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The influence of boundary condition functions on the quality of the solution and its sensitivity to coefficients of *k*-ε turbulence models

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Abstract: The paper is devoted to the problem of boundary conditions influence on the quality of the solution obtained with use of *k*-*ε* turbulence models. There are calculation results for different boundary conditions and two methods: standard *k*-*ε* and RNG *k*-*ε* in the paper. The flow parameters obtained from the calculation are compared with our own measurement results. Moreover, the influence of input data on the inflow edge on sensitivity coefficients is shown and analysed in the paper. The research is performed for components of velocity and turbulence kinetic energy.

Keywords: CFD, the *k*-*ε* turbulence models, sensitivity analysis, square cylinder.

1. Introduction

The problem of the solution quality is described in many papers, for example in Hrenya et al. [4], Shih et al. [5] and Shimada and Ishihara [6] which are concerned with the analysis of model coefficients or improving the followed turbulence model based on the *k*-*ε* method. This paper is devoted to the problem of boundary conditions influence on the solution quality and its sensitivity to coefficients of the *k*-*ε* method. The aim of the paper is to show the importance of the correct description of boundary conditions at the inflow edge of a calculation domain.

The inflow parameters are usually described at inflow edges by a few functions of flow parameters. The solution results depend on both the values of inflow parameters and their derivatives. Lack of fulfillment of derivatives continuity significantly influences the obtained solution. Here, the problem is presented for a two-dimensional incompressible steady flow around a square cylinder. The research is limited to the following flow parameters: components of velocity and turbulence kinetic energy.

2. Description of the research problem

2.1. Research methods

The subject of this research is a two-dimensional incompressible steady flow around a square cylinder. The set of this model in a calculation domain is shown in Fig. 1. The mesh of FVM contains 80075 cells and it is more dense on walls and around the square. Calculations have been made using the Fluent program for two versions of the *k-ε* models: standard and RNG ones. The assumed model coefficients are as follows: $C_{\varepsilon 1} = 1.44$, $C_{\varepsilon 2} = 1.92$, $C_{\mu} = 0.09$, $\sigma_k = 1.0$ and $\sigma_{\varepsilon} = 1.3$ for the standard *k-ε* model and $C_{\varepsilon 1} = 1.42$, $C_{\varepsilon 2} = 1.68$ and $C_{\mu} = 0.0845$ for the RNG *k-ε* model.

In order to check the quality of calculation results measurement results from the wind tunnel are used. These measurements have been carried out in the wind tunnel of the Wind Engineering Laboratory in Cracow University of Technology by the author with associates. The model was set at the ground of the wind tunnel. The length of the model was $b = 2050$ mm and the flow in the middle-plane may be treated as two-dimensional one. The research in the wind tunnel is described in the following papers: Błazik-Borowa [1] and Błazik-Borowa et al. [2].

The equations of the *k-ε* methods contain semi-empirical coefficients which significantly influence the calculations results. The sensitivity analysis serves to check the influence of small changes of the model parameters on the problem solution and it is described using sensitivity coefficients, which may be calculated from formula:

$$\tilde{w}_m = \frac{w_2 - w_1}{\Delta C_m} \tag{1}$$

where w – analysed flow parameter, ΔC_m – increment of the C_m parameter, w_1 – results of calculations at $C_m - \Delta C_m / 2$ and w_2 – results of calculations at $C_m + \Delta C_m / 2$.

The methods of calculations of sensitivity coefficients, examples of sensitivity analysis and applications of the sensitivity analysis results are presented in the papers by Błazik-Borowa [1] and Błazik-Borowa et al. [3]. Sensitivity coefficients depend on approximation methods, the quality of the mesh, etc. Here, the consequence of the boundary conditions is checked for the sensitivity of flow parameters on model coefficients.

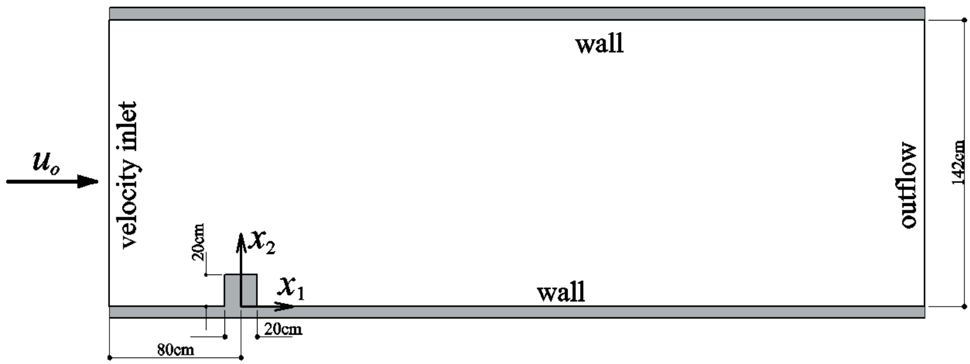

Fig. 1. The calculation domain with the description of boundary conditions.

