Unconventional building forms roofed with innovative structures arranged on regular surfaces with the negative Gaussian curvature

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Abstract: A novel method for shaping innovative building forms, roofed with diversified complex continuous and discontinuous folded structures composed of many transformed corrugated shell units, is presented in the paper. The units are defined on the basis of specific reference polyhedral networks and arranged on arbitrary reference surfaces characterized by the negative Gaussian curvature. The method is presented using several computer models of complex building structures with folded plane-walled elevations. The proposed method significantly supplements the previous method developed for modelling building free forms, roofed with shell structures arranged in conformity with surfaces having the positive Gaussian curvature. Some basic rules using parameterization and governing the creation of the multi-plane elevations, ribbed continuous and discontinuous roof shell structures, arranged in different unconventional and visually attractive patterns, were developed. The elaborated specific sets of division coefficients are taken as parameters for the designed building structures. These sets determine unconventional polyhedral networks, which are composed of several specific sets that allow to define a polygonal eaves network, a reference surface and, finally, individual shell units of the roof structures. The developed method is presented using the example of three novel forms defined by means of the appropriately selected diversified sets of values of the division coefficients. The elaborated new forms confirm the innovative nature of the achieved results. By imposing appropriate proportions between the values of these division coefficients, the developed method enables the creation of two different groups continuous and discontinuous complex shell roof structures.

Keywords: novel building form, complex shell roof, parametric modelling, nominally flat thin-walled folded sheeting, multi-plane elevation.
1. Introduction

Nominally, flat folded metal sheets tend to take many different attractive ruled shell shapes as a result of their spatial elastic transformations. This property results from their relatively low transverse bending and longitudinal torsional stiffness [1], [2], Fig.1. Some significant geometric and material limitations in the formation of the transformed corrugated steel sheeting cause difficulties for their use by designers to shape transformed roofs or elevations. The limitations often result in significant values of stresses and strains for the cases of ill-considered transformations and their high degrees [3]. The intentional spatial shape deformations are called initial transformations and have to be effective, that is, transversal freedom with increments of the transformed sheeting must be ensured by imposing special boundary conditions [4], [5].

![Complete shell unit composed of two nominally plane folded sheets connected with themselves along their longitudinal edges and transformed with two skew straight directrices. Source: own study](image)

When carrying out an analysis related to the geometric and mechanical properties of the plane corrugated sheeting transformed elastically into different roof shells, it is necessary to take into account the values of the initial bending, twisting, displacements, deformations, stresses, and strains diversified for all subsequent folds of all transformed sheeting [6]-[7]. The geometric and mechanical limitations of the sheets require composing multiple complete transformed shell sheeting units into a single edge structure to obtain medium- and large-spanned roofs [8].

The expected straightness of the longitudinal axes of all transformed shell folds and the longitudinal borders of all complete sheets, along with the whole shell sheeting after transforming, are other basic limitations in the use of the flat folded sheets. The developed models of the designed transformed shell units should be shaped in the form of various ruled surface sectors limited by spatial quadrangles with rectangular or similar corners [1], [8]. The more the corner angles differ from right angles, the greater the oblique cuts of the transverse edges of the transformed sheeting, passing parallel to the roof directrices supporting the sheeting.

The simplest way to model the complex building structures is to combine various polyhedrons into one folded, complex, polyhedral elevation structure roofed with special sums of many single parabolic-hyperbolic shell sectors. The edge lines of these sectors form closed spatial quadrangles and are contained in the planes of the polyhedra. Façade walls are also contained in these planes. To achieve a wide variety of the designed complex building forms, the façade elevation walls are folded and inclined to the vertical [8], Fig. 2.
To model a novel complex building structure, a polyhedral reference network needs to be defined [8]. The network should be shaped based on many quadruples of vertices, defining regular tetrahedral meshes. They are joined with corresponding common triangular sides to form a spatial structure that fills the three-dimensional space tightly. The facade walls of each complex form are contained in the planes of the created tetrahedral meshes.

Complete segments of closed spatial quadrangles, which define the edge lines of all single transformed roof shells, are contained within the planes mentioned above. This results in a novel polygonal eaves network, which is formed by the sum of all employed quadrangles. By parameterizing the essential dimensions of the reference polyhedral network and the eaves polygonal network, as well as their elements, the geometric properties of these networks can be diversified. This leads to the creation of complex building free forms characterized by multi-wall folded facades and multi-shell ribbed continuous or discontinuous roofs [8].

2. Critical analysis

Winter [9], Parker [10], Gergely et al. [11], Egger et al. [12] and many other researchers have demonstrated great theoretical possibilities in the search for various ruled forms of transformed folded sheeting, as well as their simple structures composed of open thin-walled corrugated sheeting covered with flat smooth sheets on both the top and bottom sides. However, the tests and analysis conducted by Bryan and Davis [13] indicate that several significant material and technological limitations greatly restrict the diversification of the transformation types and degrees. As a result, the achieved corrugated shell forms are reduced to shallow hyperbolic paraboloids known as hypars.

Biswa and Iffland [14] expanded the range of the designed complex, ribbed structures consisting of multiple identical transformed shell corrugated units. They developed two simple systems of congruent shell units distributed on a sphere using bundles of planes. In these systems, the complete transformed shells are constructed from revolved hyperboloids or right hyperbolic paraboloids confined by spatial straight quadrangles.

Abdel [15] highlighted the potential to broaden the range of single transformed folded sheeting to include cylindrical and conical surfaces through transverse bending of flat folded sheets. Abdel and Mungan [16] present various unconventional building forms and structural systems designed for these forms. Some of these shapes can be incorporated into a design process of transformed roof shell sheeting. Similarly, certain types of the structural systems outlined in the comprehensive classification provided by Saitoh [17] can also be used for shaping the transformed shell sheeting.
A novel method developed by Reichhart [7] significantly increases the diversity of creating unconventional roof shells and their structures. These structures are composed of nominally flat sheeting that is transformed into various unconventional folded shell shapes using a few rigid skew directrices. The wide range of the shell roof forms achieved by this method is a result of the properties of the thin-walled single-layer sheeting. This type of sheeting allows for bending, twisting, and a combination of both to adapt to the supporting conditions determined by the roof skew directrices.

