ST_{N_A+1}, \forall k : k = 1,..K \tag{2}$$

where: $J(t)$ – set of activities executed in the period [t-1, t],
r_{ik} – demand of activity i for resource type k= 1...K,
K – number of types of resources,
a_k – number of available resources type k.

The applied optimization criterion is the maximization of the sum of discounted cash flows from the perspective of the contractor (Eq. 3).

$$F = \sum_{i=1}^{N_A} (CFA_i \cdot e^{-\alpha \cdot ST_i}) + \sum_{m=1}^{N_M} (CFM_m \cdot e^{-\alpha \cdot MT_m}) \tag{3}$$

where: F – objective function, sum of discounted cash flows,
CFA_i – contractor's expenses related to the execution of activity i,
m - index of project stage (milestone), m = 1, …, N_M,
N_M – number of project stages,
CFM_m – client's payments for the completion of the m stage,
α – discount rate,
MT_m – completion date for the m stage in the current schedule.

The model does not contain periodic payments. It has been assumed that all the contractor's costs may be directly assigned to particular activities. Cash flows related to the project include cash inflows on account of the client's payments for the completion of project stages CFM_m as well as cash outflows associated with the involvement of resources and the performance of activities CFA_i, for instance for the purchase, transport of materials etc. necessary to complete the activity. It has been assumed that CFA_i expenses are borne exactly at the time ST_i in which the start of activity i is planned, while the client's payments CFM_m are made upon the completion of a given stage MT_m in the planned schedule.

Stage project settlements between the client and the contractor are used (Eq. 4–5).

$$MT_m = \max_{i \in MA_m}(FT_i), \qquad \forall m : m = 1,...,N_M \tag{4}$$

$$CFM_m = MP_m - MC_m \cdot \max(MT_m - MD_m, 0), \qquad \forall m : m = 1,...,N_M \tag{5}$$

where: FT_i – finish time of activity i ($FT_i = ST_i + d_i$),
MA_m – set of activities to be performed in the m stage of the project,
MP_m – amount of client's payment to the contractor for the completion of the m stage of the project,
MD_m – agreed due date for the m stage of the project,
MC_m – agreed unit penalty for exceeding the due date for the m stage of the project MD_m.

Groups of activities MA_m to be completed in a given stage of the project are determined. Same as for each stage due dates MD_m, the amount of the client's payments for the timely completion of the works MP_m, and principles for charging agreed penalties in the case of delays in the completion of stages, with determined unit penalties MC_m, agreed, for instance during negotiations between the client and the contractor.

Payments acquired by the contractor from the client for project stages are financial funds which may be allocated for current operations, for instance the purchase of materials necessary to complete subsequent activities, the employees' salaries etc. Increased payments from the client are not envisaged in the case of the completion of milestones earlier than planned in the agreement. In this case, an earlier acquisition of cash, the discounted value of which is higher, is the "bonus" for the contractor. The need to bear earlier expenses is compensated for the client by the introduction of agreed penalties for the lack of punctuality in the completion of project stages as well as the possibility to control the course of works during the project's execution.

The proposed model of stage settlements for project works may be useful in practice and beneficial both for the client and for the contractor. Its application may lead to a reduction in the lack of punctuality in the completion of project works which is a significant problem occurring during the execution of practical projects.

3. TECHNIQUES OF GENERATING SCHEDULES

A direct representation of solution for RCPSP is the vector of starting times of activities which may be used, for instance, to determine the sum of discounted cash flows of the project. When creating the schedule, solution is coded using other representations, more convenient to local search for solutions, such as an activity list, namely the permutation of activity numbers taking into account precedence relations. The activity list is transformed into a feasible schedule (taking into account resource and precedence constraints) in direct representation with the use of the Schedule Generation Scheme (SGS) which include serial SGS and parallel SGS (Kolisch, 1996). The schedule may be determined by way of planning activities from the beginning of the activity list (forward scheduling) or from the end of the list by planning the activities as late as possible with the agreed due date for the project (backward scheduling)

The schedule determined with the use of SGS procedures may be improved in the case of the problem of the maximization of the sum of negative and positive discounted cash flows (as for the analyzed problem). The growth in NPV brings the earliest possible start of activities (stages) with assigned cash inflows and the latest possible start of activities (stages) with assigned cash outflows. Bi-directional SGS (Selle & Zimmermann, 2003), iterative shift algorithms which shift activities with negative cash flows to the right (for forward schedules) and/or activities with positive cash flows to the left (for backward schedules) etc. are used as techniques of generating solutions (Vanhoucke, Demeulemeester & Herroelen, 2001).

