

Cement Concrete Mixture Performance Characterization

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Abstract: The cementitious composite nature of concrete makes very difficult directly ascertaining each mixture-factors' contribution to a given concrete mixture performance characteristics but also doubly difficult to accurately balance mutually exclusive requirements for performance (workability, strength, durability) and sustainability (the economic and efficient use of materials) for mixture proportioning based on recipes of previously produced concretes. This study sought to quantify individual mixture-factors' contribution to a given concrete mixture's performance characteristics. Proposed multi-parametric exponential mixture-response models were fitted to available test-performance data sets of HPC mixtures proportioned based on the best combined grade aggregate (minimum void) to generate mixture-strength and mixture-porosity development (age-mixture response relationships) profiles of HPC mixtures and deemed robust enough to yield reliable determination of mixture-response rate-parameters S_o , S_p , S_i and P_o , P_p , P_i as functions of mixture-factors that permitted reliable quantification of contributions to HPC mixture performance of individual mixture-factors and optimization of mixture properties under study over the study domain. Mixture-response sensitivity analysis models (or mixture response trace plots) to allow construction of mixture-factor envelopes and ultimately optimized mixture-response models to facilitate selection of optimal mixture-factors and optimal tailoring of HPC mixture requirements to HPC mixture performance were developed and used to obtain optimized adapted HPC mixtures from available high performance concrete (HPC) mixture design recipes investigated in the study over the study domain. Adapted HPC mixture design recipes yielded alternative mixture compositions with improved performance and efficiency characteristics with statistical performance metrics MAPE, NMBE and RMSE values of 7.6%, -3.7% and 6.5 MPa, respectively.

Keywords: response models, mixture-design, mixture-factors, mixture-strength, mixture-porosity, physical properties.

List of Abbreviations

The following symbols are used in this paper:

N – the total number of mixtures,

S_{ref} – mixture-strength response of a known reference mixture,

S_{target} – mixture-strength targeted optimized response of a mixture,

P_{ref} – mixture-porosity response of a known reference mixture,

P_{target} – mixture-porosity targeted optimized response of a mixture,

R_i – linearly weighted summated response function of mixture,

r_i – individual normalized response function of a mixture,

T – the target value for the response functions facilitates the determination of values of the weights,

- $f_i(a)$ – maxima (or minima) of response trace-plot function,
 $f_{ref}(a)$ – maxima (or minima) of response trace-plot function for a reference mixture,
 w_i – the weight for individual normalized response function of a mixture,
 w_i^{UB} – upper bound weight-value for individual normalized response function i ,
 w_i^{LB} – lower bound weight-value for individual normalized response function i ,
 π_i^{UB} – upper bound mixture-factor value i ,
 π_i^{LB} – lower bound mixture-factor value i ,
 σ – envelope standard deviation,
 α_{cement} – packing density of cement,
 α_{sand} – packing density of sand,
 α_{RHA} – packing density of RHA,
 $\alpha_{silica-fume}$ – packing density of silica-fume.

1. Introduction

1.1. High Performance Concretes

The American Concrete Institute [1] defines high performance concrete (HPC) as concrete meeting special combinations of performance, durability and uniformity requirements that cannot always be achieved customarily using conventional constituents and production practices. It is basically constituted of the same materials as conventional (normal) concrete but also incorporates supplementary cementitious materials (SCM) and high-performance admixtures to obtain cold-cast HPC mixtures with the desired performance indicators. This family of cement concretes comprises of (on the basis of mixture-strength), high-strength (≥ 50 -90 MPa), very high-strength (≥ 90 -130 MPa) and ultra-high-strength (≥ 130 -200 MPa) concrete (modified from Büyüköztürk et al [2]).

Normal strength concrete structures have a mass to strength ratio of 40-120 kg/MNm while that for HPC structures averages 15 kg/MNm [3] but HPC mixtures have the disadvantage of high and expensive contents of cement and additives and hence the dominant HPC products on the market are almost exclusively pre-packaged, proprietary and expensive with patented aggregate and concrete formulation (information on their compositions not readily available) and almost impossible to modify or customize or adapt for alternative mixture designs for a specific individual design, construction or architectural description.

Currently, many HPC mixtures are proportioned based on prior history of production (historical information or experience) or the cost-prohibitive trial mixtures (trial-and-error) or by prescribing the limits (maximum or minimum) on the mixture-factors that circumscribe the desired performance indicators but rarely based on the actual needs of the mixture and the locally available materials and do not often involve properly balancing mutually exclusive requirements for performance (workability, strength, durability) and sustainability (the economic and efficient use of materials) [4, 5]. Most prescription-based design studies reported in the literature have not yielded expected end results because they tended to deliberately promote oversized mixtures by using cement content as a safety factor [6] while the trial-and-error modification of existing HPC recipes although very popular for HPC mixture design in the HPC production sector is a hit-or-miss affair, expensive, wasteful and un-optimizable because of the many varied material-inputs involved and the different sources for the material-inputs [7, 8]. Effective promotion of green concrete incorporating mineral additives is required in order to minimize the environment threat due to waste disposal from mineral additives and reduce cement consumption.

1.2. Cement Concrete Mixture Matrix Structure

Nearly a century after Abrams [9] proposed the water-to-binder ratio law, much has been contributed by cement concrete mixture researchers to broaden the understanding of how the fresh and hardened-state properties of concrete are controlled by the relative proportions of concrete constituent components—cement, coarse and fine aggregates, water, and various additives—while elevating concrete to the status of the most dominant construction material for 21st century infrastructural needs. Important advances in admixture technology over the past decades and a recognition that coarse aggregates represent the weakest link in concrete and that they can be taken out to have only sand as the main aggregate (Fig. 1) have led to the development of a new generation of cement concrete mixtures with low water-binder ratio, low matrix porosity and high particle packing density that lead to far improved rheological, mechanical and durability (extended service lifetime) properties than obtains with conventional cement concretes (CCC) of similar unit weight [10]. Such cement concrete mixtures (collectively termed as high-performance concretes or engineered ‘high-tech’ concretes designed to meet project-specific needs) incorporate fine-grained additives (fga) also known as supplementary cementitious materials (SCMs) and high-range water reducers (see Fig. 1). The incorporation of fine-grained additives produces a wide and continuous grain size distribution that helps in optimizing packing density of the matrix, creates a more uniform stress distribution when the matrix is loaded and hence a strong matrix; the smaller grains serve as a lubricant that reduces the inter-particle friction and hence improved workability of the mixture; the fine-grains also lower the porosity of the system by filling the voids of the mixture with fine particles while expelling water from the voids to allow the water to be more homogeneously distributed in the system and hence, again, improve the workability of the mixture and produce a strong mixture matrix; the super-plasticizer helps disperse cement and filler particles, improve the lubrication and reduce the inter-particle friction and improve the workability of the mixture [10].