Abramczyk [1] conducted tests on the effective bending, twisting and bend-twist transformations of a number of single folded sheets with different profiles, as well as their single-layer strips. An important discovery was made regarding the contraction of each effectively transformed folded sheet. The effective transformation of a folded sheet aims to minimize stresses and strains, which requires ensuring freedom in the transverse width increments of the sheet during the initial design step. This property is achieved by introducing a line of striction of a ruled surface that models a sheet. The striction line must run half along the length of each fold of the transformed sheeting, where its shell folds have different directions. The developed method facilitates the use of different ruled surfaces in the rational design of transformed folded shell roofs.

Walentyński et al. [18] has tested curvilinear thin-walled folded sheets and curved folded panels for creating shell roof sheeting. It was elaborated a method for creating mechanical computer models to simulate the mechanical performance of these panels.

Prokopska and Abramczyk [19] have developed a system for creating simple sets of reference tetrahedrons that can be used to model complete free forms with multi-plane folded oblique elevations. These free forms are roofed with transformed folded shell units, which are modelled using various quarters of hyperbolic paraboloids arranged symmetrically. The system enables one to achieve a variety of diversified free form configurations.

Abramczyk and Prokopska proposed a general parametric sector of a warped surface, [20], which is bounded by a closed spatial quadrangle with sides defined with a set of four planes. This system of many quadrangles of planes determines and divides each designed shell ribbed roof structure into multiple complete shell segments, arranged regularly in accordance with a reference surface characterized by the positive Gaussian curvature in the three-dimensional space. However, there is a gap in the current knowledge because there is no analogous comprehensive method for shaping continuous and discontinuous shell roof structures, whose complete shell units are arranged on reference surfaces characterized by negative Gaussian curvature. There is a need for a detailed description of the procedure for shaping such complex building forms. This issue is analyzed in the present article.

Pottman [21] has described several methods for shaping various systems of smooth plane and shell sectors arranged according to the geometric properties of different regular surfaces. However, if we want to shape transformed folded shell sheeting using these methods, significant modifications must be made to account for the geometric and material constraints of the folded sheets.

Samyn [22] developed a method for creating transformed folded shells using aluminium or PVC. The method differs from previous methods, which involved joining adjacent sheets along their longitudinal borders using bolts or rivets. Instead, Samyn’s method requires the sheets to be joined by welding.

Folded shell sheeting can also be used to construct supports for folded panels arranged according to some convex shells. Żwirek [23] performed forced plastic deformations on thin-walled folded sheets to obtain doubly-curved bases that support the convex shells composed of the panels.
3. Objective of the research

The aim is to analyse the possibilities of shaping unconventional folded building forms, specifically their complex multi-plane façades and multi-shell roofs. These shell sectors are arranged in the three-dimensional space, following regular surfaces with the negative Gaussian curvature. Through this analysis, an innovative method is presented, which involves search for computational models of such forms. To achieve these goals, a novel procedure was developed for creating specific types of auxiliary reference polyhedral network and polyhedral eaves networks.

The algorithm used in this method is different from other algorithms used so far because the characteristic vertices of the developed models are positioned between the vertices of the reference polyhedral network. The roof structures are grouped into basic continuous ribbed and discontinuous shell structures derived from the basic ones. The invented algorithm allows for arranging many façade walls and many roof shell units in relation to the vertices of the polyhedral reference network using respective discrete values of division coefficients. These coefficients define the adopted proportions between the distances of the appropriate vertices of the polygonal eaves network and the distances of the selected pairs of the vertices of the reference polyhedral networks \( \Gamma \).

The novel procedure carried out is related to the differentiation of the values of the division coefficients and enables us to expect some differentiation of the obtained basic and derivative building forms. The proposed method is described in detail using a few specific examples. Several different forms, obtainable through the procedure, are provided to demonstrate a wide range of possibilities offered by the method.

The main relationships governing the position of each mesh \( B_{ij} \) and shell unit \( \Omega_{ij} \) in a polygonal eaves network \( B \) and shell roof structure \( \Omega \) deserve special emphasis. The specific sets of values of the partition coefficients presented, which define the required positions of the vertices of each developed reference polyhedral network, each reference surface, and each reference polygonal eaves network, play a distinctive role in shaping rational and attractive basic and derivative configurations of the proposed general building forms.

The algorithm of the presented method is supported by the authors’ novel computer program written in the AutoLISP programming language, specifically designed for the AutoCAD graphics editor. Each CAD object built using this program can be used as a geometric model of a building, which is helpful in shaping its structural system.

4. Methodology

Creating each polyhedral network \( \Gamma \) composed of a number of appropriately arranged tetrahedra \( \Gamma_{ij} \) begins with the first central tetrahedron \( \Gamma_{11} \), defined by four arbitrary points \( W_{AB11}, W_{CD11}, W_{AD11} \) and \( W_{BC11} \) which serve as vertices of \( \Gamma \), Fig. 3.a. The location of the vertices is determined using two arbitrary oblique straight lines \( u_{11} \) and \( v_{11} \) that are perpendicular to each other. These lines are referred to as the axes of \( \Gamma_{11} \).