In a schedule suitable for the problem with the maximization of the sum of discounted cash flows analyzed in the article, the cash inflows, namely the client's payments for the completion of the agreed stages of works, should be acquired as soon as possible, while cash outflows, namely the contractor's expenses related to commenced activities, should be borne as late as possible. The growth in NPV brings the postponement these activities in time (according to the principle *As Late As Possible* – ALAP), the delayed start of which does not change the completion times of project stages.

According to the author's knowledge, there are no procedures generating schedules dedicated for the examined matter. As a result, dedicated techniques for building solutions prepared by the author are suggested, based on known procedures, namely:

- the right shift of activities with a fixed resource allocation,
- backward scheduling with the optimization of completion times for the agreed project stages,
- the justification technique taking into account the due dates for the agreed project stages.

Subsequent sub-chapters describe particular techniques of generating schedules. Let us use an example to illustrate their effect as well as to explain the problem of the financial optimization of a multi-stage project.

Let the project consist of 8 activities performed with the use of one type of resource with availability equal to 10. This project in the Activity-On-Node representation (AON) is presented in Fig. 1 (nodes 0 and 9 represent dummy activities).

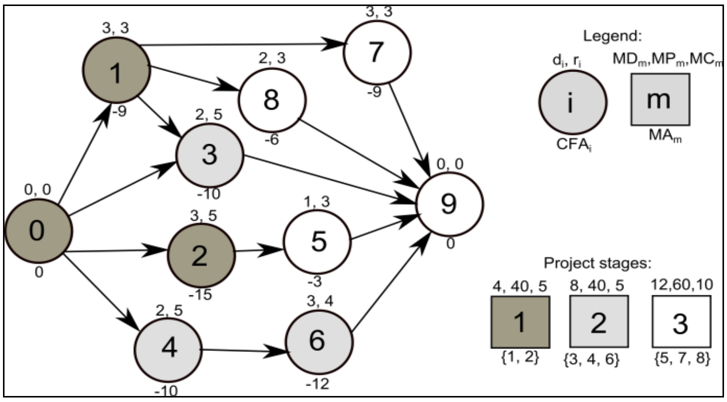

Fig. 1. Exemplary project in AON representation

The project has 3 defined stages, which consist of the following activities: MA_1 ={0, 1, 2}, MA_2 ={3, 4, 6}, MA_3 ={5, 7, 8, 9}. The due dates for stages are: MD_1 = 4, MD_2 = 8 and MD_3 = 12. The client's payments for the completed stages are determined on the basis of the amounts MP_1 = 40, MP_2 = 40, MP_3 = 60, which may be reduced by the costs of possible delays calculated on the basis of unit costs MC_1 = 5, MC_2 = 5, MC_3 = 10. The discount rate α = 0.01 is used in the calculations of the value of discounted cash flows.

The contractor of the project constructs a schedule for determined cash flows related to the project in which, from the contractor's perspective, the sum of discounted cash flows is maximized – the *F* function, taking into account precedence constraints and resource constraints. The schedules are built for solutions presented in the representation of the activity lists. Let us assume that the activity list {1, 5, 2, 3, 4, 6, 7, 8} is processed. The forward schedule generated using serial SGS for this activity list is presented in Fig. 2.

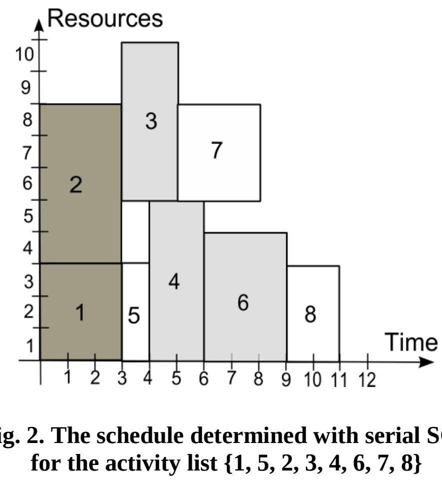

Fig. 2. The schedule determined with serial SGS for the activity list {1, 5, 2, 3, 4, 6, 7, 8}