Nevertheless, predicting common time-dependent mechanical behaviours of cement concretes remains inherently complex because of the heterogeneous nature of cement concrete, forcing concrete practitioners (engineers, concrete producers and researchers) to rely heavily on prior history of production (historical information or experience) and/or the cost-prohibitive trial mixtures (trial-and-error).

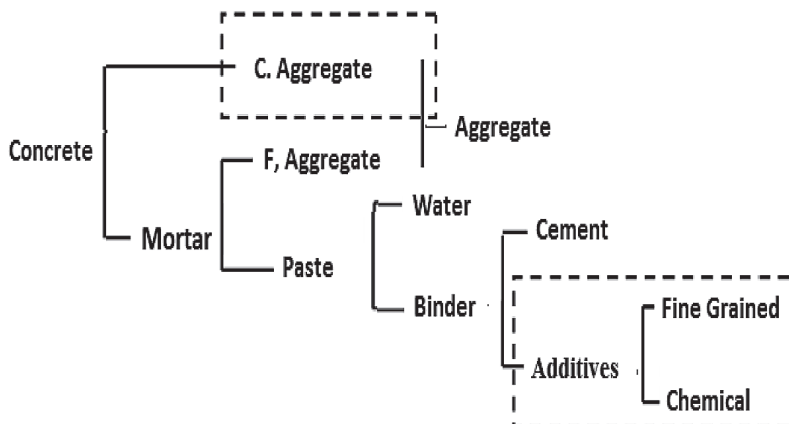


Fig. 1. Cement Concrete Mixture Matrix Structure

1.3. Cement Concrete Mixture Performance Characterization

Non-optimized (under-designed or over-designed) HPC mixtures are almost always uneconomical and inefficient [11, 12]. Efficient HPC mixtures with cement content balanced to achieve the desired performance while minimizing the risk of problems arising from high cement content (such as the obvious increased cost, the negative environmental effects [13], shrinkage and cracking problems) and reduced resource requirements in infrastructural applications can only be achieved with optimization of HPC mixtures on a quantitative basis [14]. The traditional method for optimizing HPC mixtures to achieve the desired performance, involving systematically varying individual mixture-factors in small increments and studying the resultant effect, is time-consuming, requiring multi-trial batches and hence expensive and inefficient. In the traditional method, the basis for selecting SCM dosage is often arbitrary and often focuses on a specific set of requirements such as strength or durability but as the demand for large volumes of concrete and faster speeds of construction grows and budgets grow tighter, greater attention is beginning to be paid in designing mixtures that are more efficient in their usage of materials without compromising engineering performance through optimisation of mixtures that makes it easy to take full advantage of the wide range of material and construction combinations and options not obtainable under non-optimized, high-energy and resource-consuming HPC mixtures. Optimized design of HPC mixtures involves consideration of more varied material constituents (and potentially more interactions among the material constituents), all optimized to determine the most economical and practical mixture-material constituent quantities to produce concrete of desired fresh-state properties (workability, pumpability, finishability, and consistency) and required hardened state properties (strength and durability-related properties such as water-tightness, wear and sulphate resistance, etc) consistent with particular conditions of use. Current mixture design of HPC production focuses on optimizing mixture properties (workability, strength and durability) of concrete in fresh and hardened states by optimizing the particle packing density of the granular ingredients of HPC as the accepted key mixture design consideration although the most common industry practice is to simply modify, by trial and error, existing HPC mixture recipes [12, 15, 16]. Sabir [17] has defined HPCs as cement concrete mixtures in which each granular ingredient performs effectively to contribute towards the HPC mixture's fresh and hardened state properties. The focus of this work is the quantification of the contribution of each HPC mixture-factor towards an HPC mixture's fresh and hardened state properties and the optimization of these contributions to allow optimal tailoring of HPC mixture requirements to HPC mixture performance to achieve efficiency and economy in HPC mixture design using cold-cast HPC mixture test-performance data sets available in cement concrete mixture research literature but also to allow modification or customization of available high performance concrete (HPC) formulations to meet specific infrastructural applications.

2. Methods

2.1. Parameterized Mixture-Strength and Mixture-Porosity Response Models

One-hundred thirty-four (134) sets of test-data and test-results for HPC mixtures from available cement concrete mixture research literature were employed to construct parameterized mixture-strength and mixture-porosity response models following the determination of model rate-parameters S_o , S_p , S_i and P_o , P_p , P_i . Cement concrete mixture-composition

optimization models were then built using the developed parameterized mixture-strength and mixture-porosity response models and constructed mixture-factor envelopes.

2.2. Proposed Parameterized Mixture-Strength and Mixture-Porosity Response Models

Porosity and compressive strength of cement concrete mixtures are interconnected and directly influence the failure behaviour of cement concretes. The time-dependent mechanical behaviour of concrete mixtures are characterized by an inner interval with early accelerated responses and a later outer interval with responses stabilized to a constant rate of increase which suggests a multiple-time scale problem and amenable to matching-approximations analysis [18]. The following uniform composite approximations are offered for the general response models for estimating concrete mixture-strength response S_t and concrete mixture-porosity response P_t after t days of curing:

$$S_t = [S_o + S_p t][1 - e^{-(S_i/S_o)t}]$$

$$P_t = [P_o - P_p t]/[1 - e^{-(P_i/P_o)t^{1/2}}]$$

where: S_o , S_p , S_i and P_o , P_p , P_i are mixture-strength and mixture-porosity response rate-parameters, respectively, that are functions of mixture-factors of the cement concrete mixture-matrix structure properties (see Fig. 1).

The proposed mixture-response models only require a determinations of the rate-parameters S_o , S_p , S_i and P_o , P_p , P_i from available test data or their evaluation through response prediction models based mixture-factors screened from the cement concrete mixture-matrix structure properties.