The arbitrary distance \( d_{uv11} \) between \( u_{11} \) and \( v_{11} \) measured along the straight line perpendicular to \( u_{11} \) and \( v_{11} \), needs to be determined. To define the positions of the vertices of \( \Gamma_{12} \), \( \Gamma_{13} \), \( \Gamma_{14} \), \( \Gamma_{15} \), \( \Gamma_{16} \), \( \Gamma_{17} \), \( \Gamma_{18} \), and \( \Gamma_{19} \), the distance \( d_{uv11} \) between \( W_{BC11} \) and \( W_{AD11} \) also need to be determined.

The subsequent tetrahedral meshes \( \Gamma_{ij} \) of \( \Gamma \) are positioned in two orthogonal directions of \( \Gamma_{11} \), according to the directions of the axes \( v_{11}, v_{12} \) and \( v_{12} \), Fig. 3.b, and the axes \( u_{11}, u_{21} \) and \( u_{21} \), Fig. 3.c, as follows: For the tetrahedron \( \Gamma_{12} \), its three vertices are set to be identical
to the vertices $W_{AB11}$, $W_{CD11}$ and $W_{BC11}$ of $\Gamma_{11}$. The axis $u_{12}$ of $\Gamma_{12}$ is also identical to $u_{11}$. The location of the fourth vertex $W_{BC12}$ of $\Gamma_{12}$ (that does not belong to $\Gamma_{11}$ and defines the axis $v_{12}$) is determined so that it is contained in the first plane of symmetry of $\Gamma_{11}$, distant from $W_{BC11}$ by the adopted value of $d_{v12}$ and distant from the axis $u_{11}$ by the height $d_{BC11}$ of the triangle $W_{BC11}W_{CD11}W_{AB11}$. The third tetrahedron $\Gamma_{12L}$ is symmetrical to $\Gamma_{12}$ with respect to the second plane of symmetry of $\Gamma_{11}$. The second plane of symmetry of $\Gamma_{11}$ is defined by the points $W_{AD11}$, $W_{BC11}$ and the midpoint of the segment $W_{AB11}W_{CD11}$. The first plane of symmetry of $\Gamma_{11}$ is defined by the points $W_{AB11}$, $W_{CD11}$ and the midpoint of the segment $W_{AD11}W_{BC11}$.

Fig. 3. The main steps in shaping a polyhedral reference network $\Gamma$ and reference surface $\omega_r$ are as follows: (a) the first tetrahedron $\Gamma_{11}$ of $\Gamma$, (b) the first orthogonal strip $\Gamma_{11j}$ containing the second tetrahedron $\Gamma_{12}$ of $\Gamma$, (c) the second orthogonal strip $\Gamma_{11i}$ containing the subsequent tetrahedron $\Gamma_{21}$ of $\Gamma$, and (d) reference surface $\omega_r$ and the diagonal tetrahedron $\Gamma_{22}$ of $\Gamma$. Source: own study.
All tetrahedra $Γ_{ij}$ created so far belong to the first orthogonal strip. Similarly, all tetrahedra $Γ_{i1}$ of the second searched orthogonal strip can be formed in the same manner as $Γ_{ij}$ tetrahedra belonging to the first strip. Here’s how it is done: For the tetrahedron $Γ_{21}$, its three vertices are set to be identical to the vertices $W_{AD11}$, $W_{BC11}$ and $W_{CD11}$ of $Γ_{I1}$. The axis $v_{21}$ of $Γ_{21}$ is also identical to $v_{11}$. The location of the fourth vertex $W_{CD21}$ of $Γ_{21}$ (which is not part of $Γ_{I1}$ and defines the axis $u_{21}$) is determined in such a way that it lies in the first plane of symmetry of $Γ_{I1}$, at a distance of $dt_{21}$ from $W_{CD11}$, and at a distance of $dc_{CD21} = dc_{CD11}$ (the height of the triangle $W_{BC11}W_{CD11}W_{AD11}$) from the axis $v_{11}$. The third tetrahedron $Γ_{21l}$ of this orthogonal strip is symmetrical to $Γ_{21}$ with respect to the second plane of symmetry of $Γ_{I1}$’s, as defined above.

All tetrahedra are arranged in diagonal strips $Γ_{ij}$ (where $i, j \neq 1$ and $i = j$) and are defined by the vertices of two aforementioned orthogonal strips composed of the tetrahedrons $Γ_{i1}$ and $Γ_{ij}$, or the previously created diagonal tetrahedra. Figure 3.d presents an example of a diagonal tetrahedron $Γ_{ij}$ for $i = j = 2$. In the presented method, selected proportions $dd_{ij} = dv_{12} / dv_{11}$ and $dd_{BC12} = dc_{BC12} / dc_{BC11}$ are used, which allows for better control of the shape of $Γ, B$, and $Σ$. Here, $dc_{BC12}$ refers to the distance of $W_{BC12}$ from $u_{12}$ and $dc_{BC11}$ refers to the distance of $W_{BC11}$ from $u_{11}$. $Σ$ represents the model of the entire building, as shown in Fig. 4.c. By adopting such proportions for all orthogonal tetrahedral meshes, it becomes possible to parametrize and control the shapes of $Γ, B$, and $Σ$. For the mesh $Γ_{12}$ used in the following section, $dd_{12} = 1$ and $dd_{BC12} = 1$. The parametrization should result in dividing the desired forms $Σ$ into several distinct groups with similar geometric properties.

Since the employed method for parameterizing $Γ$ involves defining a set of division coefficients that express the proportions between the distances of the vertices of the subsequently created $Γ_{ij}$’s meshes, it is important to consider the corresponding proportions between the lengths of the axes of the subsequent meshes. For the created mesh $Γ_{21}$, $dd_{21} = dc_{CD21} = 1$, see Figs 3-4, where: $dd_{21} = dv_{12} / dv_{11}$ and $dd_{BC21} = dc_{BC21} / dc_{BC11}$.