The F objective function for the schedule from Fig. 2 is calculated as follows:

$$\sum_{i=1}^{N_A}(CFA_i \cdot e^{-\alpha \cdot ST_i}) = -\frac{9}{e^{0.01\cdot 0}} - \frac{15}{e^{0.01\cdot 0}} - \frac{10}{e^{0.01\cdot 3}} - \frac{10}{e^{0.01\cdot 4}} - \frac{3}{e^{0.01\cdot 3}} - \frac{12}{e^{0.01\cdot 6}} - \frac{9}{e^{0.01\cdot 5}} - \frac{6}{e^{0.01\cdot 9}}$$

$$= -71.57,$$

$$\sum_{m=1}^{N_M}(CFM_m \cdot e^{-\alpha \cdot MT_m}) = \frac{40}{e^{0.01\cdot 3}} + \frac{40-5}{e^{0.01\cdot 9}} + \frac{60}{e^{0.01\cdot 11}} = 124.56,$$

$$F = -71.57 + 124.56 = 52.99.$$

The first and the third stage in the schedule are completed before the agreed due date which is beneficial for the contractor in relation with the increase in the discounted value of the client's payments. However, an untimely completion of the second stage of project stage is planned ($MT_2 = 9$ with the agreed due date $MD_2 = 8$), which diminishes the client's payments for this stage. Additionally, the growth in the project's NPV is possible resulting from starting the activities as late as possible keeping the due dates for stages.

3.1. Activity right-shift procedure

The first proposed method of generating schedules for the examined problem of the financial optimization of a multi-stage project is the activity right-shift procedure. Activity shifts for the S forward schedule take place with a fixed allocation of resources, for which it is easy to take into account resource constraints, it is possible to define in an unambiguous manner what changes to the schedule will be caused by the shifted start of each activity. Subsequent iterations analyze the right-shift of subsequent activities examined in the descending order of their starting times in the analyzed schedule. A given activity is shifted as long as this operation increases the value of the F objective function (Klimek & Łebkowski, 2015).

A variety of resource allocations to activities, characterized by various properties affecting the right shifts of activities and the quality of generated solutions measured by the value of the F objective function, may be generated for a given schedule (Klimek & Łebkowski, 2015). The problem of resource allocation for RCPSP is a strong NP-Hard problem, already with one type of resources (Leus & Herroelen, 2004; Deblaere, Demeulemeester, Herroelen & Van De Vonder, 2006). The allocation of resources to activities is analyzed with proactive, robust scheduling in which the objective, among others, is to minimize the number of additional arcs (Deblaere et al., 2006; Klimek & Łebkowski, 2011, 2013). A review of resource allocation procedures is presented in study (Deblaere et al., 2006).

Resource allocations with the smallest number of additional arcs are usually preferred for the analyzed problem. Additional order constraints resulting from the adopted resource allocation may diminish the number of activities the shift of which increases the value of the project's objective function. Right-shift procedures will be analyzed for the schedule from Fig. 2.

The allocation of resources generated with the use of a simple allocation procedure (Artigues, Michelon & Reusser, 2003), in which the activities are allocated to the first free chains related to subsequent resources, is presented in Fig. 3a.

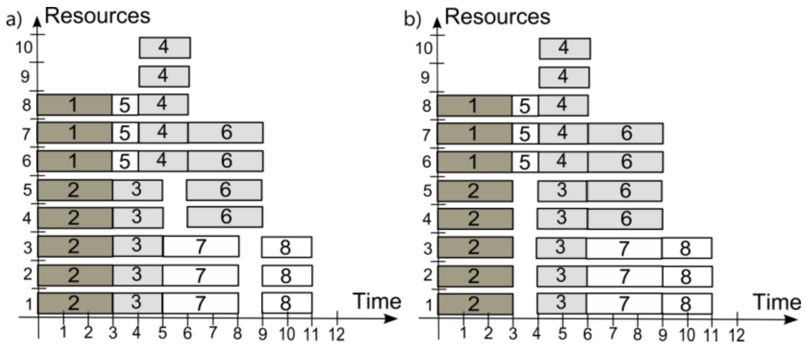

Fig. 3. Schedules: a) schedule with resource allocation, b) schedule determined with the use of the right shift procedure

Right-shifts of subsequent activities are analyzed for the allocation of resources from Fig. 3a: the shift of activities 8 and 6 does not increase the *F* value, the shift of activity 7 by one time unit increases *F* from 52.99 to 53.07, the shift of activity 4 does not increase the *F* value, the shift of activity 3 by one time unit increases *F* from 53.07 to 53.17, the shift of activities 5, 2 and 1 does not increase the *F* value, the shift algorithm finishes its operation. The schedule with right-shifts with $F = 53.17$ is presented in Fig. 3b.

A greater right-shift of activities 3 and 7 is impossible for the allocation of resources determined presented in Fig. 3a. Such operation is possible for the allocation found with the use of the RALS procedure (Resource Allocation with Local Search), in which an allocation which enables right-shifts of the highest number of activities improving the project's NPV is requested (Klimek & Łebkowski, 2015). The schedule with resource allocation determined with the use of the RALS procedure is presented in Fig. 4a.