2.2. The description of inflow boundary conditions

The boundary conditions are located as it is shown in Figure 1. The input flow parameters may be given only at the edge with the boundary condition called inlet velocity. The calculations have been made for four sets of functions of the inflow parameters:

Case No 1

Parameters are constants and they are equal to values measured in the middle part of inflow plate of the wind tunnel, i.e: input velocity u_o = 10 m/s, turbulence intensity I_u = 0.052, turbulence kinetic energy k = 0.2704 m^2/s^2 and dissipation of turbulence kinetic energy ε = 0.0162.

Case No 2

The functions of velocity u_o and turbulence intensity I_u are determined on the basis of measurement results. Velocity is described by the following relationships:

$$u_o(x_2) = u_h \left(\frac{x_2}{h}\right)^{0.15} \text{ for } 0 < x_2 < \delta \text{ and } H - \delta < x_2 < H \tag{2}$$

$$u_o(x_2) = 10\text{m/s} \text{ for } \delta < x_2 < H - \delta \tag{3}$$

where u_h = 9.67 m/s – velocity at the upper edges of the square, h = 20 cm – height of the square, H = 142 cm – height of the working section of the wind tunnel, δ = 25 cm – thickness of the boundary layer.

The turbulence intensity is expressed by the functions:

$$I_u(x_2) = 0.87x_2^2 \frac{1}{\text{m}^2} - 0.63x_2 \frac{1}{\text{m}} + 0.15 \text{ for } 0 < x_2 < \delta \text{ and } H - \delta < x_2 < H \tag{4}$$

$$I_u(x_2) = 0.24x_2^2 \frac{1}{\text{m}^2} - 0.18x_2 \frac{1}{\text{m}^2} + 0.07$$

$$\text{for } \delta < x_2 < 0.6625\text{m and } H - 0.6625\text{m} < x_2 < H - \delta \tag{5}$$

$$I_u(x_2) = 0.052 \text{ for } 0.6625\text{m} < x_2 < H - 0.6625\text{m}. \tag{6}$$

Turbulence kinetic energy and its dissipation are calculated on the basis of velocity and turbulence intensity on the basis of the following formulae:

$$k(x_2) = 0.5\left(u_o(x_2) I_u(x_2)\right)^2 \tag{7}$$

$$\varepsilon = \frac{C_\mu^{0.75} k^{1.5}}{L_x} \tag{8}$$

where L_x = 142 cm – length of turbulence scale.

Case No 3

Velocity is described by a set of functions with fulfillment of derivatives continuity. They are expressed by relationships:

$$u_o(x_2) = 316x_2 \frac{1}{\text{s}} \text{ for } 0 < x_2 < 0.025\text{m and } H - 0.025\text{m} < x_2 < H \tag{9}$$

$$u_o(x_2) = -\frac{0.015778}{x_2 - 0.017937\text{m}}\frac{\text{m}^2}{\text{s}} - 0.292975 x_2 \frac{1}{\text{s}} + 10.141232\frac{\text{m}}{\text{s}}$$

$$\text{for} \quad 0.025\text{m} < x_2 < \delta \text{ and } H - \delta < x_2 < H - 0.025\text{m} \tag{10}$$

$$u_o(x_2) = 10\text{m/s} \text{ for } \delta < x_2 < H - \delta \,. \tag{11}$$

Other parameters are calculated in the same way as in the case No 3.

Case No 4

Velocity is described as in the case No 3. Turbulence kinetic energy is described by a few polynomials with different degrees, which are determined on the basis of measurement results. They are written as:

$$k(x_2) = 22.759 x_2 \frac{\text{m}}{\text{s}^2} \text{ for } 0 < x_2 < 0.025\text{m and } H - 0.025\text{m} < x_2 < H \tag{13}$$

$$k(x_2) = -133.1091 x_2^4 \frac{1}{\text{m}^2\text{s}^2} + 125.9460 x_2^3 \frac{1}{\text{ms}^2} - 34.3476 x_2^2 \frac{1}{\text{s}^2} - 0.6692 x_2 \frac{\text{m}}{\text{s}^2} + 0.5630\frac{\text{m}^2}{\text{s}^2}$$

$$\text{for } 0.025\text{m} < x_2 < 0.35\text{m and } H - 0.35\text{m} < x_2 < H - 0.025\text{m} \tag{14}$$

$$k(x_2) = 1.0788 x_2^2 \frac{1}{\text{s}^2} - 0.6692 x_2 \frac{\text{m}}{\text{s}^2} + 0.2163\frac{\text{m}^2}{\text{s}^2}$$

$$\text{for } 0.35\text{m} < x_2 < 0.6625\text{m and } H - 0.6625\text{m} < x_2 < H - 0.35\text{m} \tag{15}$$

$$k(x_2) = 0.2704\text{m}^2/\text{s}^2 \text{ for } 0.6625\text{m} < x_2 < H - 0.6625\text{m}\,. \tag{16}$$

The dissipation rate of kinetic turbulence energy is calculated from Eq. 7.

