LIMITING VALUE OF COCKROFT-LATHAM INTEGRAL FOR COMMERCIAL PLASTICINE

Abstract
The paper presents the results of experimental and numerical research in the scope of commercial plasticine cracking. The purpose of the study was to determine the limit value of the Cockroft-Latham integral. The value of the integral was determined on the basis of the stretching test and computer simulations. Experimental studies utilized axially symmetrical samples made of commercial black and white wax-based plasticine. Samples were cooled to 0, 5, 10, 15 and 20°C. After the completion of experimental studies, finite element numerical simulation was performed under the conditions of 3-dimensional state of deformation in DEFORM 3D simulation software. Based on the results of experimental and numerical studies, the Cockroft-Latham limit value was calculated.

1. INTRODUCTION

Over the last few years, entrepreneurs in the forging industry have empowered their tendency to reduce costs and production preparation time, while emphasizing the high quality of the final product. Companies following this trend are required to optimize design processes which are often difficult and time-consuming. Limitations occurring during optimization process are mainly due to complex and costly experimental tests, often impossible to perform in industrial conditions.
Therefore, new methods are being sought to streamline the design of metal forming processes that will reduce the costs of industrial research using actual materials. Such methods allow conducting research on the metal forming process in laboratory conditions.

The methods used by companies to optimize the process of constructing tools and technologies of plastic processing include computer modelling as well as physical modelling.

Computer modelling (numerical) is an approximation of a discrete model to an actual model. The discrete model is composed of finite number of elements and nodes. Each element and node is described by physical differential equations. The main advantage of computer modelling is the ability to produce results for complex shapes for which analytical calculations can not be made and simulations are easy to perform (Balasundar, Raghu, & Sudhakara, 2009; Dębski, Lonkwic & Rozylo, 2015; Gontarz & Winiarski, 2015; Rozylo & Wojcik, 2017; Lis, Pater & Wojcik, 2016b). The main limitation of the numerical method is the uncertainty of the obtained results.

Physical modelling makes it simpler to analyse and optimize the design of plastic technological processing. The physical simulation method allows the substitution of actual material with model material (lead, wax, resin, plasticiine) (Dziubińska & Gontarz, 2015). This technique is based on the analysis of physically similar phenomena that occur in the actual object as well as in the model. Replacing the actual material with the model material allows for the use of tools made of much cheaper and more easily machined materials (Kowalczyk, 1995; Świątkowski, 1994a,b).

The physical modelling method is based on the similarity of actual and model material. The main similarity criteria include the similarity of flow curves for model and actual material, the similarity of friction conditions, the similarity of tool shapes, the similarity of process kinematics (Gontarz, Łukasik, & Pater, 2003).

2. THE ANALYSIS OF THE STATE OF THE PROBLEM

Physical modelling of solid shaping in hot forming processes allows for analysis of the process. In many metal forming processes the final product suffers from certain limitations, one of them is the cracking component occurring during the plastic processing. These cracks are often invisible to the human eye, but have a very negative effect on the finished element (Pater, 2010).

The fracture phenomenon is dependent on structural phenomena occurring in the material during shaping. Two different ways of cracking of the material are distinguished – brittle and ductile cracking. The most frequently encountered form of cracking in the hot metal forming is ductile cracking (Arikawa & Kakimotoa, 2014; Fuertesa, Leóna, Luisa, Luria, Puertas, & Salcedoa, 2015; Gontarz & Piesiak, 2010; Pires, Song, & Wu, 2016). Plastic cracking occurs
by the formation of cracks and narrowing of the matrix material. In the case of model materials, cracks are represented by voids and gas bubbles inside the material. The formation of ductile scrap is accompanied by an increase in plastic deformation of the material.

The research object is commercial wax based plasticine, produced by "Primo" in white and black colour. Plasticine is a mixture of clays, silts, oils, waxes, and colouring pigments. It is characterized by very high plasticity at room temperature. This material is classified in the group of non-metallic model materials. The main advantage of using this model mass is the possibility of multiple use (material that has not been contaminated with oils, dust, etc.). Plasticine has been used repeatedly for experimental analysis of various plastic forming technological processes (Assempour & Razi, 2002; Komori & Mizuno, 2009).

In the scientific literature, the results of empirical studies on mechanical properties of materials used for physical modelling have been repeatedly described (Moon & Van Tyne, 2000; Rasty & Sofuoglu, 2000) Laboratory analyses omitted issues related to cracking of model materials in the forming processing.