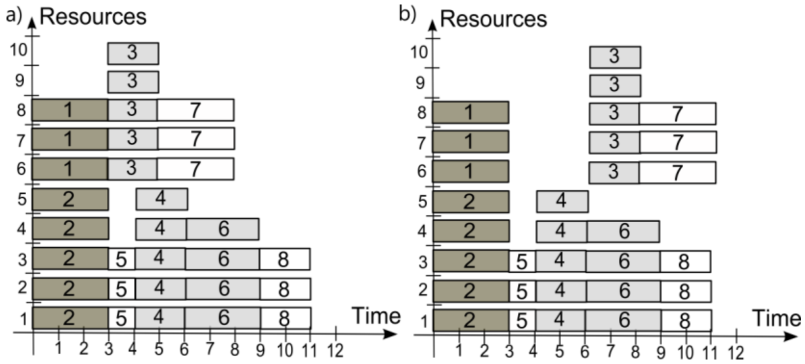

Fig. 4. Schedules: a) schedule with resource allocation determined with the use of the RALS procedure, b) schedule corrected with the use of the right-shift procedure

For resource allocation presented in Fig. 4a, the right-shift procedure will shift activity 7 by three time units (increase in *F* from 52.99 to 53.24) and will shift activity 3 by three time units (increase in *F* from 53.24 to 53.53). The schedule with right-shifts with *F* = 53.53 is presented in Fig. 4b.

The right-shift procedure does not introduce changes to the completion times of the project's stages. It may be effective when correcting schedules with the most beneficial completion times for the milestones. The optimum solution with *F* = 58.73 for the analyzed example (Fig. 5b) is found, for instance for a schedule determined with serial SGS for the activity list {1, 2, 3, 4, 5, 6, 7, 8} with resource allocation generated with the RALS procedure (Fig. 5a).

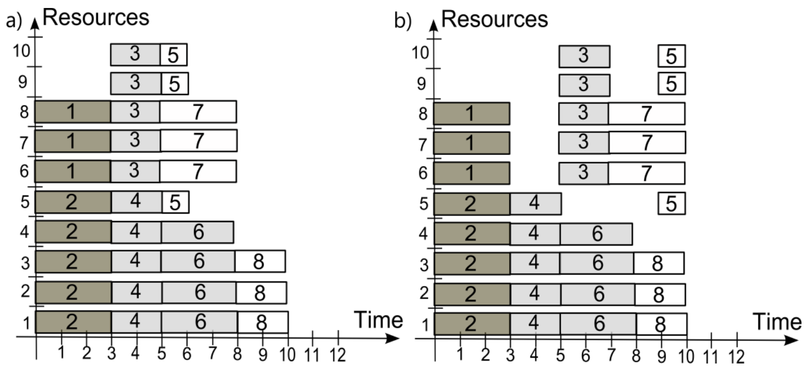

Fig. 5. Schedules: a) schedule generated for activity list {1, 2, 3, 4, 5, 6, 7, 8} with resource allocation determined with the use of the RALS procedure, b) schedule corrected with the use of the right-shift procedure

3.2. Backward scheduling

Another proposed procedure of generating solutions dedicated to the examined problem is backward scheduling with the optimization of completion times for project stages (Klimek & Łebkowski, 2017). This procedure includes serial SGS with backward planning of activities performed for the selected completion times for the project's stages MT_m iteratively shifted to the left. Shift operations are performed for subsequent stages, from the first to the last.

A baseline schedule is created for the processed activity list, assuming the planned completion dates for the stages MT_m equal to the agreed due dates MD_m. The completion dates for stages MT_m in the baseline schedule may be greater than the agreed due dates MD_m, if it is not possible to generate a feasible schedule with $MT_m = MD_m$ for each $m = 1, .., N_M$ for the processed activity list. The shift of a given stage takes place as long as this operation increases the value of the F objective function. Subsequent schedules created by the optimization procedure for completion dates for the agreed project stages for the analyzed project and the activity list {1, 5, 2, 3, 4, 6, 7, 8} are presented in Fig. 6a–c.