Fig. 2 shows the profiles of input velocity and turbulence kinetic energy. The velocity graphs are similar, but the values of turbulence kinetic energy are quite different. The values for the case No 4 are close to measurements, but a physical relationship between flow parameters is not kept. It is caused by assumption functions for velocity and turbulence kinetic energy profiles which are in agreement with measurements. The errors in measurements cause the relation between input flow parameters not to be fully in agreement with equations of the turbulence model.

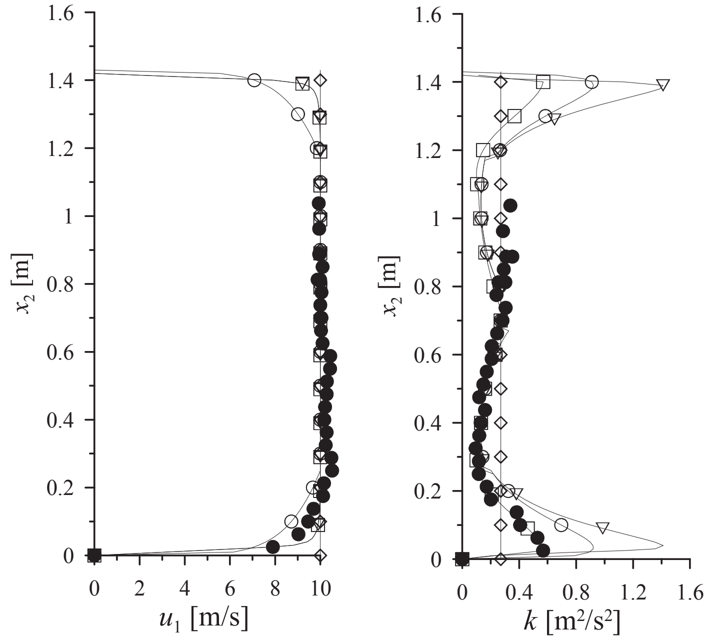

Fig. 2. The graphs of input velocity u_1 and turbulence kinetic energy k; ● – measurement results, data for: ◇ – case No 1, O – case No 2; ▽ – case No 3, □ – case No 4.

3. Presentation and discussion of results

3.1. The analysis of flow parameters

The calculation and measurement results are shown in Figs 3, 4, 5, 6, 7 and 8. The figures contain the graphs of calculation results for all cases and for two k-ε methods: the standard one and RNG one. The calculation results are compared with our own measurement results. It arises from the research that:

- the best results are obtained for the case No 4;
- standard k-ε method is more sensitive to boundary conditions than RNG version;
- differences between calculation results for all cases are bigger for turbulence kinetic energy than for velocity, but it is noted that the input profiles of turbulence kinetic energy vary more in the shapes and values than the graphs of velocities;
- fulfillment of derivatives continuity has a positive influence on the quality of results, but the exact description of input flow parameter values is more important;
- more exact description of the values of turbulence kinetic energy causes significant improvement of results for the u_2 components of velocity.