In a general case, $dd_{ij} = dd_{ui1} = 1$, $dd_{BCij} = dd_{CD11} = 1$ for $Γ_{ij}$ and $Γ_{i1}$. The above activities should be repeated to ensure that all subsequent meshes $Γ_{ij}$ of the orthogonal strips are congruent to each other.

For all diagonal tetrahedra, there are $W_{ADij} = W_{BCi-1j-1}$, $W_{BCij} = W_{ADi-1j}$, $W_{CDij} = W_{CDi-1j-1}$. The developed procedure allows one to locate $W_{ADij}$, $W_{BCij}$, $W_{CDij}$ and $W_{ABij}$ at any points of the respective side edges of $Γ$. Thus, they do not have to only be located at the vertices of the previously created tetrahedra. This modification related to the properties of $Γ$ leads to fundamental changes in the proportions between the overall dimensions and the size of the elements of the building model shaped. The description of the modified procedure for shaping such networks $Γ$ goes beyond the scope of this article.

The polyhedral network $Γ$ presented in Figs 3-4 consists of four symmetrical parts $Γ_k$ ($k = 1$ to 4), where the first quarter $Γ_1$ is composed of $Γ_{ij}$ ($i, j = 1$ to 2). Based on the symmetrical reference network $Γ$, a specific polygonal eaves network $B_v$ can be created to define a multi-shell roof structure $Σ$. To determine the eaves network $B_v$, appropriate relationships between the location of the vertices of $Σ$ and $B_v$, in relation to the selected pairs of the $Γ$’s vertices, need to be adopted. According to the method’s assumptions, these relationships are defined using the division coefficients of the pairs {$W_{ABij}, W_{ADij}$}, {$W_{ABij}, W_{BCij}$}, {$W_{ADij}, W_{BCij}$} of each $Γ_{ij}$ by the vertices of each $B_{vij} = <A_{ij}B_{ij}C_{ij}D_{ij}>$, each base quadrangle $<P_{ij}P_{Bij}P_{Cij}P_{Dij}>$, Fig. 4.a, and each plane quadrangle $<S_{ij}S_{Bij}S_{Cij}S_{Dij}>$ of the reference surface $α$ used, Figs 3.a, 4.
The steps in shaping a whole building structure $\Sigma$: (a) the reference polyhedral network $\Gamma$, (b) network $B_v$ and reference surface $\omega_r$, and (c) the shell roof structure $\Omega$. Source: own study

The set \{dd$_{SAij}$, dd$_{SBij}$, dd$_{SCij}$ and dd$_{SDij}$\} of division coefficients is used to determine the positions of the points $S_{Aij}$, $S_{Bij}$, $S_{Cij}$ and $S_{Dij}$ located on the side edges of $\Gamma$ and defining a reference surface $\omega_r$. To search for the network $B_v$, another set \{dd$_{Aij}$, dd$_{Bij}$, dd$_{Cij}$ and dd$_{Dij}$\} of the division coefficients is employed to define the positions of the points $A_{ij}$, $B_{ij}$, $C_{ij}$ and $D_{ij}$ on the side edges of $\Gamma$. These points constitute the vertices of $B_v$, Fig. 4.a-c. On the basis of the network $B_v$, the model $\Sigma$ of the searched complex folded building form is created as follows. The walls of $\Sigma$ should be included in the planes of $\Gamma$. The elevation edges of $\Sigma$ should be included in the side edges of $\Gamma$. The eaves line $B_{vij}$ of each single shell $\Omega_{ij}$ of the $\Omega$'s roof structure must be contained in the planes of $\Gamma$. The vertices of each eaves net $B_v$ belong to the side edges of $\Gamma$. In the presented procedure, the considered vertices of $\omega_r$ and $B_v$ have to be located between the vertices of $\Gamma$. 

Fig. 4. The steps in shaping a whole building structure $\Sigma$: (a) the reference polyhedral network $\Gamma$, (b) network $B_v$ and reference surface $\omega_r$, and (c) the shell roof structure $\Omega$. Source: own study
The bases of the created forms are flat and horizontal. The arbitrary level of each base plane $P_b$ must belong to the interval $<0, d_{uv}>$. The positions of all points $P_{Aij}, P_{Bij}, P_{Cij}$ and $P_{Dij}$ of each base, belonging to the edges of the facade walls, can be obtained as a result of the intersection of the plane $P_b$ with all $\Gamma$’s side edges. The methodology for studying the shaping of unconventional folded building forms is presented in Figure 5.

Fig. 5. Methodology of the research

For the examined vertices of $\omega$, $B_v$ and $\Sigma$, all acceptable values of the above-mentioned two types of the division coefficients must be taken from the range $(0, 1)$. In addition, the arbitrary reference surface $\omega_r$, defined by all tetrads of the points $S_{Aij}, S_{Bij}, S_{Cij}$ and $S_{Dij},$ has to have negative Gaussian curvature. This assumption results in the specific geometric properties of $B_v$ and $\Sigma$, proving the innovative nature of the performed analysis and the developed procedures. In the case of other articles, the values of the division
coefficients employed are within the range (1, ∞) and the reference surface is convex and characterized by the positive Gaussian curvature.

Each set of the division coefficients taken into account should determine the mutual position of all vertices of the reference polyhedral \( \Gamma \) network, the polygonal \( B_r \) network, the base plane \( P_b \) and the reference surface \( \omega_r \). The adopted proportions define the values of the distances between the above-mentioned vertices, so the coordinates of the vertices can be calculated in the three-dimensional space. In order to illustrate the impact of adopting different sets of values of the above-mentioned coefficients on the shape of \( \Gamma, B_r \) and \( \Sigma \), a few examples of complex folded base continuous building forms and their discontinuous derivatives with roof shell units arranged on surfaces characterized by the negative Gaussian curvature are presented in the next section.