In the paper it was considered advisable to conduct experimental and theoretical studies in the scope of determination of the Cockroft-Latham limit value for commercial plasticine. The Cockroft-Latham criterion accurately determines the moment of separation of the material in which narrowing and plastic separation occurs. This criterion is described by the equation:

\[
\frac{\varphi}{\varphi^*} \int_{\sigma_i}^{\sigma_1} d\varphi = C
\]  

(1)

where:  
\(\sigma_i\) – reduced stress [MPa],  
\(\sigma_1\) – highest main stress [MPa],  
\(\varphi^*\) – limiting cracking deformation,  
\(C\) – material constant.

The plasticine mass used was previously plastometrically examined (Lis, Pater & Wojcik, 2016a). Plastometric studies have shown that the material is highly sensitive to temperature variations and deformation rates. The drop in temperature had a significant effect on the increase of plasticizing stresses. The highest values of plasticizing stresses were observed at 0°C (with the highest stress strain rate \(\sigma_{pl} = 0.67\text{MPa}\)). Fig. 1 shows the flow curves for plasticine tested at a temperature range of 0 to 20 °C.

Studied materials were described by the constitutive equations, the white plasticine described by the equation (2), while the equation (3) describes the black plasticine:
\[
\sigma_p = 0.48057 \cdot e^{-0.03127 \cdot \exp(0.08705 \cdot \varepsilon)} \cdot \dot{\varepsilon}^{0.24508 - 0.00267 \cdot \varepsilon} \cdot \exp(-0.03283 \cdot \varepsilon)
\]  
\[
\sigma_p = 0.6817 \cdot e^{-0.0711 \cdot \exp(0.07203 \cdot \varepsilon)} \cdot \dot{\varepsilon}^{0.2701 + 0.0033 \cdot \varepsilon} \cdot \exp(-0.07358 \cdot \varepsilon)
\]

where: \( \sigma_p \) – plasticizing stresses [MPa], 
\( \varepsilon \) – reduced strain, 
\( \dot{\varepsilon} \) – strain rate [s\(^{-1}\)], 
\( T \) – test sample temperature [°C].

Fig. 1. Plasticine flow curves at temperatures of 0, 5, 10, 15, 20 °C
2.1. Experimental research

In order to determine the limit value of the Cockroft-Latham integral, the axially symmetrical samples were subjected to the tensile test. The geometrical dimensions of the test samples are shown in Fig. 2. Experimental studies were carried out using the Instron 3369 universal testing machine. The testing machine is characterized by a maximum load of 50 kN and a speed range of 1 to 500 mm/min. The measuring device allows for the accuracy of the measurement to be within ± 0.5% of the obtained value. The stand is equipped with software that allows registering the displacement of the slider and the force as a function of time.

![Fig. 2. Shape and dimensions of the sample used for tensile tests](image)

In order to determine the influence of temperature on the variation of the limit value of the Cockroft-Latham integral, the studies were conducted for five different temperatures 0, 5, 10, 15, 20°C. The test was repeated three times for each test temperature. A total of 30 axially symmetric samples were used. The literature shows a number of methods for the preparation of samples made of a model material (Altan & Vazquez, 2000; Assempour & Razi, 2003). Samples used for testing were performed according to the following procedure.

The first step in the preparation of the samples was to heat the plasticine billets to a temperature of about 30–35°C, which facilitated the repeated manual processing and shaping of the test material. Use of this stage allows for removal of the air bubbles caused by the production process. The presence of air bubbles negatively affects the subsequent processing of the material and the quality of the final sample.

In the second stage, cylindrical blocks with a diameter of 32 mm and a height of about 60 mm were formed. Then the shaped body was subjected to an extrusion process which resulted in a bar of 10 mm in diameter (Fig. 3).

In the next step, the obtained bar was divided into smaller cylindrical samples of 120 mm length.

Such semi-finished products were subjected to a cross wedge rolling process.