2.3 Proposed Mixture-Response Rate-Parameter Models

The aggregative effect of the enormous number of cement concrete mixture-constituents and properties contributing to the mixture-strength and mixture-porosity (and other desired concrete attributes) can be captured through the three rate-parameters as follows:

$$S_o = \prod_{j=1}^m \pi_j^{\alpha_{oj}}; \quad S_p = \prod_{j=1}^m \pi_j^{\alpha_{pj}}; \quad S_i = \prod_{j=1}^m \pi_j^{\alpha_{ij}}$$

$$P_o = \prod_{j=1}^m \pi_j^{\beta_{oj}}; \quad P_p = \prod_{j=1}^m \pi_j^{\beta_{pj}}; \quad P_i = \prod_{j=1}^m \pi_j^{\beta_{ij}}$$

where m is the number of independent parameterized mixture variables considered, π_j is the j^{th} independent parameterized mixture variable; α_{oj} , α_{pj} , α_{ij} and β_{oj} , β_{pj} , β_{ij} are the exponents to be determined for strength and porosity responses, respectively, through regression analysis. Independent parameterized mixture variables screened from the cement concrete mixture-matrix structure (see Fig. 1) include:

$$\pi_1 \text{ RHA surface area factor, } \frac{100 + \% \text{ rhas}_A}{100}$$

$$\pi_2 \text{ water content factor, kg/cu. m}$$

$$\pi_3 \text{ cement content factor, kg/cu. m}$$

- π_4 silica-fume factor, $\frac{100 + \%sf}{100}$
- π_5 fga factor, $\frac{100 + \% fga/[w/b + 1]}{100}$
- π_6 super-plasticizer content factor, kg/cu. m
- π_7 coarse aggregate-to-binder ratio
- π_8 water-to-binder ratio factor, $\frac{w}{b}$
- π_9 entrapped air factor, $\frac{100 + \% \text{ Entrapped Air}}{100}$
- π_{10} mixture-slump factor, mm
- π_{11} sand percentage in the mixture-aggregate, %
- π_{12} sand packing factor, $\frac{\alpha_{\text{cement}} + \alpha_{\text{sand}}}{\alpha_{\text{cement}}}$
- π_{13} RHA packing factor, $\frac{\alpha_{\text{cement}} + \alpha_{\text{RHA}}}{\alpha_{\text{cement}}}$
- π_{14} silica-fume packing factor, $\frac{\alpha_{\text{cement}} + \alpha_{\text{SF}}}{\alpha_{\text{cement}}}$

2.4. Mixture-Response Model Calibrations

HPC mixture test-data from tests by Azizinamini [19], Domone and Soutsos [20] and Nguyen [21] were used to calibrate the proposed models. MATLAB®'s non-linear least-squares (nlinfit) regression analysis and MICROSOFT EXCEL®'s linear least squares (linest) regression analysis programmes were employed to fit concrete test-data to the proposed parameterized mixture-strength and mixture-porosity response models and to perform rate-parametric analyses to obtain model coefficients, respectively, as follows:

Mixture-Strength Response Rate-Parameters

$$S_o = \pi_1^{0.06} \pi_2^{0.11} \pi_3^{0.77} \pi_4^{1.81} \pi_5^{0.39} \pi_6^{0.00} \pi_7^{1.06} \pi_8^{-0.61} \pi_9^{-3.28} \pi_{10}^{0.29} \pi_{11}^{0.23}$$

$$S_p = \pi_1^{-0.03} \pi_2^{-0.51} \pi_3^{-1.16} \pi_4^{-4.48} \pi_5^{2.56} \pi_6^{-0.03} \pi_7^{-2.06} \pi_8^{-1.55} \pi_9^{3.91} \pi_{10}^{-1.49} \pi_{11}^{1.32}$$

$$S_i = \pi_1^{0.02} \pi_2^{0.41} \pi_3^{-0.23} \pi_4^{-2.85} \pi_5^{0.43} \pi_6^{-0.02} \pi_7^{0.86} \pi_8^{-2.06} \pi_9^{2.15} \pi_{10}^{-0.79} \pi_{11}^{1.27}$$

Mixture-Porosity Response Rate-Parameters

$$P_o = \pi_3^{0.76} \pi_4^{-1.33} \pi_6^{-4.87} \pi_8^{-0.04} \pi_{11}^{0.01} \pi_{12}^{-0.32} \pi_{13}^{1.03} \pi_{14}^{1.48}$$

$$P_p = \pi_3^{28.7} \pi_4^{-29.5} \pi_6^{-31.1} \pi_8^{13.9} \pi_{11}^{-3.66} \pi_{12}^{-31.1} \pi_{13}^{12.2} \pi_{14}^{25.9}$$

$$P_i = \pi_3^{2.11} \pi_4^{-2.75} \pi_6^{-11.9} \pi_8^{-0.03} \pi_{11}^{0.14} \pi_{12}^{-2.26} \pi_{13}^{1.47} \pi_{14}^{1.7}$$

Predicted results using the developed parameterized mixture-strength response and mixture-porosity response models and those predicted by mixture-strength response models proposed by Sarkar et al. [22] and Rajasekaran [23] are shown in Table 1, Table 2, Table 3 and Table 4.