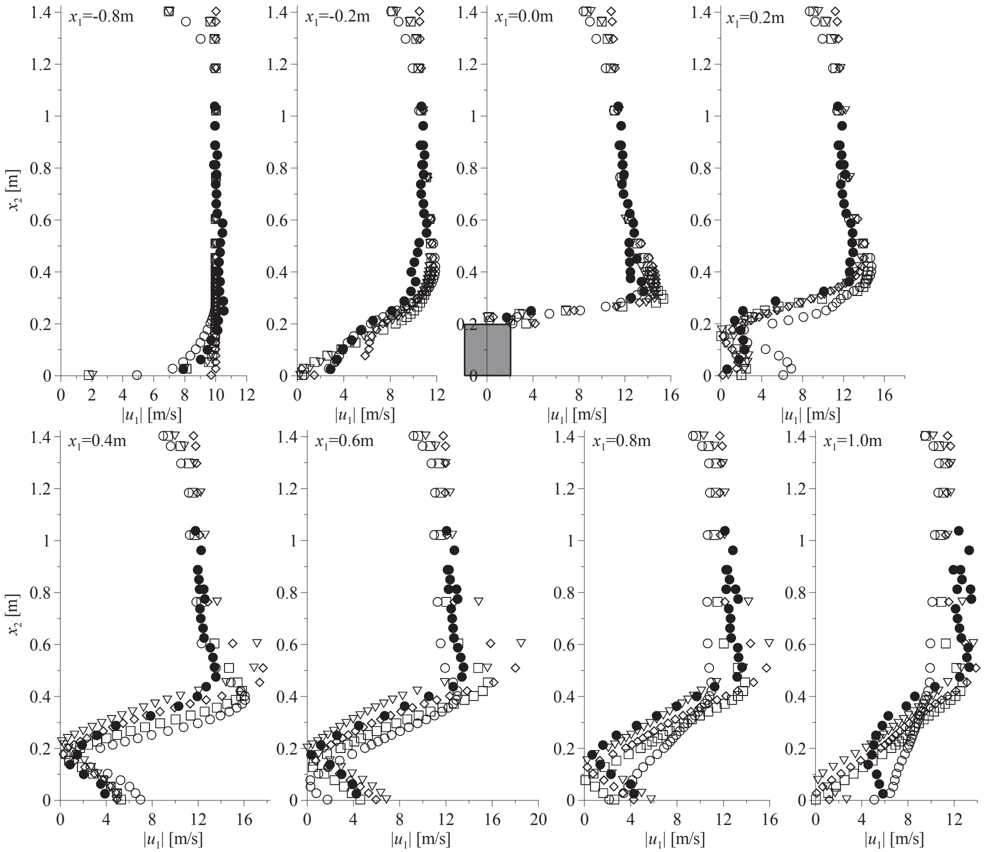

Fig. 3. The graphs of the component of velocity u_1 obtained from the standard k-ε method; ● – measurement results, data for: ◇ – case No 1, ○ – case No 2; ▽ – case No 3, □ – case No 4.

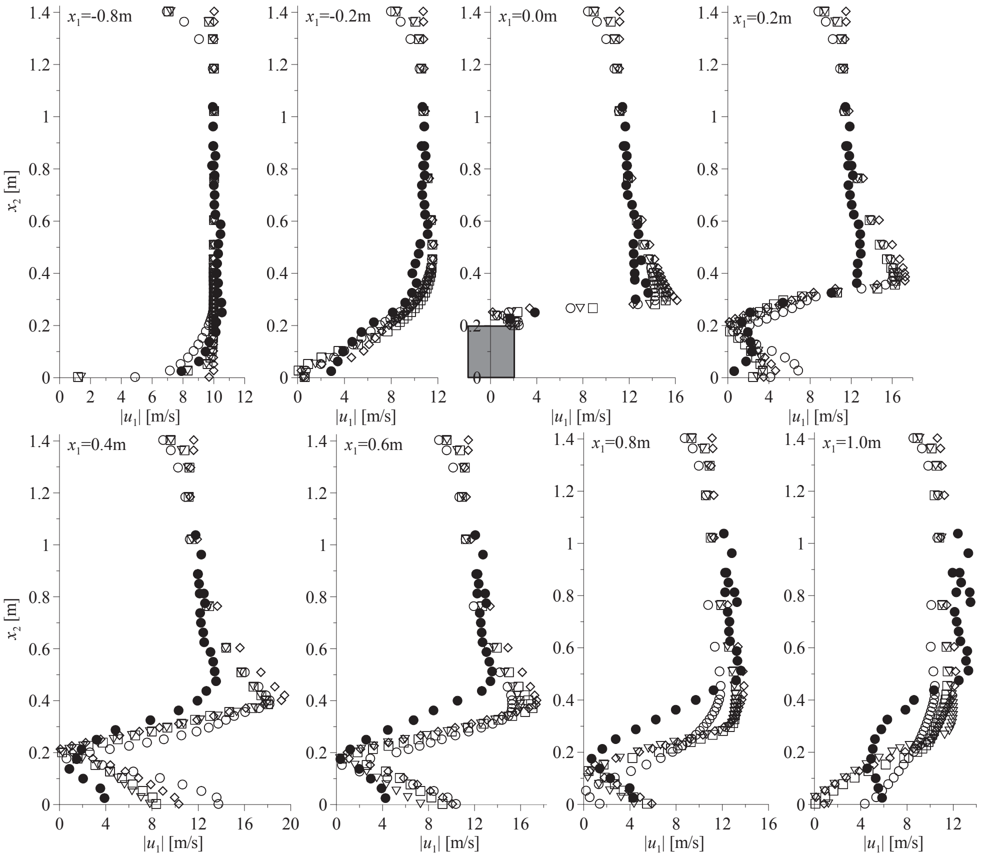

Fig. 4. The graphs of the component of velocity u_1 obtained from the RNG k-ε method; ● – measurement results, data for: ◇ – case No 1, ○ – case No 2; ▽ – case No 3, □ – case No 4.