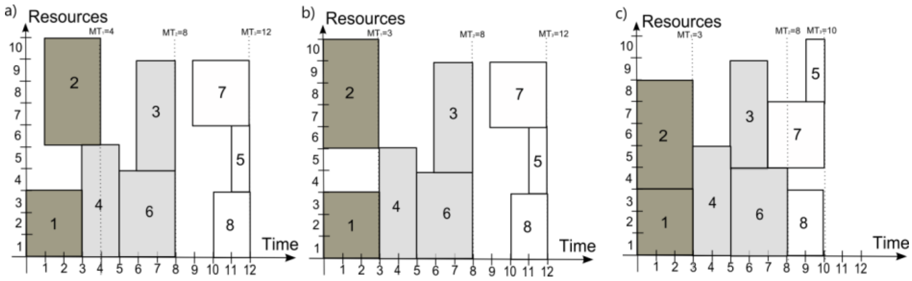

Fig. 6. Schedules: a) backward schedule generated for activity list {1, 5, 2, 3, 4, 6, 7, 8} using initial completion dates for stages: $MT_1 = 4$, $MT_2 = 8$, $MT_3 = 12$, b) schedule determined as a result of optimization of the completion date for the first stage: $MT_1 = 3$, $MT_2 = 8$, $MT_3 = 12$, c) Schedule determined as a result of optimization of completion dates for three project stages: $MT_1 = 3$, $MT_2 = 8$, $MT_3 = 10$

The backward schedule determined for the activity list {1, 5, 2, 3, 4, 6, 7, 8}, with completion dates for stages such as the agreed due dates, is presented in Fig. 6a. The value of the objective function for this schedule $F = 57.84$. The left-shift procedure of the project's stages starts with shifting the due date for the first stage. Assuming $MT_1 = 3$, a schedule with a greater value of the objective function $F = 58.08$ is generated, for $MT_1 = 2$ it is impossible to create a feasible schedule, the procedure proceeds to the optimization of the due date for the second stage MT_2. It is impossible to generate a feasible schedule for $MT_2 = 7$, the procedure proceeds to a left-shift of the project's third stage. In the third stage, assuming $MT_3 = 11$, the value of the objective function increases $F = 58.45$, assuming $MT_3 = 10$, the value of the objective function increases

$F = 58.73$. It is impossible to generate a feasible schedule within the due date $MT_3 = 9$, the procedure ends the first iteration of shifting the project's stages and starts its operation again from the first stage with assumed $MT_1 = 3$, $MT_2 = 8$, $MT_3 = 10$. No changes are made in the second iteration of the procedure, the algorithm finishes its operation.

The schedule determined after a unit shift of the first stage with the value of the objective function $F = 58.08$ is presented in Fig. 6b. The final schedule generated with the optimization of completion dates for all stages of the project with value of the objective function $F = 58.73$ is presented in Fig. 6c.

3.3. Justification techniques

Generating solutions relevant to the analyzed problem is also possible with the use of the justification techniques (Valls, Ballestin & Quintanilla, 2005): right justification (abbreviated to RJ) and left justification (abbreviated to LJ), which are used for RCPSP, among others, for the problem of the minimization of the project's duration or for the problem with defined due dates for activities. Justification techniques transform the schedule and often improve its quality. The justification of a given activity to the right (to the left) consists in determining the latest possible (earliest possible) starting time for such an activity, taking into account order constraints and resource constraints. LJ and LJ techniques are often combined, for instance double justification is used, RJ+LJ or LJ+RJ. The conducted analysis of justification techniques for exemplary schedules has shown that solutions of good quality are generated with the use of triple justification RJ+LJ+RJ. An effective strategy for the order of shifting the activities has been adopted in the case of justification, namely justification by extremes: the right (left) justification includes subsequent activities with the maximum finish time (minimum starting time) in the justified schedule. The RJ technique is modified: activities are shifted so as not to delay the current completion dates for the project's stages MT_m.

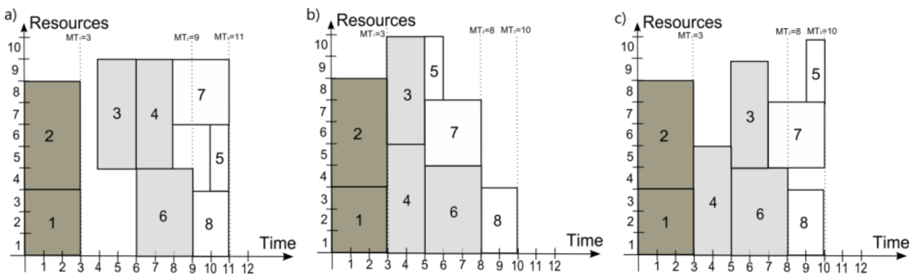

Fig. 7. Schedules: a) schedule from Fig. 2 after using RJ, b) schedule from Fig. 2 after using RJ+LJ, c) Schedule from Fig. 2 after using RJ+LJ+RJ

Let us assume that the schedule from Fig. 2 generated forward is amended with the use of serial SGS for the activity list {1, 5, 2, 3, 4, 6, 7, 8}. Subsequent transformations of the schedule with the use of triple justification RJ+LJ+RJ are presented in Fig. 7a–c.