Table 1. Mixture-data, test- and model-results for high strength HPC mixtures

Source	Mixture ID	$\frac{w}{b}$	c. aggr. kg/cu. m	water kg/cu. m	cement kg/cu. m	fa kg/cu. m	sf kg/cu. m	Sp kg/cu.m	Entra-ined Air %	slump mm	*Test Result MPa	Model Result MPa
	MS1‡	0.228	655.6	137.9	484.4	85.7	33.9	15.1	2.3	260	68.3	76.6
	MS3‡	0.241	623.6	163.0	550.2	87.0	38.6	10.1	1.5	267	64.1	71.9
	MS13‡	0.282	625.9	150.5	449.8	52.9	31.4	13.4	7.0	267	66.0	59.2
	MS12‡	0.248	596.9	170.3	537.9	85.1	64.7	10.3	2.1	279	71.7	69.9
	T1	0.357	685.9	155.1	328.5	82.2	23.1	2.7	5.4	89	51.4	56.4
	T3	0.265	668.1	130.1	382.6	82.1	26.6	7.4	7.4	267	67.1	63.6
	T4	0.269	747.0	112.5	338.8	56.4	23.9	6.3	4.5	222	70.7	74.6
	T5‡	0.359	680.5	169.6	390.2	55.7	27.1	3.0	2.2	152	58.2	57.3
	T6	0.294	697.1	134.0	360.7	69.5	25.5	4.8	5.4	140	64.8	62.2
	T7‡	0.400	709.0	163.0	329.6	54.9	23.3	2.5	3.5	140	50.4	53.6
	T8	0.309	686.4	138.8	355.3	68.4	25.1	4.8	6.4	178	60.7	58.5
	T10‡	0.280	675.8	116.3	314.9	78.8	22.2	6.2	10.3	235	58.1	58.2
	T11‡	0.364	646.7	178.8	382.1	82.1	26.5	3.1	5.1	114	51.4	51.5
Azizinamini [19]	T12	0.301	666.9	131.2	345.1	66.4	24.4	4.6	8.5	229	61.4	57.0
	T14‡	0.293	723.8	139.1	374.6	72.1	28.0	5.0	6.7	216	66.2	63.5
	T15‡	0.300	650.9	133.3	341.9	79.1	24.1	4.7	10.0	229	55.6	54.1
	T17	0.317	716.7	135.5	333.9	69.6	23.6	4.5	5.1	191	59.8	61.3
	T18	0.345	672.8	154.0	353.4	68.1	25.0	2.8	6.6	165	52.7	53.0
	MS2	0.229	656.8	144.1	485.7	85.9	58.1	9.4	1.8	267	83.8	79.9
	MS4	0.233	677.5	134.9	486.9	57.3	34.1	8.7	2.7	229	82.6	76.7
	MS5	0.231	644.3	147.9	518.2	72.0	49.2	12.8	1.9	267	77.6	77.3
	MS6	0.229	644.9	147.0	546.5	57.4	38.3	16.1	NR	254	71.4	-
	MS7	0.226	662.7	136.7	488.2	57.4	58.4	15.1	2.5	267	88.0	81.4
	MS8	0.223	631.3	151.1	551.9	58.0	66.4	10.1	1.9	241	84.1	78.2
	MS9	0.219	606.4	152.9	546.8	86.5	65.7	17.5	2.8	267	83.9	75.2
	MS10	0.234	640.8	148.6	515.5	71.7	47.8	12.7	2.3	267	81.5	75.0
	MS11	0.232	620.6	154.4	542.5	57.0	65.3	16.6	2.3	267	80.6	73.9
	MS14	0.229	662.7	138.2	488.5	57.4	58.5	9.1	2.3	229	86.9	78.5
	MS15	0.212	653.8	137.6	525.8	73.1	50.0	13.0	2.3	229	83.6	82.3
	MS16	0.221	655.0	144.4	555.1	58.3	38.9	9.8	2.3	254	88.1	78.8

Table 1. (continued)

Source	Mixture ID	$\frac{w}{b}$	c. aggr. kg/cu. m	water kg/cu. m	Cement kg/cu. m	fa kg/cu. m	Sf kg/cu. m	sp kg/cu. m	Entra-ined Air %	slump mm	*Test Result MPa	Model Result MPa
	MS17	0.211	630.7	142.7	551.2	87.2	38.6	16.9	2.3	267	78.5	77.6
	MS18	0.242	658.0	146.7	486.5	86.1	34.0	9.1	2.3	254	81.6	73.7
	MS19	0.233	627.7	144.2	476.8	84.4	57.1	15.5	2.3	267	76.5	72.1
	MS20	0.206	655.6	134.0	527.6	73.3	50.1	13.0	2.3	254	78.9	83.0
	MS21	0.224	642.0	145.1	512.0	87.5	49.8	13.2	2.3	248	79.5	76.6
	MS22	0.206	650.3	133.7	523.0	77.9	49.7	9.8	2.3	254	84.5	83.1
	MS23	0.238	661.5	144.3	488.2	71.9	46.6	12.1	2.3	254	79.4	77.0
	MS24	0.230	639.6	146.1	514.7	71.6	48.9	15.9	2.3	248	78.4	74.5
	MS25	0.216	641.4	138.2	519.6	71.7	49.0	12.7	2.3	254	83.4	78.9
	MS26	0.221	655.6	138.2	519.4	57.7	49.4	12.5	NR	229	84.8	-
	MS27	0.225	651.4	140.7	516.4	71.8	36.1	12.5	2.3	254	84.6	77.0
	MS28	0.223	630.1	150.3	550.6	72.5	52.2	13.5	2.3	254	76.9	76.5
	MS29	0.244	632.5	158.4	516.1	71.7	61.9	13.0	2.3	254	84.4	72.9
	MS30	0.214	651.4	138.2	523.9	72.9	49.8	12.9	2.3	229	89.0	79.6
	T2‡	0.306	706.6	141.3	365.7	70.4	25.8	4.9	2.3	152	67.9	63.3
	T6	0.294	697.1	134.0	360.7	69.5	25.5	4.8	2.3	140	64.8	62.2
	T9	0.264	709.0	126.4	394.7	56.3	27.4	7.2	2.3	229	75.2	71.7
	T13	0.276	686.4	135.5	393.4	70.3	27.3	5.2	2.3	171	71.8	64.7
	T16	0.302	711.4	138.4	363.3	70.0	25.7	6.9	2.3	203	76.5	64.3
	T19	0.267	712.6	124.2	368.9	71.0	26.0	5.0	2.3	114	76.3	70.2
	T20	0.289	716.7	129.2	365.6	56.2	25.8	4.8	2.3	203	70.2	68.1

Note: 1.0 lb/cu. yd = 0.5933 kg/m³ 1.0 in = 25.4 mm

1.0 psi = 0.006895 N/mm² 1.0 oz/cu. yd = 0.03708 kg/m³

‡ mixture used in constructing rate-parameter model sand content in aggregate: 54.5% w/b: water-to-binder ratio

c. aggr: coarse-aggregate fa: fly-ash sf: silica-fume sp: super-plasticizer NR:

Not Recorded *28-day mix-compressive strength

Table 2. Mixture-data, test- and model-results for higher strength HPC mixtures

Source	Mixture ID	w/b	binder kg/cu. m	water kg/cu. m	†% cement	†% fa	†% sf	†% ggbs	†% sp	slump mm	*Test Result MPa	Model Result MPa	SAJ Model Result MPa	SLNN Model Result MPa
Domone and Soutsos [20]	H2‡	0.32	454	145.3	100				0.80		90.0	91.0		
	K4	0.32	454	143.5	95		5		0.89		94.5	95.3	90.1	113.5
	K5	0.32	454	143.5	90		10		0.97		106.0	99.3	91.3	117.3
	K6‡	0.32	454	143.5	85		15		1.05		107.5	102.7	95.0	120.0
	K7	0.29	492	142.7	95		5		1.08		99.5	99.2	95.1	119.0
	K8	0.29	492	142.7	90		10		1.08		114.5	103.4	96.5	122.4
	K9	0.29	492	142.7	85		15		1.08		118.5	107.2	100.3	124.5
	K10	0.26	510	132.6	95		5		1.32		110.0	104.6	100.2	122.1
	K11	0.26	510	132.6	90		10		1.22		113.5	109.2	101.7	124.5
	K12	0.26	510	132.6	85		15		1.12		115.0	113.4	105.7	125.5
	K13‡	0.23	547	125.8	95		5		1.67		111.5	110.5	105.4	128.3
	K14	0.23	547	125.8	90		10		1.67	150	116.0	115.6	107.3	132.2
	K15‡	0.23	547	125.8	85		15		1.67		125.0	120.2	111.8	134.8
	H3	0.29	492	142.7	100				1.50		96.5	94.3		119.2
	H4	0.26	510	132.6	100				2.33		110.0	99.6		128.6
	H5‡	0.23	547	125.8	100				0.46		106.5	105.1		109.5
	L3‡	0.26	510	132.6	38		5	57	0.63		93.5	96.2		
	L4	0.26	510	132.6	36		10	54	1.22		94.0	96.1		
	L5‡	0.26	510	132.6	54	36	10		1.22		90.0	110.0		
	L6‡	0.20	590	118.0	90		10		2.22		118.0	123.4	113.2	141.1
	L7‡	0.20	590	118.0	40		10	50	2.22		113.5	116.5		
	L8‡	0.20	590	118.0	60		10	30	2.22		115.0	129.4		
	L9	0.20	590	118.0	60	30	10		2.22		103.5	116.5		
	I8‡	0.26	510	132.6	80	20			0.80		94.0	108.3		
	J4‡	0.32	454	145.3	90		10		0.50		90.0	95.3	90.1	112.1
	J7	0.26	510	132.6	90		10		0.97		105.0	104.9	95.1	121.5
	J8‡	0.26	510	132.6	70		30		0.74		105.0	109.5	101.7	119.2

Note: 1.0 lb/cu. yd = 0.5933 kg/m³ 1.0 in = 25.4 mm 1.0 psi = 0.006895 N/mm³ 1.0 oz/cu.

yd = 0.03708 kg/m³ coarse aggregate = 1115 kg/m³

fine aggregate (sand) = 670 kg/m³ ‡mixture used in constructing rate-parameter model SAJ:

Sarkar et. al [22] SLNN Rajasekaran [23]

w/b: water-to-binder ratio c. aggr: coarse-aggregate fa: fly-ash sf: silica-fume ggbs: ground-granulated blast-furnance slag sp: super-plasticizer sand content in aggregate: 37.5% †%: as a percentage of binder *28-day mix-compressive strength

Table 3. Mixture-data, test- and model-results for ultra-high strength HPC mixtures

Source	Mixture ID	$\frac{w}{b}$	binder kg/cu. m	water kg/cu. m	cement kg/cu. m	††%rha	††%sf	†%sp	slump mm	*Test Result MPa	Model Result MPa
	‡SF20		1062	159.3	885		20	0.76		168	169
	SF15RHA5	0.18	1062	159.3	885	5	15	1.15	210-230	174	170
	SF10RHA10		1062	159.3	885	10	10	1.15		184	168
	SF5RHA15		1062	159.3	885	15	5	1.15		176	165
	‡RHA20(3.6)		1062	159.3	885	20		1.15		176	162
	RHA20(5.6)		1062	159.3	885	20		1.75		132	163
	‡RHA20(6.3)		1062	159.3	885	20		1.20		156	162
	RHA20(9.0)		1062	159.3	885	20		1.15		174	162
	RHA20(5.6)		1062	159.3	885	20		0.89		180	162
Nguyen [21]	‡REF		1140	205.3	1140			0.90		162	155
	‡SF20		1062	159.3	885		20	0.76		164	169
	‡REF		1140	205.2	1140			0.90		163	155
	RHA10(5.6)		1110	200.0	1110	10		1.15		170	165
	SF10		1110	181.8	885		10	0.76		163	165
	SF20		1062	191.2	885		20	0.76		164	169
	‡RHA20(5.6)		1062	137.7	885	20		1.15		174	164
	SF30		995	191.2	765		30	0.76		142	168
	‡SF10		1110	137.7	1010		10	0.76		170	164
	SF10RHA10(5.6)		1062	181.8	885	10	10	1.15		185	167
	SF10RHA20(5.6)		995	137.7	765	20	10	1.15		166	161
	‡SF10RHA30(5.6)		903	116.1	645	30	10	1.15		154	151

Note:SF(A)RHA(B)(C): A: %sf B: %rha C: rha grain size 1.0 lb/cu. yd = 0.5933 kg/m³ 1.0 in = 25.4 mm 1.0 psi = 0.006895N/mm²

1.0 oz/cu. yd = 0.03708 kg/m³ ‡mixture used in constructing rate-parameter model

rha surface area \approx 3x(sf surface area) \approx 62x(cement surface area) w/b: water-to-binder ratio rha: rice-husk ash sf: silica-fume sp: super-plasticizer sand content in aggregate: 100% †%: as a percentage of binder ††%: as a percentage of cement *28-day mix-compressive strength

Table 4. Mixture-Data, Test- and Parameterized Mixture-Porosity Model-Results for Ultra-High Strength HPC Mixtures