Two adjacent tetrahedra of each polyhedral network \( \Gamma \), created in the above way, have one common face and three common vertices. Two adjacent closed spatial quadrangles of each polygonal eaves network \( B_r \), have one edge or vertex in common. Roof shell structures \( \Sigma \) constructed with the help of the method are called complex basic shells and are characterized by the continuity of the entire complex roof shell.

In general case, some roof structures are created so that their meshes do not have common sides or vertices. This property is characteristic of the roof shell structures derived from the aforementioned basic ones. These structures will be presented in the following section through a few specific examples. The roof structures created in this manner are referred to as complex derivative roof structures. They are characterized by multiple areas of discontinuity between their shell units. The empty spaces between the roof shell sectors can be designated for window openings.

5. Results

The algorithm of the method is presented using above-mentioned examples of parametric modelling of various complex roof forms covered with continuous and discontinuous shell structures using the same reference polyhedral network \( \Gamma \). The parameters used to define \( \Gamma \) are as follows. The first parameter is the distance \( d_{v11} \) of two vertices of the axis \( v_{11} \). The second one is the ratio \( d_{d_{u11}} \) of the distance \( d_{u11} \) between the skew axes \( u_{11} \) and \( v_{11} \), and \( d_{v11} \). The third parameter is the ratio \( d_{d_{u11}} \) of the distance \( d_{u11} \) of two vertices of \( u_{11} \) to \( d_{v11} \). The values of all similar parameters that define all tetrahedral meshes \( \Gamma_{ij} \) and \( \Gamma_{i1} \) of \( \Gamma \) belonging to the two orthogonal strips of \( \Gamma \) are equal to 20,000 mm, 5 and 1, respectively.

Thus, the examined network \( \Gamma \) is composed of tetrahedrons \( \Gamma_{ij} \) and \( \Gamma_{i1} \), which are congruent to each other and positioned in two orthogonal strips. Additionally, it is characterized by two mutually perpendicular planes of symmetry defined by the principal axes of \([x, y, z]\). The vertices of other meshes \( \Gamma_{ij} \) (\( i \neq 1 \) or \( j \neq 1 \)) located diagonally in relation to \( \Gamma_{11} \) are taken at the corresponding vertices of the orthogonal tetrahedral meshes \( \Gamma_{ij} \) and \( \Gamma_{i1} \).
Fig. 6. The designed reference network $\Gamma$ and the eaves network $B_v$, of the basic complex building structure $\Sigma$: (a) the front view, (b) the side view, (c) the top view, and (d) axonometric view. Source: own study

The coordinates of the vertices of the examined network $\Gamma$, shown in Fig. 6, in the orthogonal coordinate system $[x, y, z]$ are given in Tab. 1. The published values refer to one quarter $\Gamma_1$ of the symmetrical network $\Gamma$, located between the principal planes $(x, z)$ and $(y, z)$ in the dihedral angle containing the positive senses of $x$ and $y$. 
Table 1. The coordinates of the vertices $W_{ABij}$, $W_{CDij}$, $W_{ADij}$, $W_{BCij}$ (for $i, j = 1, 2$) of the polyhedral reference network $\Gamma$. Source: own study

<table>
<thead>
<tr>
<th>Vertex</th>
<th>$x$-coordinate [mm]</th>
<th>$y$-coordinate [mm]</th>
<th>$z$-coordinate [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{AB11}$</td>
<td>-10,000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$W_{CD11}$</td>
<td>10,000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$W_{AD11}$</td>
<td>0</td>
<td>-10,000</td>
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</tr>
<tr>
<td>$W_{BC11}$</td>
<td>0</td>
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<td>50,000</td>
</tr>
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<td>0</td>
</tr>
<tr>
<td>$W_{CD12}$</td>
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<td>0</td>
</tr>
<tr>
<td>$W_{AD12}$</td>
<td>0</td>
<td>10,000</td>
<td>50,000</td>
</tr>
<tr>
<td>$W_{BC12}$</td>
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<td>28,793</td>
<td>43,160</td>
</tr>
<tr>
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<td>0</td>
</tr>
<tr>
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<td>0</td>
<td>6,840</td>
</tr>
<tr>
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<td>0</td>
<td>50,000</td>
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<td>$W_{BC21}$</td>
<td>0</td>
<td>28,793</td>
<td>43,160</td>
</tr>
</tbody>
</table>

During the second step of the algorithm, the positions of the subsequent tetrads of the vertices $S_{Aij}$, $S_{Bij}$, $S_{Cij}$ and $S_{Dij}$ of a reference surface $\omega_r$, located on the side edges of $\Gamma$, are defined using the division coefficients $dd_{SAij}$, $dd_{SBij}$, $dd_{SCij}$ and $dd_{SDij}$. The adopted values of these division coefficients, taken for the exemplary basic continuous structure, shown in Fig. 6.a-d, are given in Tab. 2.

Table 2. The initial data – division coefficients defining the positions of the points $S_{Aij}$, $S_{Bij}$, $S_{Cij}$ and $S_{Dij}$ of the reference surface $\omega_r$ for $i, j = 1, 2$. Source: own study

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$dd_{SA11}$</td>
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</tr>
<tr>
<td>$dd_{SB11}$</td>
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<tr>
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<tr>
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</tr>
<tr>
<td>$dd_{SD12}$</td>
<td>0.489</td>
</tr>
<tr>
<td>$dd_{SA21}$</td>
<td>0.489</td>
</tr>
<tr>
<td>$dd_{SB21}$</td>
<td>0.489</td>
</tr>
<tr>
<td>$dd_{SC21}$</td>
<td>0.332</td>
</tr>
<tr>
<td>$dd_{SD21}$</td>
<td>0.332</td>
</tr>
<tr>
<td>$dd_{SA22}$</td>
<td>0.489</td>
</tr>
<tr>
<td>$dd_{SB22}$</td>
<td>0.623</td>
</tr>
<tr>
<td>$dd_{SC22}$</td>
<td>0.332</td>
</tr>
<tr>
<td>$dd_{SD22}$</td>
<td>0.332</td>
</tr>
</tbody>
</table>

The above values are related to one symmetrical quarter of $\omega_r$ contained in the subspace limited by the planes $(x, z)$ and $(y, z)$ defined by positive senses of the axes $x$ and $y$. The values of the distinguished points of this $\omega_r$’s quarter are published in Tab. 3.
Unconventional building forms roofed with innovative structures arranged on regular surfaces...