Technique RJ is used at the beginning and it includes activities with the maximum finish time – subsequent activities 8, 6, 7, 4, 3, 5, 2, 1 (the activity with the higher number is analyzed earlier with an equal finish time) In the modified RJ, the latest possible starting time is determined for the justified activity, taking into account the precedence relations and resource relations as well as the current completion time of a project stage to which this activity belongs (MT_1 = 3, MT_2 = 9, MT_3 = 11). The schedule presented in Fig. 7a with better quality F = 53.72, due to a delayed start of activities 3, 4, 5 and 7, is created as a result of RJ.

Technique LJ is performed for the schedule after RJ. LJ includes activities with the minimum starting time in the schedule from Fig. 7a – subsequent activities 1, 2, 3, 4, 7, 5, 8 (the activity with the lower number is analyzed earlier with an equal starting time). The earliest possible starting time is determined for the justified activity during LJ, taking into account the precedence relations and resource relations. The schedule presented in Fig. 7b with a higher value of the F objective function is created as a result of LJ, due to an earlier completion of the second and third stage (MT_2 = 8, MT_3 = 10) despite an earlier start of activities 3, 4, 5, 6, 7 and 8.

The schedule after RJ+LJ may be improved by the repeated application of the modified RJ. RJ includes activities with the maximum finish time in the schedule from Fig. 7b – subsequent activities 8, 7, 6, 5, 4, 3, 2, 1 for which the starting time is determined taking into account current completion dates for project stages MT_1 = 2, MT_2 = 8, MT_3 = 10). The schedule presented in Fig. 7c with the highest value F = 58.73 is created as a result of RJ, due to a delayed start of activities 3, 5 and 7.

4. EXPERIMENTS

The simple experiments were performed with the use of an application implemented in the C# language run on a PC computer with an Intel Core processor i7-4770 CPU 3.4 GHz, 8 GB RAM. 480 test instances from the set J30 (projects with 30 activities) as well as 480 instances from the set J90 (projects with 90 activities) were used from the PSPLIB (Kolisch & Sprecher, 1997). Three agreed stages are defined for each project from PSPLIB created on the basis of the S schedule generated with the use of serial SGS for the activity list {1, 2, …, 30} for the set J30 or {1, 2, …, 90} for the set J90. The makespan of the project in the S baseline schedule is calculated and marked with T.

The agreed due dates for the stages are determined as $MD_1 = T/3$, $MD_2 = 2T/3$ as well as $MD_3 = T$. Then, sets of activities to be executed in particular stages are created and determined on the basis of the S schedule (Klimek & Łebkowski, 2017):

- the set MA_1 contains all activities the completion time of which is lower or equal to MD_1,
- the set MA_2 contains all activities the completion time of which is lower or equal to MD_2 and greater than MD_1,
- the set MA_3 includes the remaining activities which do not belong to MA_1 or MA_2.

The data for determining the cash flows are determined for each test instance as follows:

- the amounts of the client's agreed payments are $MP_1 = 60$, $MP_2 = 60$, $MP_3 = 120$,
- the agreed unit penalties $MC_1 = 1.5$, $MC_2 = 1.5$, $MC_3 = 3$,
- the costs of the execution of activities CFA_i are calculated in proportion to the total demand for resources and the duration of a given activity, while the sum CFA_i for all activities amounts to 100.

The discount rate adopted in experiments $\alpha = 0.01$.

The aim of the experiments is to evaluate the effectiveness of the developed techniques of generating solutions. Random sampling is used – 1000 activity lists are generated and schedules are created for them with the use of serial SGS or parallel SGS as well as the activity right-shift procedure with a fixed allocation of resources, backward scheduling with the optimization of completion dates for agreed project stages or the modified justification. A schedule with the highest value of the F objective function from among the 1000 analyzed solutions is determined for each technique. The experiments are conducted two times due to the stochastic nature of the calculations. The results of experiments are presented in Table 1.

