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APPLICATION OF FINITE DIFFERENCE METHOD FOR MEASUREMENT SIMULATION IN ULTRASOUND TRANSMISSION TOMOGRAPHY

Abstract

In this work, we present a computer simulation model that generates the propagation of sound waves to solve a forward problem in ultrasound transmission tomography. The simulator can be used to create data sets used in the supervised learning process. A solution to the "free-space" boundary problem was proposed, and the memory consumption was significantly optimized from $O(n^2)$ to $O(n)$. The given method of simulating wave scattering enables the control of the noise extinction time within the tomographic probe and the permeability of the sound wave. The presented version of the script simulates the classic variant of a circular probe with evenly distributed sensors around the circumference.

1. INTRODUCTION

1.1. Ultrasound Transmission Tomography

Measurement methods using the information contained in the ultrasonic signal after its passage through the medium under test are called ultrasonic transmission methods (Polakowski, Rymarczyk & Sikora, 2020). The main advantage of ultrasonic testing is the non-invasive measurement in the tested environment, not causing any changes in physical and chemical parameters that could interfere with the measurement results. In addition, because ultrasound waves belong to the category of short waves, they possess propagation and radiation properties such that they can be treated as rays. The wavelengths of these waves depend on the medium they are radiated into and range from a few micrometres in liquids to tens of centimetres in metals. Therefore, they can be used to measure the attenuation coefficient and transit time of the ultrasonic signal in the medium subjected to their influence.

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Moreover, with the help of ultrasound, it is possible to make multiple measurements without fear of damage or irradiation of the tested objects. Measurements of such parameters as signal transit time, damping factor and its derivative after frequency allow, after appropriate reconstruction transformations, to image the internal structure of the tested medium and such flow parameters as its instantaneous velocity, average velocity or velocity profile. Differences in local values of specific acoustic parameters are the basis for such imaging. The image obtained using appropriate reconstruction methods presents the distribution of local values of selected acoustic parameters, obtained from the measurement of data with the scanning technique from as many directions as possible after the passage of ultrasonic impulses through the surveyed medium. In addition, this technique allows obtaining quantitative images of the internal structure, in which numerical values of each pixel describe such physical properties of the studied objects as flow velocity, temperature distribution, density, and viscosity (Antunes dos Santos Júnior, 2012). A characteristic feature of ultrasonic techniques is that the measurement of only two to three selected acoustic parameters can be the basis for a whole range of different measurement technologies because ultrasonic waves interact with the tested environment in many different ways (Polakowski & Sikora, 2016).

1.2. Finite difference approaches

This method was proposed by A. Thom in the twenties of the twentieth century, under the name of the "square method", to solve the nonlinear hydrodynamic equation. Since then, the method has found applications in solving various problems. Finite difference techniques are based on approximations that allow the differential equation to be replaced by finite difference equations. These approximations have an algebraic form. They bind the value of the dependent variable at the solution region point with the values at several adjacent points. The Finite Difference Method (FDM) is one of the most frequently used methods of approximating partial differential equations using a system of algebraic equations, which is usually solved using a computer (Degroot-Hedlin, 2008).

The areas in which the output equations are determined coincide with the solution grid, and the derivatives of the solution sought are approximated by appropriate difference quotients, using only values in the grid nodes. The so constructed differential scheme is used to determine the value of the approximate solution in the mesh nodes, and it leads directly to a system of equations with a special structure related to the local character of the approximation of the differential operator.

The classical method of finite differences is an approximate method of discrete solving of boundary problems described by ordinary or partial differential equations. The idea of the method is to replace the differential operators with appropriate differential operators, defined on a discrete and regular set of points; this set was called a mesh, and its elements were called nodes. As a result, the initial-boundary problem is reduced to a system of equations in which the values of the function and, in some cases, their derivatives are the unknowns (Bilbao, 2013).

The generalization of the classical method of finite differences is the method with an arbitrarily irregular mesh of nodes, also used to solve problems formulated in the variational form. The research conducted as part of the work was limited to applying the classical finite difference method to solve partial differential equations; solving ordinary or partial differential equations is elementary. Examples of applications can be found in textbooks on numerical methods. During the research, the basic issue was the correct setting of the tasks, where an important element is the uniqueness of the solution and its continuous dependence on the right sides of the equations and boundary conditions that guarantee the stability of the differential problem. In addition, the research showed whether the approximate solutions converge to the exact solution and the speed of this convergence. In this way, information about the numerical correctness of the respective algorithms was also taken into account (Li Li, Shao & Li, 2019).