Table 3. The coordinates of the selected points defining the reference surface \( \omega \) for \( i, j = 1, 2 \).

Source: own study

<table>
<thead>
<tr>
<th>Points</th>
<th>( x )-coordinate [mm]</th>
<th>( y )-coordinate [mm]</th>
<th>( z )-coordinate [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_{D11} )</td>
<td>5,107.3</td>
<td>-4,892.7</td>
<td>24,463.6</td>
</tr>
<tr>
<td>( S_{C11} )</td>
<td>5,107.3</td>
<td>4,892.7</td>
<td>24,463.6</td>
</tr>
<tr>
<td>( S_{B11} )</td>
<td>-5,221.5</td>
<td>4,778.5</td>
<td>23,892.6</td>
</tr>
<tr>
<td>( S_{A11} )</td>
<td>-5,221.5</td>
<td>-4,778.5</td>
<td>23,892.6</td>
</tr>
<tr>
<td>( S_{D12} )</td>
<td>5,107.3</td>
<td>4,892.7</td>
<td>24,463.6</td>
</tr>
<tr>
<td>( S_{C12} )</td>
<td>3,772.1</td>
<td>17,932.4</td>
<td>26,879.2</td>
</tr>
<tr>
<td>( S_{B12} )</td>
<td>-3,880.6</td>
<td>17,620.2</td>
<td>26,411.2</td>
</tr>
<tr>
<td>( S_{A12} )</td>
<td>-5,221.5</td>
<td>4,778.5</td>
<td>23,892.6</td>
</tr>
<tr>
<td>( S_{D21} )</td>
<td>19,237.5</td>
<td>-3,318.8</td>
<td>21,164.1</td>
</tr>
<tr>
<td>( S_{C21} )</td>
<td>19,224.6</td>
<td>3,323.3</td>
<td>21,183.4</td>
</tr>
<tr>
<td>( S_{B21} )</td>
<td>5,107.3</td>
<td>4,892.7</td>
<td>24,463.6</td>
</tr>
<tr>
<td>( S_{A21} )</td>
<td>5,107.3</td>
<td>-4,892.7</td>
<td>24,463.6</td>
</tr>
<tr>
<td>( S_{D22} )</td>
<td>19,224.6</td>
<td>3,323.3</td>
<td>21,183.4</td>
</tr>
<tr>
<td>( S_{C22} )</td>
<td>15,836.3</td>
<td>12,957.2</td>
<td>23,184.0</td>
</tr>
<tr>
<td>( S_{B22} )</td>
<td>3,772.1</td>
<td>17,932.4</td>
<td>26,879.2</td>
</tr>
<tr>
<td>( S_{A22} )</td>
<td>5,107.3</td>
<td>4,892.7</td>
<td>24,463.6</td>
</tr>
</tbody>
</table>

At the third step of the method’s algorithm, the locations of the vertices \( A_{ij}, B_{ij}, C_{ij} \) and \( D_{ij} \) of the subsequent meshes \( B_{vij} \) of \( B_v \), which define the designed roof shell \( \Omega \) are determined using the respective division coefficients \( dd_{Aij}, dd_{Bij}, dd_{Cij} \) and \( dd_{Dij} \). The division coefficients values used for the examined basic structure \( \Sigma \) roofed with \( \Omega \), as shown in Fig. 6.a-d, are listed in Tab. 4. The coordinates of the vertices \( A_{ij}, B_{ij}, C_{ij} \) and \( D_{ij} \) of the developed \( B_v \) are provided in Tab. 5.

Table 4. The values of the division coefficients employed to calculate the coordinates of the vertices of the basic structure \( \Omega \) for \( i,j =1,2 \). Source: own study

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( dd_{D11} )</td>
<td>0.505</td>
</tr>
<tr>
<td>( dd_{C11} )</td>
<td>0.474</td>
</tr>
<tr>
<td>( dd_{B11} )</td>
<td>0.493</td>
</tr>
<tr>
<td>( dd_{A11} )</td>
<td>0.462</td>
</tr>
<tr>
<td>( dd_{D12} )</td>
<td>0.474</td>
</tr>
<tr>
<td>( dd_{C12} )</td>
<td>0.638</td>
</tr>
<tr>
<td>( dd_{B12} )</td>
<td>0.597</td>
</tr>
<tr>
<td>( dd_{A12} )</td>
<td>0.493</td>
</tr>
<tr>
<td>( dd_{D21} )</td>
<td>0.317</td>
</tr>
<tr>
<td>( dd_{C21} )</td>
<td>0.347</td>
</tr>
<tr>
<td>( dd_{B21} )</td>
<td>0.474</td>
</tr>
<tr>
<td>( dd_{A21} )</td>
<td>0.505</td>
</tr>
<tr>
<td>( dd_{D22} )</td>
<td>0.347</td>
</tr>
<tr>
<td>( dd_{C22} )</td>
<td>0.435</td>
</tr>
<tr>
<td>( dd_{B22} )</td>
<td>0.638</td>
</tr>
<tr>
<td>( dd_{A22} )</td>
<td>0.474</td>
</tr>
</tbody>
</table>
Table 5. The coordinates of the vertices $A_{ij}$, $B_{ij}$, $C_{ij}$ and $D_{ij}$ (for $i, j = 1, 2$) of the basic structure $\Omega_1$ for $i,j =1,2$. Source: own study