Tab. 1. Results of experiments

		FS		FS + RS		BS	FS+RJ		FS+RJ+LJ+RJ	
		ser	par	ser	par	ser	ser	par	ser	par
J30	Av_F, 1st run	75.12	74.08	75.56	74.65	77.22	76.55	75.69	77.33	76.87
	Av_F, 2nd run	75.20	74.06	75.63	74.60	77.23	76.62	75.66	77.36	76.85
	Nr_best , 1st run	0	0	19	18	300	194	137	308	253
	Nr_best, 2nd run	0	0	12	17	291	185	134	311	245
	CPU time [sec.]	0.01	0.04	0.14	0.16	0.19	0.04	0.07	0.09	0.13
J90	Av_F, 1st run	42.25	44.32	42.75	44.88	51.31	45.67	47.63	51.66	51.35
	Av_F, 2nd run	42.18	44.28	42.68	44.84	51.36	45.63	47.62	51.56	51.41
	Nr_best, 1st run	0	0	0	0	145	19	8	192	111
	Nr_best, 2nd run	0	0	0	0	151	22	9	179	123
	CPU time [sec.]	0.05	0.37	2.08	2.26	0.96	0.30	0.63	0.65	0.97

where: Av_F – the average value of the objective function after 1000 runs, Nr_best – the number of solutions identical to the best solution found by all algorithms analysed (for 480 test instances), FS – forward scheduling, RS – activity right-shift procedure, BS – backward scheduling, ser – serial SGS, par – parallel SGS.

The highest average value of the *F* objective function, both for projects from the set J30 and from the set J90, was achieved for a procedure using the triple justification RJ+LJ+RJ with serial SGS. This procedure found 308 or 311 (192 or 179) best solutions in two experimental studies from among solutions generated by all analyzed algorithms for the 480 examined test instances from the set J30 (J90).

Effective techniques of generating of solutions include backward scheduling with the optimization of completion times for project stages or the triple justification of solutions. The least effective and the most costly procedure in terms of computation is the activity right-shift procedure.

Better schedules for the set J30 are generated with the use of serial SGS. Solutions of a higher quality for the set J90 are generated with the use of parallel SGS, while the application of the triple justification brings better results for schedules created with the use of serial SGS.

5. SUMMARY

The article analyses the problem of the maximization of discounted cash flows of a multi-stage project from the perspective of the contractor. The problem and the techniques of generating relevant schedules are illustrated. An experimental analysis of the developed procedures, which determined effective techniques of generating solutions, was conducted, namely backward scheduling with the optimization of completion dates for project stages, triple justification.

The proposed model of stage settlements for project works may be useful in practice and beneficial both for the client and for the contractor. Its application may lead to a reduction in the lack of punctuality in the completion of project works which is a significant problem occurring during the execution of practical projects. Further works focus on comparing the effectiveness of various justification strategies as well as on the use of effective techniques of generating solutions in more advanced metaheuristics, namely simulated annealing.

REFERENCES

Artigues, C., Michelon, P., & Reusser, S. (2003). Insertion techniques for static and dynamic resource-constrained project. *European Journal of Operational Research*, *149*(2), 249–267. doi:10.1016/S0377-2217(02)00758-0

Bahrami, F., & Moslehi, G. (2013). Study of payment scheduling problem to achieve client-contractor agreement International. *Journal of Advanced Manufacturing Technology*, *64*(1), 497–511. doi:10.1007/s00170-012-4023-5

Dayanand, N., & Padman, R. (1997). On modelling payments in projects. *Journal of the Operational Research Society*, *48*(9), 906-918. doi:10.1057/palgrave.jors.2600440

Dayanand, N., & Padman, R. (2001). Project contracts and payment schedules: The client's problem. *Management Science*, *47*(12), 1654-1667. doi: 10.1287/mnsc.47.12.1654.10242

Deblaere, F., Demeulemeester, E., Herroelen, W., & Van De Vonder, S. (2006). Proactive resource allocation heuristics for robust project scheduling. *Research report KBI_0608*, K.U. Leuven. doi:10.2139/ssrn.870228

Hartmann, S., & Briskorn, D. (2012). A Survey of Variants and Extensions of the Resource-Constrained Project Scheduling Problem. *European Journal of Operational Research*, *207*(1), 1–14. doi:10.1016/j.ejor.2009.11.005

He, Z., Liu, R., & Jia, T. (2012). Metaheuristics for multi-mode capital-constrained project payment scheduling. *European Journal of Operational Research*, *223*(3), 605–613. doi:10.1016/j.ejor.2012.07.014

He, Z., Wang, N., Jia, T., & Xu, Y. (2009). Simulated annealing and tabu search for multimode project payment scheduling. *European Journal of Operational Research*, *198*(3), 688–696. doi:10.1016/j.ejor.2008.10.005

He, Z., & Xu, Y. (2008). Multi-mode project payment scheduling problems with bonus penalty structure. *European Journal of Operational Research*, *189*(3), 1191–1207. doi:10.1016/ j.ejor.2006.07.053