Source	Mixture ID	Sand%	†%w/b	Cement				†Packing Density, α				‡Test Result %	Model Result %	
				kg/cu. m	†%sf	†%sp	†%RHA	Cement	Sand	RHA	SF			
	REF	100	18.0	1140.0	0.0	0.9	0.0	0.399				7.5	8.67	
	RHA20	100	18.0	885.0	0.0	1.15	20.0	0.399	0.478	0.364		5.76	5.87	
	SF20	100	18.0	885.0	20.0	0.76	0.0	0.399				0.64	4.55	4.56
	S4	0	25.0	1140.0	0.0	0.8	0.0	0.399					10.61	9.18
	S5	0	25.0	1076.3	0.0	0.8	5.0	0.399					10.87	8.90
	S6	0	25.0	1012.5	0.0	0.8	10.0	0.399					11.68	8.91
	S7	0	25.0	885.0	0.0	0.8	20.0	0.399	0.364			8.95	8.99	
Nguyen [21]	S8	0	25.0	1076.3	5.0	0.8	0.0	0.399					7.52	6.86
	S9	0	25.0	1012.5	10.0	0.8	0.0	0.399					6.23	5.22
	S10	0	25.0	885.0	20.0	0.8	0.0	0.399					4.53	2.94
	S11	0	40.0	1140.0	0.0	0.8	0.0	0.399					17.46	16.98
	S12	0	40.0	1076.3	0.0	0.8	5.0	0.399					16.18	18.88
	S13	0	40.0	1012.5	0.0	0.8	10.0	0.399					16.7	18.92
	S14	0	40.0	885.0	0.0	0.8	20.0	0.399					20.93	19.13
	S15	0	40.0	1076.3	5.0	0.8	0.0	0.399				0.64	13.27	14.81
	S16	0	40.0	1012.5	10.0	0.8	0.0	0.399					12.91	11.53
	S17	0	40.0	885.0	20.0	0.8	0.0	0.399					13.26	6.81

†Packing density, α (based on LPM—Linear Packing Model [24]) ‡Total porosity of samples measured by mercury intrusion porosimetry (MIP) at 28 days [24]

2.5. Proposed Parameterized Mixture Optimization Algorithm

Using the developed parameterized mixture-strength and mixture-porosity response models and an optimization scheme based on a linearly weighted least-squares algorithm, mixture optimization models were constructed for mixtures investigated in the study as follows:

Defining a linearly weighted summated mixture response (WSMR) function

$$R_i = \sum_{j=1,1}^N w_j \bar{R}_{ji}$$

with individual normalized mixture response functions

$$\bar{R}_i = \frac{1}{N} \frac{f_i(a)}{f_{i(\text{ref})}(a)}$$

where are maxima (or minima) of mixture response trace plot (RTP) or mixture-response sensitivity analysis functions [25] shown in Fig. 2, Fig. 3, Fig. 4 and Fig. 5 to facilitate determination of the N mixture-factor weights, , provided and by minimizing the squared sum of deviations between the weighted-values and the target value, T

$$\sum_{i=1}^N [w_i \bar{R}_i - T]^2 \rightarrow \min.$$

Differentiating the squared sum of deviations with respect to corresponding weights, the following system of equations is obtained:

$$\begin{bmatrix} 2(\bar{R}_1)^2 & 0 & \dots & \dots & 0 & 0 \\ 0 & 2(\bar{R}_2)^2 & \dots & \dots & \text{symm} & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots & 2(\bar{R}_{N-1})^2 & 0 \\ 0 & 0 & \dots & \dots & 0 & 2(\bar{R}_N)^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \dots \\ \dots \\ \dots \\ w_{N-1} \\ w_N \end{bmatrix} = \begin{bmatrix} 2TR_1 \\ 2TR_2 \\ \dots \\ \dots \\ 2TR_{N-1} \\ 2TR_N \end{bmatrix}$$

that facilitates the determination of values of the mixture-factor weights, w_i , and their envelope standard deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^N [w_i \bar{R}_i - T]^2}{N-1}}$$