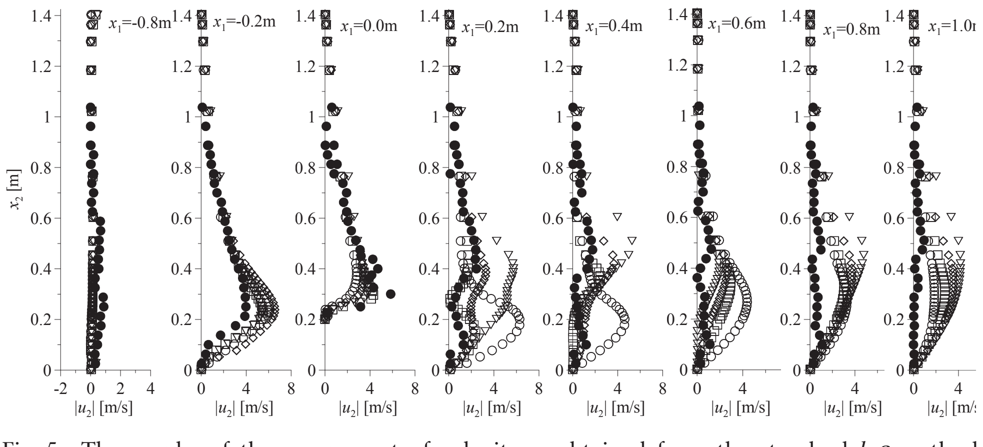

Fig. 5. The graphs of the component of velocity u_2 obtained from the standard k-ε method; ● – measurement results, data for: ◇ – case No 1, ○ – case No 2; ▽ – case No 3, □ – case No 4.

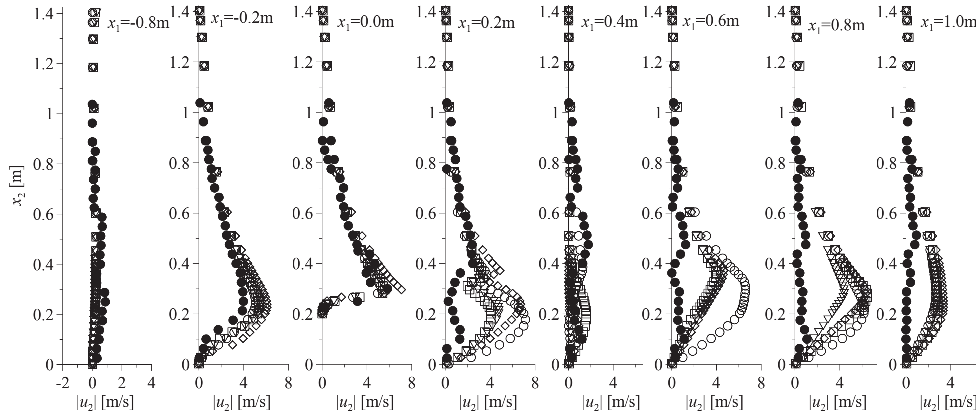

Fig. 6. The graphs of the component of velocity u_2 obtained from the RNG k-ε method; ● – measurement results, data for: ◇ – case No 1, ○ – case No 2; ▽ – case No 3, □ – case No 4.