Herroelen, W., Reyck, B. D., & Demeulemeester, E. (1997). Project network models with discounted cash flows: A guided tour through recent developments. *European Journal of Operational Research*, *100*(1), 97–121. doi:10.1016/S0377-2217(96)00112-9

Klimek, M. (2017). Priority algorithms for the problem of financial optimisation of a multi stage project. *Applied Computer Science*, *13*(4), 20–34. doi:10.23743/acs-2017-26

Klimek, M., & Łebkowski, P. (2011). Resource allocation for robust project scheduling. *Bulletin of the Polish Academy of Sciences: Technical Sciences*, *59*(1), 51–55. doi:10.2478/v10175-011-0008-z

Klimek, M., & Łebkowski, P. (2013). Robustness of schedules for project scheduling problem with cash flow optimization. *Bulletin of the Polish Academy of Sciences: Technical Sciences*, *61*(4), 1005–1015. doi:10.2478/bpasts-2013-0108

Klimek, M., & Łebkowski, P. (2015). Heuristics for project scheduling with discounted cash flows optimization. *Bulletin of the Polish Academy of Sciences: Technical Sciences*, *63*(3), 613–622. doi:10.1515/bpasts-2015-0072

Klimek, M., & Łebkowski, P. (2017). Financial optimisation of the scheduling for the multi-stage project. *Bulletin of the Polish Academy of Sciences: Technical Sciences*, *65*(6), 899–908. doi:10.1515/bpasts-2017-0097

Kolisch, R. (1996). Serial and parallel resource-constrained project scheduling methods revisited: Theory and computation. *European Journal of Operational Research*, *90*(2), 320–333. doi:10.1016/0377-2217(95)00357-6

Kolisch, R., & Sprecher, A. (1997). PSPLIB - a project scheduling library. *European Journal of Operational Research*, *96*(1), 205–216. doi:0.1016/S0377-2217(96)00170-1

Leus, R., & Herroelen, W. (2004). Stability and resource allocation in project planning. *IIE Transaction*, *36*(7), 667–682. doi:10.1080/07408170490447348

Leyman, P., & Vanhoucke, M. (2016). Payment models and net present value optimization for resource-constrained project scheduling. *Computers & Industrial Engineering*, *91*, 139–153. doi:10.1016/j.cie.2015.11.008

Leyman, P., & Vanhoucke, M. (2017). Capital- and resource-constrained project scheduling with net present value optimization. *European Journal of Operational Research*, *256*(3), 757–776. doi:10.1016/j.ejor.2016.07.019

Mika, M., Waligóra, G., & Węglarz, J. (2005). Simulated annealing and tabu search for multi-mode resource-constrained project scheduling with positive discounted cash flows and different payment models. *European Journal of Operational Research*, *164*(3), 639–668. doi:10.1016/j.ejor.2003.10.053

Russell, A. H. (1970). Cash flows in networks. *Management Science*, *16*(5), 357–373. doi:10.1287/mnsc.16.5.357

Selle, T., & Zimmermann, J. (2003). A bidirectional heuristic for maximizing the net present value of large-scale projects subject to limited resources. *Naval Research Logistics*, *50*(2), 130-148. doi:10.1002/nav.10052

Smith-Daniels, D. E., Padman, R., & Smith-Daniels, V. L. (1996). Heuristic scheduling of capital constrained projects. *Journal of Operations Management*, *14*(3), 241–254. doi:10.1016/0272-6963(96)00004-6

Ulusoy, G., & Cebelli, S. (2000). An equitable approach to the payment scheduling problem in project management. *European Journal of Operational Research*, *127*(2), 262–278. doi:10.1016/S0377-2217(99)00499-3

Ulusoy, G., Sivrikaya-Serifoglu F., & Sahin, S. (2001). Four Payment Models for the Multi-Mode Resource Constrained Project Scheduling Problem with Discounted Cash Flows. *Annals of Operations Research*, *102*, 237-261. doi:10.1023/A:1010914417817

Valls, V., Ballestin, F., & Quintanilla, S. (2005). Justification and RCPSP: a technique that pays. *European Journal of Operational Research*, *165*(2), 375–386. doi:10.1016/j.ejor.2004.04.008

Vanhoucke, M., Demeulemeester, E., & Herroelen, W. (2003). Progress payments in project scheduling problems. *European Journal of Operational Research*, *148*(3), 604–620. doi:10.1016/S0377-2217(02)00452-6

Vanhoucke, M., Demeulemeester, E., & Herroelen, W. (2001). Maximizing the net present value of a project with linear time-dependent cash flows. *International Journal of Production Research*, *39*(14), 3159-3181. doi:10.1080/00207540110056919