In the early 1990s, when wave-based approaches started to become computationally viable, finite difference methods started to be applied to the problem of simulating acoustics in low frequencies, using mathematical formulations stemming from analogous equations in electromagnetics (Botteldooren, 1994; Chiba et al., 1993; Mickens, 1994; Ishimaru, 2017) and, independently, stemming from developments in digital waveguide sound synthesis techniques (Asadzadeh, 2020). Other well-known families of numerical methods were also applied to acoustic problems, including finite volume (in the time domain) (Botteldooren, 1994) and finite element and boundary element methods in both times- and frequency domains (Svensson, Fred & Vanderkooy, 1999). However, concerning the methods implemented for the wave equation itself (i.e., in the time domain), finite difference methods seem to have gained the most popularity over the years, e.g. (Benito et al., 2020; Sullivan & Young, 2001; Liu, Ding & Sen, 2011; Liu & Sen, 2009; Kumar, 2004), most likely due to their simplicity in formulation and ease in implementation. Seminal texts on finite difference methods (and other numerical methods) include (Thomas, 2013; Forsythe & Wasow, 1960; Knabner & Angermann, 2021). See also (Thomée, 2001) for a detailed history of finite difference methods.

2. AIM OF THE RESEARCH

A modern approach to solving inverse ultrasound tomography problems aims to use novel methods based on machine learning techniques. As those methods are excellent for almost automatic search for solutions for complex problems, most ML algorithms are the supervised methods with the means to find a proper model for a problem. It needs to provide input data and referenced output data sets. In tomography, for training, e.g. deep neural network, we need to collect a large amount of measurement data with prior knowledge of the distribution of imaged medium. It leads to the technical problem that we need another reference method to solve the same problem and an inverse problem with one method. Another approach to that impasse is to create a forward problem solver capable of simulating measurement data on defined medium distribution. That type of solver needs to meet a few requirements. It must be quick and thus simple enough to generate large data sets. Moreover, simulations need to allow for easy defining a broad range of heterogeneous distributions inside the tomographic probe. This research aimed to create a simple but sufficiently versatile framework for the quick generation of simulations suitable for tomographic applications, focusing on machine learning techniques.

In the future work, authors plan to use created simulations as a start point for various types of tomographic problems like examination of the shapes of sensitivity maps of tomographic settings, experimentations of transducers excitations patterns, application of style-transfer learning for forward and inverse problems, designing compression algorithms for ultrasound measurement data etc.

3. COMPUTER SIMULATION MODEL

Based on the simulation of acoustic wave propagation on a regular square grid, we measure the value of acoustic pressure that excites the movement of the diaphragm of the measurement sensor, causing the excitation of an electric voltage. Such an approach gives the possibility of solving a forward problem. It consequently allows the creation of learning sets used in machine learning, especially in supervised deep learning.

Simulation of acoustic wave propagation inside the tomographic probe is necessary to reflect the wave propagation and generated voltages in sensors at a given distribution of the internal medium of the probe. Such a situation is not fully possible to reproduce in real measurements and, at the same time, is very time consuming and requires a significant amount of work. Machine learning requires tagged sets with large volumes, even up to several tens of thousands. Moreover, real measurements performed by a human can be burdened with uneven distribution of objects inside the tomographic probe, which lead to biased measurement data set, which can affect the learning capabilities of neural network.

Radial models are commonly used to solve the inverse problem (Kania et al., 2019). However, despite the effectiveness of these models in imaging on real measurement systems, it is not possible to use them to simulate acoustic processes (Kania, Rymarczyk, Maj & Gołąbek, 2019). In work (Kania et al., 2020), an attempt was made to simulate acoustic phenomena using ray tracing with Fermat's principle, thus succeeding in tracing acoustic wave trajectories and reproducing the lensing phenomenon depending on the objects and the medium filling the measurement probe. However, due to technical problems, using this method for simulation is not an effective method of carrying it out. Furthermore, these problems generated the need to use finite difference methods to solve the wave equation.

3.1. Finite difference methods for the wave equation

The simulation is performed on a grid of 128x128 spatial nodes during 8000 steps, on a 40x40 cm square with a probe with a diameter of 20 cm. A sequence of 16 sensors triggered with intervals of 500 steps is simulated with the time of activation: 20 iterations and sinusoidal excitation:

$$U = Asin(wt) \tag{1}$$

where: $A = 10$,
$w = 1.0$.