<table>
<thead>
<tr>
<th>Point</th>
<th>$x$-coordinate [mm]</th>
<th>$y$-coordinate [mm]</th>
<th>$z$-coordinate [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{11}$</td>
<td>4,953.3</td>
<td>-5,046.7</td>
<td>25,233.4</td>
</tr>
<tr>
<td>$C_{11}$</td>
<td>5,261.2</td>
<td>4,738.8</td>
<td>23,693.8</td>
</tr>
<tr>
<td>$B_{11}$</td>
<td>-5,067.5</td>
<td>4,932.5</td>
<td>24,662.4</td>
</tr>
<tr>
<td>$A_{11}$</td>
<td>-5,375.4</td>
<td>-4,624.6</td>
<td>23,122.8</td>
</tr>
<tr>
<td>$D_{12}$</td>
<td>5,261.2</td>
<td>4,738.8</td>
<td>23,693.8</td>
</tr>
<tr>
<td>$C_{12}$</td>
<td>3,620.7</td>
<td>18,368.4</td>
<td>27,532.7</td>
</tr>
<tr>
<td>$B_{12}$</td>
<td>-4,032.0</td>
<td>17,184.3</td>
<td>25,757.8</td>
</tr>
<tr>
<td>$A_{12}$</td>
<td>-5,067.5</td>
<td>4,932.5</td>
<td>24,662.4</td>
</tr>
<tr>
<td>$D_{21}$</td>
<td>19,673.4</td>
<td>-3,167.4</td>
<td>20,510.6</td>
</tr>
<tr>
<td>$C_{21}$</td>
<td>18,788.6</td>
<td>3,474.7</td>
<td>21,836.9</td>
</tr>
<tr>
<td>$B_{21}$</td>
<td>5,261.2</td>
<td>4,738.8</td>
<td>23,693.8</td>
</tr>
<tr>
<td>$A_{21}$</td>
<td>4,953.3</td>
<td>-5,046.7</td>
<td>25,233.4</td>
</tr>
<tr>
<td>$D_{22}$</td>
<td>18,788.6</td>
<td>3,474.7</td>
<td>21,836.9</td>
</tr>
<tr>
<td>$C_{22}$</td>
<td>16,258.5</td>
<td>12,535.1</td>
<td>22,651.5</td>
</tr>
<tr>
<td>$B_{22}$</td>
<td>3,620.7</td>
<td>18,368.4</td>
<td>27,532.7</td>
</tr>
<tr>
<td>$A_{22}$</td>
<td>5,261.2</td>
<td>4,738.8</td>
<td>23,693.8</td>
</tr>
</tbody>
</table>

At the fourth step of the algorithm, the vertices of the plane base of the developed basic continuous structure $\Sigma$ are determined as the points of intersection of the base horizontal plane (parallel to $(x, y)$) and the side edges of the previously created mesh $\Gamma$. The $p_z$ coordinate level of the base plane is equal to 15,920 mm in $[x,y,z]$.

At the fifth step, two discontinuous roof structures $\Omega_{d1}$ and $\Omega_d$ derivative to the obtained continuous structure $\Omega$ are created. The first discontinuous derivative structure $\Omega_{d1}$, shown in Fig. 7.a-d, is created through a significant modification of $\Omega$ as follows.

Four vertices belonging to four adjacent meshes $B_{vij}$, $B_{v(i+1)j}$ and $B_{v(i+1)(j+1)}$ of the network $B_v$ (structure $\Omega$) are separated into pairs, where two vertices from opposite meshes $B_{vij}$ and $B_{v(i+1)(j+1)}$ (shell sectors $\Omega_d$ and $\Omega_{d(i+1)(j+1)}$) in a diagonal strip are shifted to the opposite side of the surface $\alpha_b$ compared to the position of other two stationary vertices belonging to other meshes $B_{v(i+1)j}$ and $B_{vij}$ (sectors $\Omega_d(i+1)$ and $\Omega_{d(i+1)}$). These stationary vertices belong to the meshes contained in a different diagonal strip of $B_v$ ($\Omega$). This action is performed for all common vertices of $B_v$ ($\Omega$). As a result of this modification, new discontinuous structures $\Omega_{d1}$ and $\Sigma_{d1}$, shown in Fig. 7.a-d, are obtained.
Unconventional building forms roofed with innovative structures arranged on regular surfaces...

Fig. 7. The designed reference network \( \Gamma \) and the eaves network \( B_{vd1} \) of the first complex derivative discontinuous structure \( \Sigma_{d1} \): (a) the front view, (b) the side view, (c) the top view, and (d) the axonometric view. \textit{Source:} own study

As a result of the displacing all sets of vertices belonging to adjacent meshes of \( B_v \) along the respective side edge of \( \Gamma \), triangular flat areas are formed in the discontinuous roof structure \( \Omega_{d1} \) (the network \( B_{vd1} \)) allowing sunlight to enter the interior of the designed building. The differentiation of the vertex positions, belonging to adjacent meshes of \( B_{vd1} \) (\( \Omega_{d1} \)) and located on the same side edge of \( \Gamma \), can be achieved by appropriately adjusting the values of the division coefficients associated with the vertices of \( B_{vd1} \) and \( \omega_i \). The division coefficient values used are presented in Tab. 6.