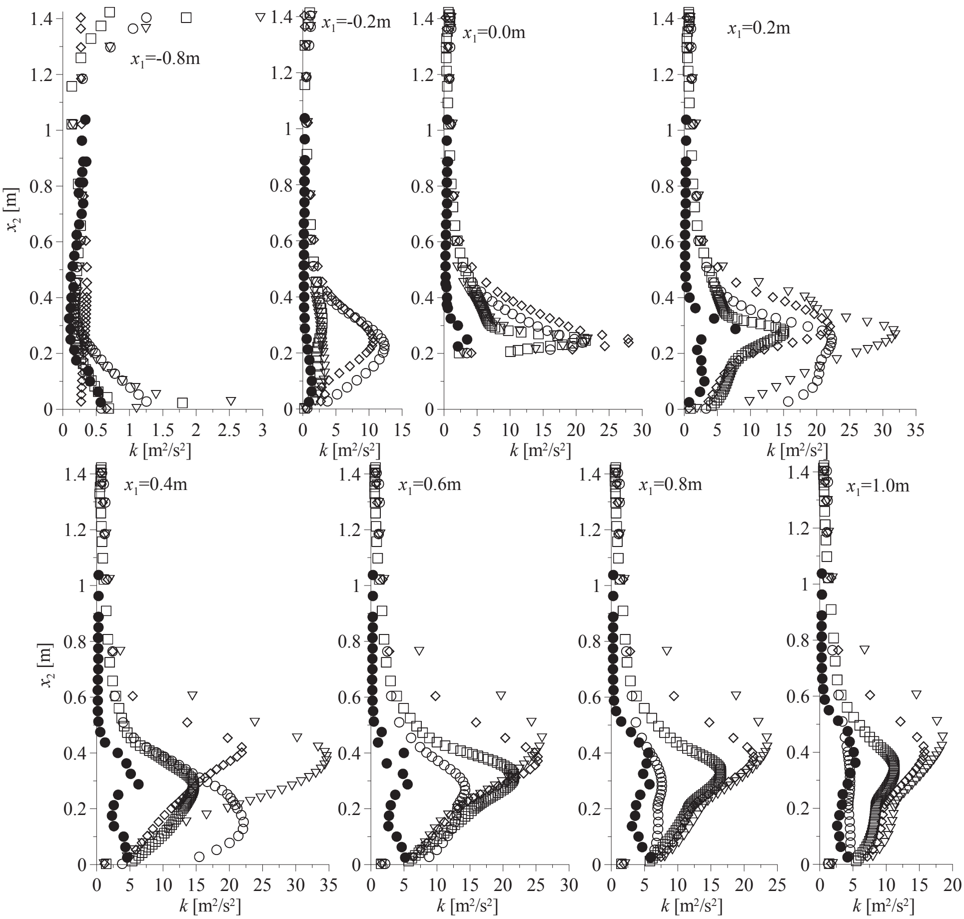

Fig. 7. The graphs of turbulence kinetic energy k obtained from the standard k-ε method; ● – measurement results, data for cases: ◇ – No 1, ○ – No 2; ▽ – No 3, □ – No 4.

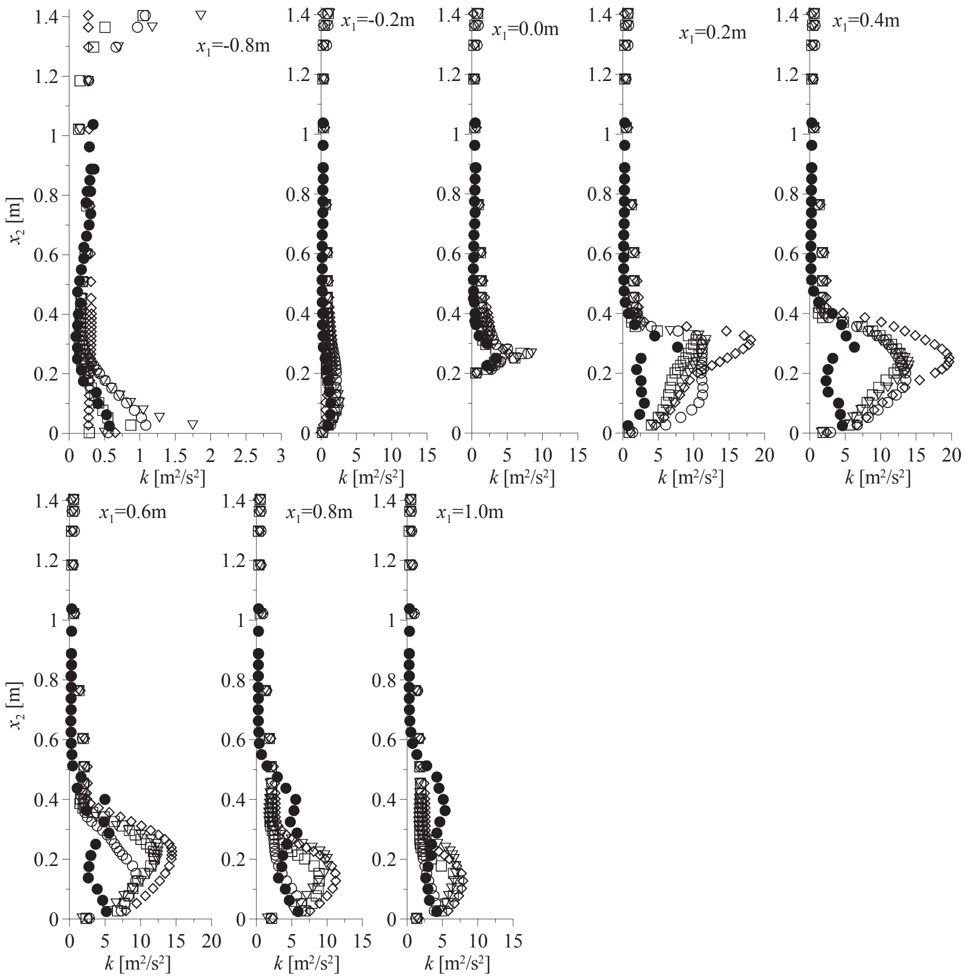

Fig. 8. The graphs of turbulence kinetic energy k obtained from the standard k-ε method; ● – measurement results, data for cases: ◇ – No 1, ○ – No 2; ▽ – No 3, □ – No 4.

3.2. The analysis of sensitivity coefficients

Figures 9 and 10 show model fields of sensitivity coefficients. Two coefficients are presented: the figures on the left side present sensitivity of the velocity components u_1 to the $C_{\varepsilon 1}$ coefficient and bitmaps on the right side show sensitivity of turbulence kinetic energy k to the $C_{\varepsilon 1}$ coefficient. The ranges of sensitivity coefficients are set at the same values in figures. The extreme values are different, but the figures are presented in such a way in order to make comparison areas of the same sensitivities.