Full simulation of the measurement sequence (8000 iterations, 16 sensors) is determined in about 7 seconds (on an i9-11900F processor) due to the stability conditions, which corresponds to approximately 12 ms of real-time.

First, matrix boundary conditions were implemented, taking into account the acoustic impedance of the "walls" of the simulated area. Through numerical experiments, it turned out that implementation of the lossy condition does not allow for the simulation of the total absorption of the wave by the border of the simulation area. By analyzing the free-space boundary problem, i.e. the condition in which the wave freely "flies" across the simulated boundary, it has been established that there are currently no typical boundary conditions for two-dimensional and/or more-dimensional problems. The most common solution to the problem is to extend, in some way, the scope of the simulation beyond the area of interest. Ultimately, therefore, the scope of the simulation was increased, and the wave energy outside the probe circle was periodically reset to zero once for a specified number of simulation steps (in this case, Fig. 1, once every 100 iterations). Increasing the simulation area allows modelling the wave "drift" outside the region of interest. After erasing the wave outside the region of interest, the wave in the probe propagates further undisturbed.

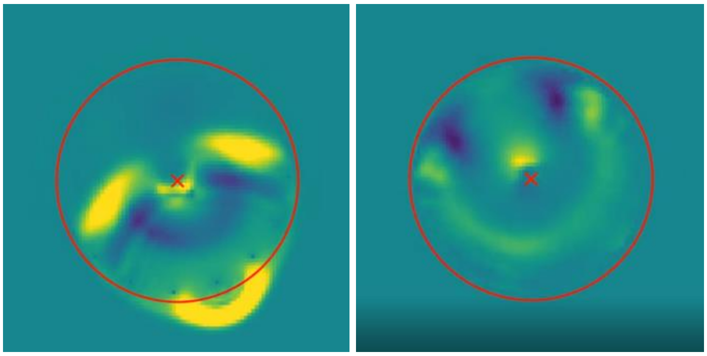

Fig. 1. Simulation results: a collision of an acoustic wave on a square object filled with air (left) and view of the wave passing behind the object (right)

The number of frames after which the deletion takes place is selected in such a way that the wave has, on the one hand, sufficient time to escape from the probe, on the other hand, so that its reflection from the border of the simulated area does not return to the interior. Therefore, it is indirectly related to the allowance to the simulated domain to be considered in determining the area of interest of the simulation. The necessity to increase the simulation area in relation to FOV causes the reconstructed to be indisputably more pixels than results from the FOV size alone. Using the implementation of the simulation, taking into account the lossy boundary conditions, based on the matrix equation, we have:

$$u^{t+1} = (\lambda B + I)^{-1}(2I + \lambda^2 L)u^t + (\lambda B - I)u^{t-1} \quad (2)$$

We must therefore store in memory two matrices of size $n^2 \times n^2$, where the simulation is performed over the area of size $n \times n$. The rigid implementation of the Dirichlet conditions allows to simplify the equation to the formula:

$$u^{t+1} = (2I + \lambda^2 L)u^t - u^{t-1} \quad (3)$$

However, we still need at least one matrix of sizes $n^2 \times n^2$, which means that the simulation still needs $O(n^2)$ memory. To reduce this problem, an equivalent implementation based on the convolutional filter was used:

$$u^{t+1} = 2u - u^{t-1} + \lambda^2 Conv(u, L) \quad (4)$$

where: $L = \begin{pmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ is a filter implementing the discrete Laplacian.

Thanks to this, apart from the necessity to store u^{t+1}, u^{t}, u^{t-1} tables in the memory, there is no need for additional memory apart from the 3x3 table of the convolutional filter. We should also note that such implementation does not require u matrix expansion into vectors, as in the case of matrix equations. In addition, by experimenting, the simulation is more stable for the "soft" Laplacian variant, which is used in the simulation:

$$L^{*} = \begin{pmatrix} 0.25 & 0.5 & 0.25 \\ 0.5 & -3 & 0.5 \\ 0.25 & 0.5 & 0.25 \end{pmatrix} \tag{5}$$

The condition for the stability of the simulation for Laplacian L is the so-called Courant-Friedrichs-Lewy condition:

$$\lambda = \frac{v\,dt}{dx} \leq \sqrt{0.5} \approx 0.0707 \tag{6}$$

where: $v -$ wave velocity at a node,
$dx -$ the actual distance between nodes,
$dt -$ actual time step length.