The obtained axially symmetric samples were then trimmed to 116.5 mm. The samples were measured and their geometrical dimensions are shown in Table 1.
Fig. 3. The order of sample preparation (starting from the left: bar extrusion, narrowing of the bar, cutting of the sample to the length)

Tab. 1. Parameters of samples used for tensile test (marking according to Fig. 2)

<table>
<thead>
<tr>
<th>White Plasticine</th>
<th>No.</th>
<th>T [°C]</th>
<th>D [mm]</th>
<th>d [mm]</th>
<th>a [mm]</th>
<th>L [mm]</th>
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<th>ΔL</th>
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<table>
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<th>D [mm]</th>
<th>d [mm]</th>
<th>a [mm]</th>
<th>L [mm]</th>
<th>Lₜ [mm]</th>
<th>ΔL</th>
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</table>
In order to properly secure the samples in testing machine, special inserts were designed and manufactured for correct grip (Fig. 4). The inserts used for the study were made by 3D printing. The 3D printing technique is carried out by placing thin layers (one on top of another) of molten plastic (in this case ABS). The tools were made on the uPrint SE printer.

![Sample mounting inserts. Samples were stretched at 300 mm/min (Fig. 5)](image)

During the test, the force, the path and the time with an acquisition rate of 100 Hz were recorded. Based on the recorded force values, the force function graphs in the stretching process were determined. The obtained experimental research results were used in numerical research.

![Slip of the charge due to tensile test](image)

### 2.2. Numerical studies

To determine the limit value of the Cockroft-Latham integral numerical modelling method was used. To perform the numerical simulation, DEFORM 3D software was used, which uses finite element method. The DEFORM simulation implemented 13 different criteria, including the Cockroft-Latham criterion.
Computer simulations were performed for the stretching of samples made from model material. The computer models used for the simulation were designed according to the dimensions of the actual samples. In order to facilitate the simulation, the computer models of the samples have been redesigned to fit the virtual handles (Fig. 6). The shape of the ends of the samples did not affect the results, due to the tensile strain occurring in the region of the constriction.

Fig. 6. Sample and mounting system in DEFORM-3D

Samples were modelled using quadratic mesh elements. The average number of elements describing the sample was 150,000. In place of the expected major deformation, these elements have been concentrated.

Computer modelling was based on the results of earlier studies (Wojcik, Lis, Pater, 2016,a), which were incorporated into the DEFORM-3D software in tabular form.

Numerical simulations were performed for the white and the black samples at five different temperatures. For simulation calculations, the tensile velocity and temperature were determined according to the experimental parameters.

Numerical studies allowed to determine the forces and map deformation, stress, strain rate and Cockroft-Latham integral (Fig. 7). In areas where narrowing occurs, the highest values of these parameters has been observed.

Fig. 7. Distribution of velocity, deformation, strain and damage criterion
3. TESTING RESULTS

Based on experimental studies and numerical simulations, Cockroft-Latham limit values were calculated, which corresponded to the moment of an actual sample slip. The results of the study are presented in Tab. 2.

<table>
<thead>
<tr>
<th>Cockroft-Latham Integral</th>
<th>Temperature</th>
<th>0 °C</th>
<th>5 °C</th>
<th>10 °C</th>
<th>15 °C</th>
<th>20 °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>0.646</td>
<td>0.786</td>
<td>1.27</td>
<td>1.377</td>
<td>1.453</td>
<td></td>
</tr>
<tr>
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<td>1.134</td>
<td>1.25</td>
<td>2.037</td>
<td>2.0</td>
<td></td>
</tr>
</tbody>
</table>

Based on the conducted studies, the limit value of the Cockroft-Latham integral (C) increases along with the increase of the temperature. The C values for the temperatures 0÷15 °C vary between 0.646÷1.453 for the white material and 0.691÷2.037 for the black material. Fig.8 shows the average values of the limiting integral dependent on temperature. The following figure shows the trend lines which are described by the equations.

The black model material:

$$C = 0.718 + 0.07T$$  \hspace{1cm} (4)

The white model material:

$$C = 0.665 + 0.44T$$  \hspace{1cm} (5)

![Fig. 8. Cockroft-Latham integral limit value for model materials, depending on temperature](image-url)
4. CONCLUSION

The study presented the methodology and the results allowing for determination of limiting value of Cockroft-Latham integral for commercial plasticine.

The used test method was based on experimental and numerical studies of the stretching of the narrowed samples. The use of samples of such shape allowed to force the slip in a predictable place. Narrowing the sample allows to eliminate the problems associated with the location of deformation in numerical studies.

The limiting value of the C-L integral for both materials increased with the decrease of temperature in which they were deformed. The values for the black model material were greater than the values of the white material.

The course of change of the C-L integral limiting value represents the change in plasticity of the material along with the increase in temperature.

REFERENCES