Since the quality of calculation results depends on the correctness of the choice of model coefficients, the increase in sensitivity of flow parameters to these coefficients means the decrease in trust of obtained results. On the other side it should be noted that the parameters of the model are the factors of derivatives in a set of equations described in the k-ε model. When the term with a given parameter is large enough, then also the value of this parameter, being only a factor, has a greater influence on the solution. It is confirmed by the comparison of the turbulence kinetic energy graphs and its sensitivity to $C_{\varepsilon 1}$.

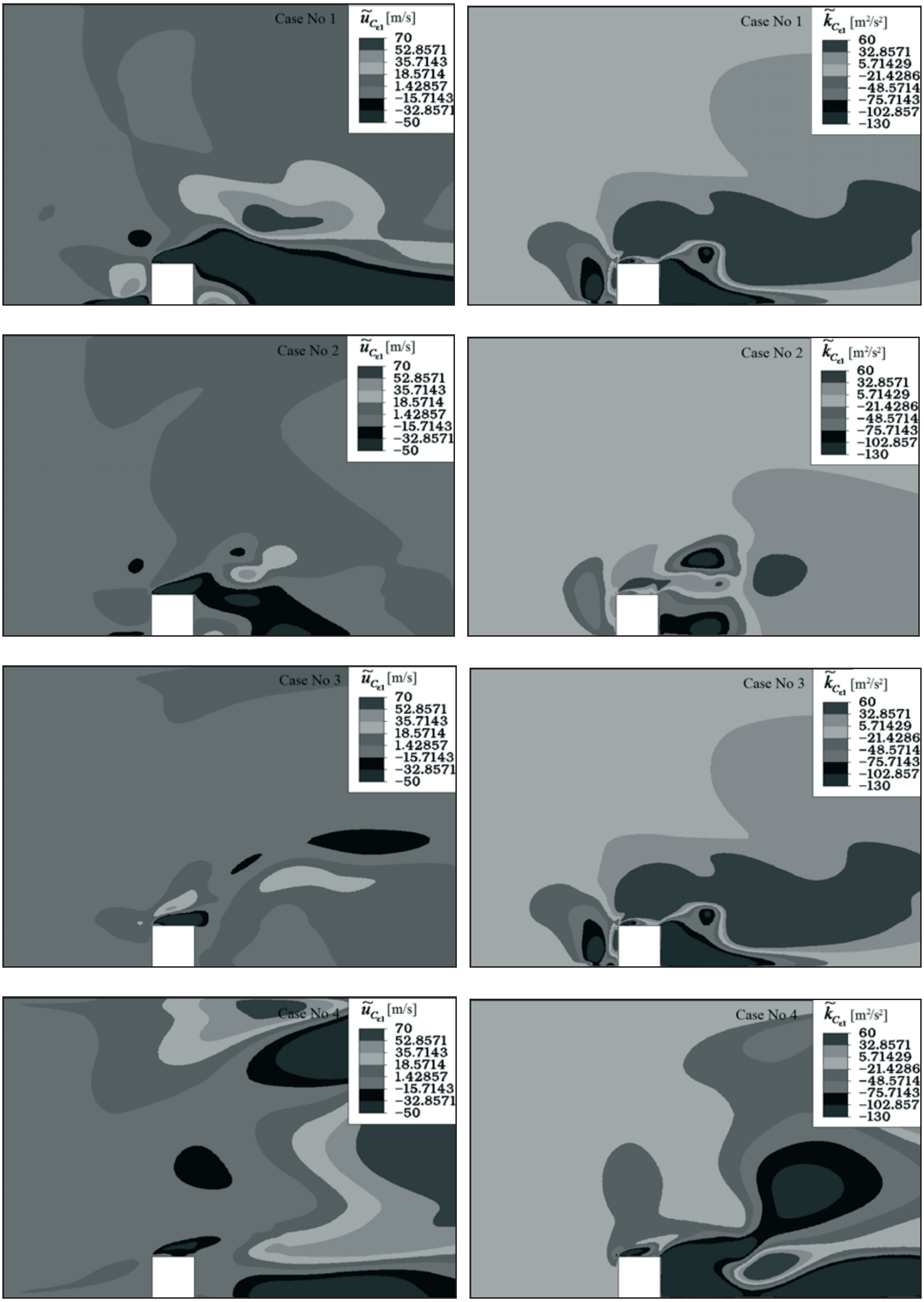

Fig. 9. Model fields of sensitivity coefficients obtained using the standard version of k-ε method.