In practice, to meet this condition, the values of dt and dx are selected so that:

$$\lambda_c = \frac{c\,dt}{dx} = 0.07 \tag{7}$$

where: c – the maximum wave speed used in the simulation.

Thanks to this, by simulating the values of $lam \in [0, \lambda_c]$ on the mesh nodes, we can be sure that the stability of the simulation will be ensured. In this context, it turns out that the use of L^{*} Laplacian allows a slight increase in the range of lam values over λ_c (Fig. 2).

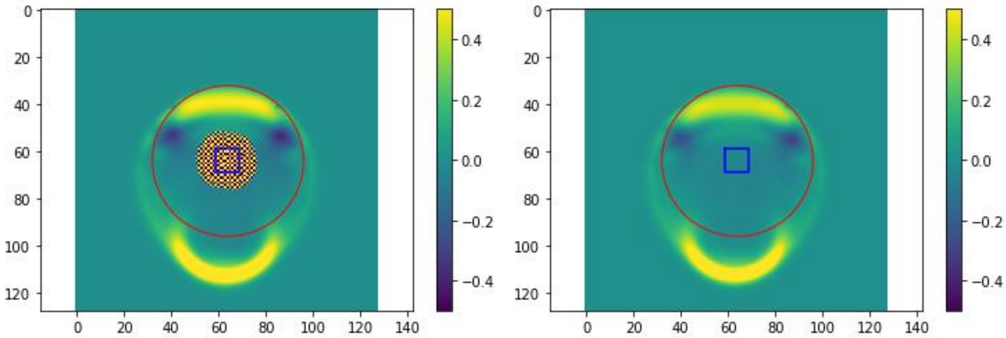

Fig. 2. Wave passage through an obstacle (the same 86th frame of simulation) not meeting the theoretically CFL condition ($lam = 1.2\lambda_c$), using Laplacian L (left, destabilization) and L^{*} (right, no destabilization artefact) – axis ticks in figures are pixel numbers (32 pixels correspond to 10 cm)

An additional aspect of the simulation is that in making a series of measurements, we must wait until after one sensor is excited until the acoustic wave inside is attenuated enough to not interfere with the wave produced by the next transmitter.

In the model currently used, the scattering factor was not considered, resulting in the probe's interior never reaching the level of complete silence. In order to control the wave scattering, Gauss Kernel Filter has been added to the simulation step. This way, by controlling the variance of the filter, the scattering of the wave can be increased (Fig. 3).

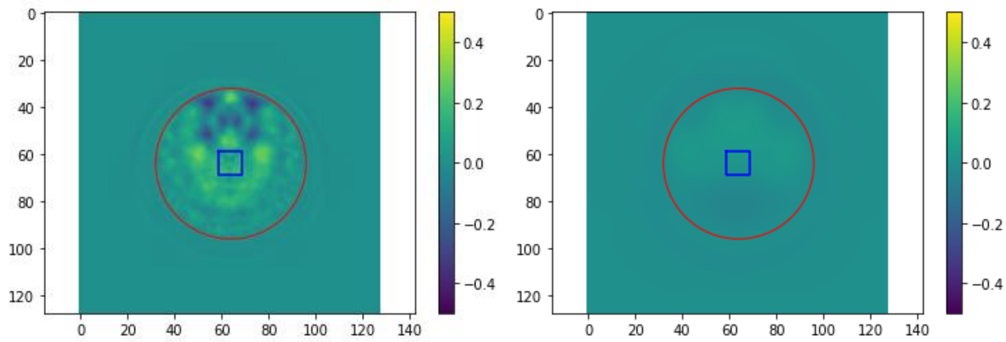

Fig. 3. View 390 of the simulation frame without dispersion $\sigma = 0$ (left) and with the same parameters but with dispersion $\sigma = 0.4$ (right) – axis ticks in figures are pixel numbers (32 pixels correspond to 10 cm)

By taking into account the dispersion, we can obtain much cleaner measurement waveforms, as the calming of the waveform does not disturb the measurement readings in the vicinity of the sensors.

The first extreme will be the variant without attenuation, where there is significant wave collimation and no mute (Fig. 4).