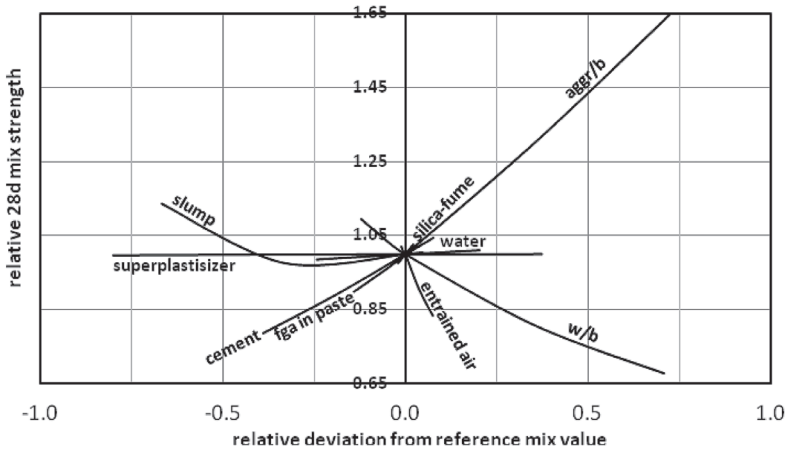


Fig. 2. Mixture-strength response trace plots of mixture M22

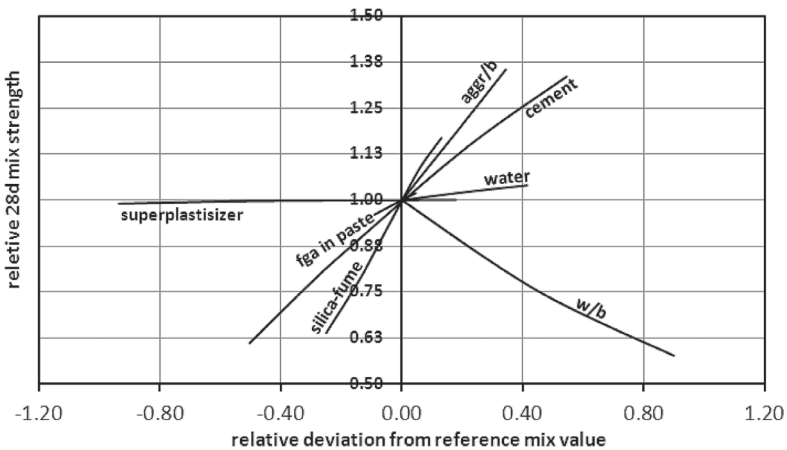


Fig. 3. Mixture-strength response trace plots of mixture L8

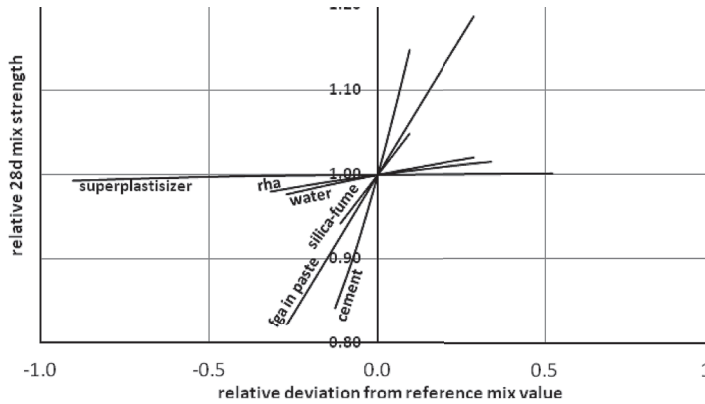


Fig. 4. Mixture-strength response trace plots of mixture RHA(5.6)

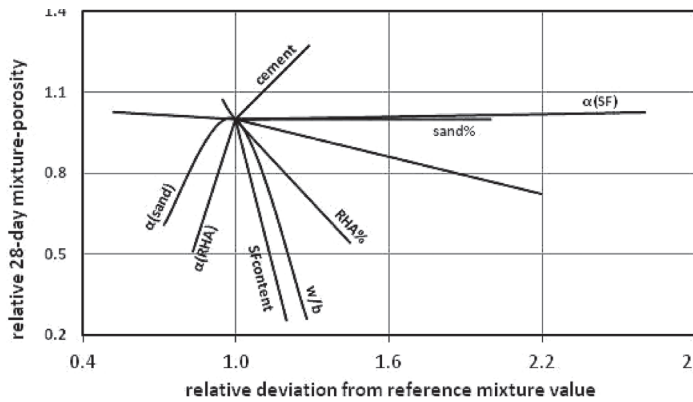


Fig. 5. Mixture-porosity response trace plots for mixture S7

The constructed mixture-factor envelopes for concrete mixtures investigated in the study are presented in Fig. 6.

The constructed mixture-factor envelopes readily yield optimized mixture-compositions and corresponding optimized mixture-responses S_{target} and P_{target} by simply interpolating between upper and lower bound values of mixture-factor weights and mixture-factor values (for a known mixture- response S_{ref} and P_{ref} of a known concrete mixture as follows:

For optimized mixture-compressive strength responses

$$S_{target} = S_{ref} \left\{ 1 + \left\{ \left[\frac{\pi_i - \pi_i^{LB}}{\pi_i^{UB} - \pi_i^{LB}} \right] \left| \frac{\sum_{i=1}^N |w_i^{UB} - w_i^{LB}|}{N} \right| \right\} \right\}$$

For optimized mixture-total porosity responses

$$P_{target} = P_{ref} \left\{ 1 - \left\{ \left[\frac{\pi_i - \pi_i^{LB}}{\pi_i^{UB} - \pi_i^{LB}} \right] \left| \frac{N}{\sum_{i=1}^N |w_i^{UB} - w_i^{LB}|} \right| \right\}^3 \right\}$$

Predicted optimized mixture-compositions and corresponding optimized mixture responses are summarized in Table 5.

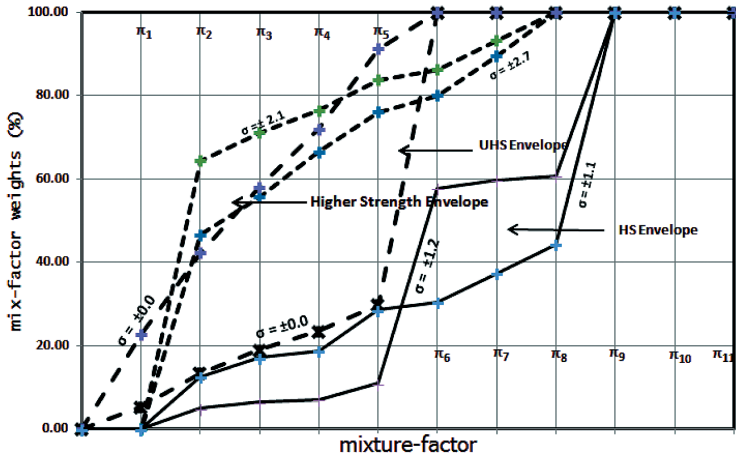


Fig. 6. Mixture-factor envelopes for studied mixture property

Table 5. Predicted optimized mixture composition and parameterized mixture-strength for HPC mixtures