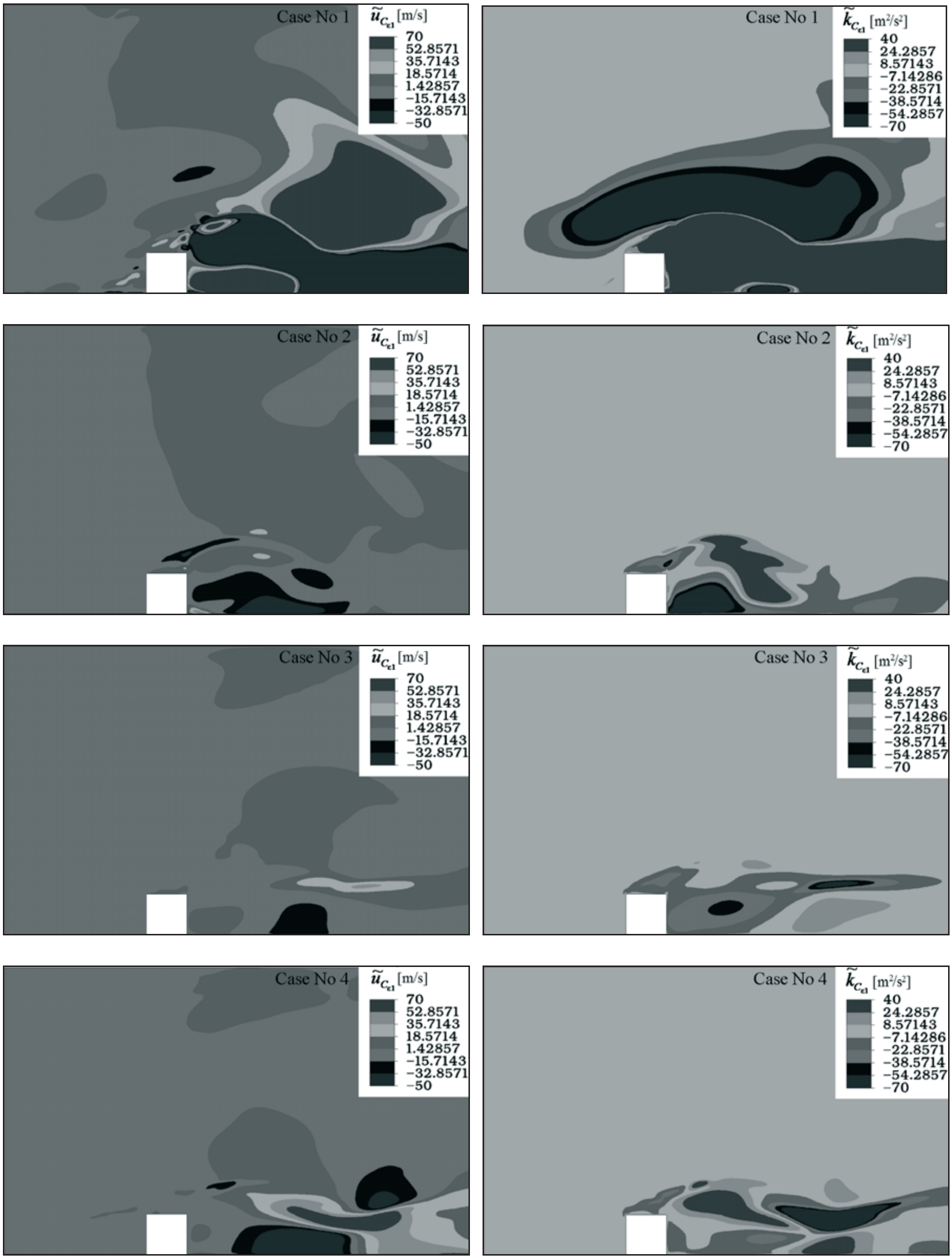

Fig. 10. Model fields of sensitivity coefficients obtained using the RNG version of k-ε method.

The fields of sensitivity coefficients show that fulfillment of derivatives continuity significantly decreases sensitivity of calculated flow parameters to the model coefficients and it means that it smooths away undesirable gradients of flow parameters functions. The increase in sensitivity is seen for the case No 4. It is probably caused by the lack of complete agreement between input flow parameters and equations of the k-ε model.

4. Conclusions

The calculation results depend on boundary conditions and their influence may be observed in the whole domain. Moreover, the presented analysis confirms that RNG method is better than standard *k*-*ε* method. The differences between calculation results for different inflow boundary conditions are similar to the differences between the solutions obtained using two versions of *k*-*ε* method. The other conclusions are as follows:

- fulfillment of derivatives continuity has a positive influence on the quality of results, but the exact description of input flow parameter values is more important;
- lack of the complete agreement between input flow parameters and equations of *k*-*ε* model causes sensitivity coefficients to grow;
- RNG method is less sensitive to accuracy of boundary conditions than standard *k*-*ε* method.

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