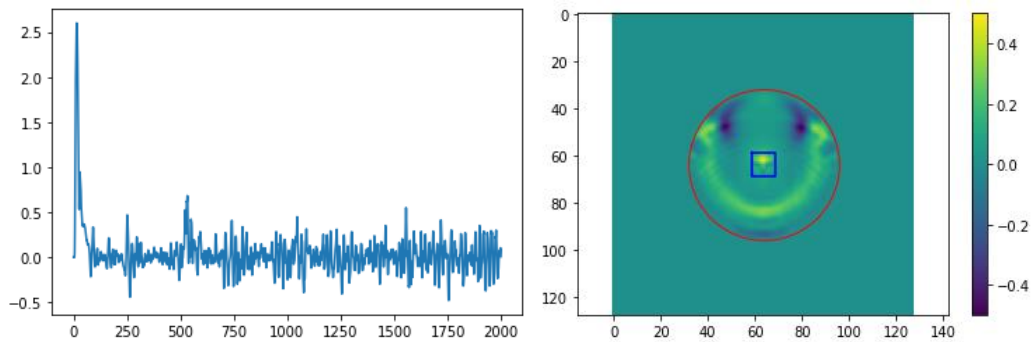

Fig. 4. Simulated measurement series of probe no. 0 for the first 2000 simulation steps at $\sigma = 0$ (left, X-axis descriptions are the number of iterations) and the effect of wave collimation on the circular edge of the probe $\sigma = 0$ (right)

On the other hand, too high attenuation causes almost undisturbed wave passage through the probe, eliminating the collimation effect, but we get much "cleaner" waveforms of the pulses (Fig. 5).

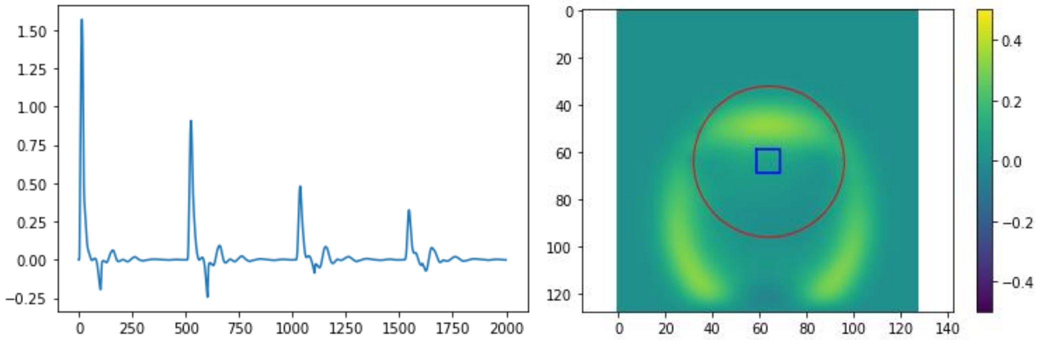

Fig. 5. Measurement of probe no. 0 for the first 2000 simulation steps at $\sigma = 1.0$ (left, X-axis descriptions are the number of iterations) and significant "blur" ($\sigma = 1.0$) of the wave eliminating collimation (right)

4. CONCLUSIONS

The results obtained so far allow for efficient and easy to perform simulations of acoustic phenomena on an arbitrary velocity distribution inside the probe, which will allow for the generation of simulated data sets of any size.

Finite difference methods comprise a simple starting point for such simulations, but they are known to suffer from approximation errors that may necessitate expensive grid refinements to achieve sufficient accuracy levels. As such, research has gone into designing finite difference methods that are highly accurate while remaining computationally efficient. A solution to the "free-space" boundary problem was proposed, and the memory consumption was significantly optimized from $O(n^2)$ to $O(n)$. The given method of simulating wave scattering enables the control of the noise extinction time within the tomographic probe and the permeability of the sound wave.

Further work will concern the verification of the simulator with real measurements (selection of parameters and signal conditioning) and the implementation of the possibility of defining custom measurement sequences, including those that enable beam-forming.

Author Contributions

Development of the concept of the simulator, research methodology, and implementation of in ultrasound transmission tomography, KK.; Development of the system concept, measurement methodology, techniques, image reconstruction and supervision, T.R.; Preparation of research methodology, literature review, formal analysis, general review, and editing of the manuscript, M.M. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest

The authors declare no conflict of interest.

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