Reference Mixture ID	Optimized Mixture factor	Sand %	w/b	c.aggr kg/cu. m	water kg/cu. m	Cement kg/cu. m	fa kg/cu. m	Sf kg/cu. m	sp kg/cu.m	rha kg/cu.m	Entrained Air %	slump mm	PMS Model Result MPa	MFE Model Result MPa	
MS5			0.219	647.1	150.4	541.9	82.2	62.7	15.4		1.8	267	81.3	87.1	
MS8			0.230	627.5	153.8	525.1	82.3	62.3	12.2		1.8	241	81.6	88.1	
MS18	fga	54.5	0.228	651.2	150.4	513.1	82.4	62.9	11.0		2.2	254	78.7	83.0	
T3			0.248	674.9	134.5	406.8	83.6	52.9	9.0		7.1	267	69.7	71.6	
T11			0.340	643.0	182.6	401.3	82.6	52.7	3.9		4.8	114	53.9	58.0	
T20			0.255	703.7	130.9	381.1	82.0	52.3	5.7		3.9	203	71.9	76.7	
H2			0.285	1081.8	148.1	453.5	28.6	37.8	3.3				100.0	98.9	
H4			0.245	1088.6	138.0	520.0		43.2	7.2				106.1	108.2	
K14	C. aggr	37.5	0.213	1083.3	128.7	493.4		111.6	8.3			150	122.8	125.6	
K15			0.210	1080.3	127.7	462.3		144.6	8.2				127.2	130.6	
L5			0.250	1090.1	138.5	281.7	162.9	108.2	4.6				107.6	119.5	
SF20				1140.0	171.0	950.0		190.0	8.7				183.4	190.3	
SF10	C. aggr	100.0	0.18	1140.0	186.5	1036.4		103.6	8.7			220	171.2	185.8	
RHA20(5.6)				1140.0	171.0	950.0		-	13.1	190.0				172.0	184.7
SF10RHA10(5.6)				1140.0	171.0	950.0		95.0	13.1	95.0				177.9	188.1
K15	Cement	37.5	0.203	1139.1	116.1	495.0		75.7	8.2			150	128.8	133.3	
L5			0.240	1131.0	126.5	303.0	175.2	27.5	4.7				118.1	114.8	
MS8	Cement	54.5	0.222	654.6	146.3	537.3	56.3		9.8		1.9	241	79.7	87.3	
SF10RHA10	Sf	100.0	0.18	1140.0	171.0	950.0		95.0	13.1	95.0		220	177.9	173.8	
	Sp			1140.0	171.0	950.0		95.0	13.1	95.0				177.9	178.6
	Water		0.285	650.8	134.2	366.8	78.7	25.5	7.1		7.1		59.7	66.2	
	Cement		0.244	646.1	123.0	400.4	77.7	25.2	7.0		7.0		65.7	66.5	
T3	Sf	54.5	0.252	708.2	123.0	362.7	77.8	48.0	7.0		7.0	267	72.7	68.3	
	Fa			606.4	112.5	314.9	52.9	22.2	2.5		1.5		67.8	63.6	
	Sp			671.0	129.7	381.4	81.9	26.5	9.4		7.4		62.4	67.3	

3. Results and Discussion

Results predicted by the proposed parameterized mixture-strength response and mixture-porosity response models are compared with mixture-test results and results of available models in Table 1, Table 2, Table 3 and Table 4. As can be observed, the proposed parameterized mixture-response models yield reasonably good results for most of the mixtures studied over the study domain (with statistical performance metrics for an unbiased estimate of the prediction ability of the models of mean absolute percentage error MAPE, normalized mean bias error NMBE and root mean square error RMSE values of 6.7%, -3.8% and 9.4MPa, respectively).

The computed mixture-factor weights and constructed mixture-factor envelopes were used to predict optimized mixture-compositions, with mixture-factors limited to the ranges in the study domain, and these were in turn used to predict optimized mixture-response(s) of interest. Optimized mixture-strength response results, presented in Table 5, indicate maximum performance of 8.7%, 12.6% and 12.6% over parent mixture-strengths of VHS, HS and UHS HPC mixtures recipes, respectively. HS concrete mixtures have the least mixture-factor envelope-area (see Fig. 6) and hence respond the least to mixture-composition optimization efforts. Optimized mixture-strength responses predicted by the mixture-factor envelope (MFE) model compare favourably with those obtained by the parameterized mixture-strength (PMS) response model (with statistical performance metrics for an unbiased estimate of the prediction ability of the models of mean absolute percentage error MAPE, normalized mean bias error NMBE and root mean square error RMSE values of 7.6%, -3.7% and 6.5MPa, respectively) and although the results for the parameterized mixture-porosity (PMP) response model are not provided, it has a lean mixture-factor envelope-area (see Fig. 6) and hence responds the least to mixture-porosity based mixture-composition optimization efforts.

The optimal values of mixture-factors for fly-ash and silica-fume yield more strength-efficient T3 mixture while optimizing UHS (high-energy and resource-consuming) HPC mixtures through graded aggregates yielded strength-efficient and cement-efficient mixtures.

HPC mixtures can be similarly optimized for workability (through the mix-slump factor) and durability (through the mix entrained-air-content factor). Attempts at modelling optimization of HPC mixture-compositions in the larger mixture-response modelling research community—even with the non-traditional advanced machine learning optimization approaches like sequential learning neural network (SLNN) or neuro-fuzzy computing techniques such as the adaptive neuro-fuzzy inference system (ANFIS)—have thus far only yielded, at best, qualitative characterizations of concrete performance or mixture optimization [26, 27].

It is acknowledged that these models are derived from laboratory-test strengths and therefore their application to field cement concrete mixtures (in-situ concretes) suspect or uncertain. Experience has, however, shown that a high percentage (up to 90%) of laboratory test strengths are attainable in-situ concrete under good field practices [28]. HPCs have already moved (transitioned) from laboratory research to industrial implementations (practical applications) and already occupy a sizeable share of the market although most of these applications have been limited to proprietary blends and non-in situ construction (commercial ready mix products and pre-cast applications) and even convenience blends but are in general more expensive (by an order of upwards of twenty) than non-proprietary conventional cement concretes mainly owing to proprietary specifications of mixture proportions of non-proprietary HPC mixtures usually being based on trial and error methods than any settled material/behavioural laws or some quantitative characterization of its performance [29, 30].

4. Conclusions

Contributions to mixture-performance by identified mixture-factors were quantitatively determined through models developed in the study which allowed optimal tailoring of mixture requirements to mixture performance of HPCs by explicitly relating performance (user) specifications to mixture (producer) requirements and make possible optimized trade-offs between them where the three main performance specifications—strength, workability, and durability variously specified (the strength, through the desired compressive strength; workability, through the desired slump; and durability, through some given exposure condition)—can be explicitly achieved through variously specified mixture requirements (compressive strength, via some water/cement ratio; workability, via some indication of water content per unit volume of concrete; and durability, via some indication of some minimum cement content and maximum water/cement ratio). The study findings suggest the performance of a known mixture (its strength, workability, and durability) can be improved by a determinable amount and an optimized mixture-composition reliably determined through mixture-factor envelopes largely by increasing the binder content of the mixture and/or the graded aggregate content of the mixture. Developed models were reliably used to optimally modify or customize available HPC formulations to yield alternative mixture compositions with improved performance and efficiency characteristics (eliminating the cost-prohibitive need for undertaking multiple trials) with statistical performance metrics MAPE, NMBE and RMSE values of 7.6%, -3.7% and 6.5MPa, respectively.

Although the study considered only two mixture-performance parameters, compressive strength and porosity, the study approach can be applied for other mixture-performance parameters (tensile strength, shrinkage and other mixture-properties attributable to high performance concretes).

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Statement of Competing Interests

The author has no competing interests relating to this work.

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