Table 6. The values of the division coefficients defining the coordinates of the vertices of the first derivative structure \( \Omega_{d1} \) for \( i,j = 1,2 \). \textit{Source:} own study

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( dd_{D11} )</td>
<td>0.493</td>
</tr>
<tr>
<td>( dd_{C11} )</td>
<td>0.462</td>
</tr>
<tr>
<td>( dd_{B11} )</td>
<td>0.505</td>
</tr>
<tr>
<td>( dd_{A11} )</td>
<td>0.474</td>
</tr>
<tr>
<td>( dd_{D12} )</td>
<td>0.462</td>
</tr>
<tr>
<td>( dd_{C12} )</td>
<td>0.640</td>
</tr>
<tr>
<td>( dd_{B12} )</td>
<td>0.602</td>
</tr>
<tr>
<td>( dd_{A12} )</td>
<td>0.505</td>
</tr>
<tr>
<td>( dd_{D21} )</td>
<td>0.347</td>
</tr>
<tr>
<td>( dd_{C21} )</td>
<td>0.317</td>
</tr>
<tr>
<td>( dd_{B21} )</td>
<td>0.505</td>
</tr>
<tr>
<td>( dd_{A21} )</td>
<td>0.474</td>
</tr>
<tr>
<td>( dd_{D22} )</td>
<td>0.343</td>
</tr>
<tr>
<td>( dd_{C22} )</td>
<td>0.435</td>
</tr>
<tr>
<td>( dd_{B22} )</td>
<td>0.640</td>
</tr>
<tr>
<td>( dd_{A22} )</td>
<td>0.462</td>
</tr>
</tbody>
</table>

The coordinates of the vertices \( A_{ij}, B_{ij}, C_{ij} \) and \( D_{ij} \) of \( \Omega_{d1} \) obtained based on these coefficients are provided in Tab. 7.
Table 7. The coordinates of the vertices $A_{ij}$, $B_{ij}$, $C_{ij}$ and $D_{ij}$ (for $i, j = 1, 2$) of the first derivative structure $\Omega_1$. Source: own study

<table>
<thead>
<tr>
<th>Vertex</th>
<th>$x$-coordinate [mm]</th>
<th>$y$-coordinate [mm]</th>
<th>$z$-coordinate [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{11}$</td>
<td>-5.375.4</td>
<td>-4.624.6</td>
<td>23,122.8</td>
</tr>
<tr>
<td>$B_{11}$</td>
<td>-5,067.5</td>
<td>4,932.5</td>
<td>24,662.4</td>
</tr>
<tr>
<td>$C_{11}$</td>
<td>5,261.2</td>
<td>4,738.8</td>
<td>23,693.8</td>
</tr>
<tr>
<td>$D_{11}$</td>
<td>4,953.3</td>
<td>-5,046.7</td>
<td>25,233.4</td>
</tr>
<tr>
<td>$A_{12}$</td>
<td>-5,375.4</td>
<td>4,624.6</td>
<td>23,122.8</td>
</tr>
<tr>
<td>$B_{12}$</td>
<td>-3,729.2</td>
<td>18,056.2</td>
<td>27,064.7</td>
</tr>
<tr>
<td>$C_{12}$</td>
<td>3,923.5</td>
<td>17,496.5</td>
<td>26,225.8</td>
</tr>
<tr>
<td>$D_{12}$</td>
<td>4,953.3</td>
<td>5,046.7</td>
<td>25,233.4</td>
</tr>
<tr>
<td>$A_{21}$</td>
<td>5,261.2</td>
<td>-4,738.8</td>
<td>23,693.8</td>
</tr>
<tr>
<td>$B_{21}$</td>
<td>4,953.3</td>
<td>5,046.7</td>
<td>25,233.4</td>
</tr>
<tr>
<td>$C_{21}$</td>
<td>19,660.5</td>
<td>3,171.9</td>
<td>20,530.0</td>
</tr>
<tr>
<td>$D_{21}$</td>
<td>18,801.5</td>
<td>-3,470.2</td>
<td>21,817.5</td>
</tr>
<tr>
<td>$A_{22}$</td>
<td>5,261.2</td>
<td>4,738.8</td>
<td>23,693.8</td>
</tr>
<tr>
<td>$B_{22}$</td>
<td>3,620.7</td>
<td>18,368.4</td>
<td>27,532.7</td>
</tr>
<tr>
<td>$C_{22}$</td>
<td>16,258.5</td>
<td>12,535.1</td>
<td>22,651.5</td>
</tr>
<tr>
<td>$D_{22}$</td>
<td>18,788.6</td>
<td>3,474.7</td>
<td>21,836.9</td>
</tr>
</tbody>
</table>

The second discontinuous structure $\Omega_{d2}$, derived from the original continuous structure $\Omega$, is obtained by displacing the vertices of all shell sectors $\Omega_i$ and $\Omega_{ij}$ of $\Omega$ located in two orthogonal strips relative to the central sector $\Omega_{11}$ along the side edges of $\Gamma$, as shown in Figure 8.a-d. However, the vertices of the diagonally located shell sectors do not change their position.

![Fig. 8. The reference network $\Gamma$ and the eaves network $B_{vd2}$ of the second complex derivative discontinuous structure $\Sigma_{d2}$: (a) the front view, (b) the side view, (c) the top view, and (d) the axonometric view. Source: own study](image-url)
Another characteristic feature of the second discontinuous derivative structure $\Sigma_{d2}$ is that the sectors $\Omega_{ij}$ of one orthogonal strips are shifted above the reference surface $\omega_r$, while the sectors of the other orthogonal strip $\Omega_{1i}$ are displaced below the surface $\omega_r$. The result of the displacement of the selected sectors $\Omega_{1i}$ and $\Omega_{ij}$ along the side edges of $\Gamma$ comes from the assumption of the appropriate differences in the values of the division coefficients related to the vertices of $B_v$ and the points of $\omega_r$. The values of these coefficients are given in the Tab. 8.

Table 8. The values of the division coefficients defining the coordinates of the vertices of the second derivative structure $\Omega_{d2}$ for $i,j = 1,2$. Source: own study

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{D11}$</td>
<td>0.462</td>
</tr>
<tr>
<td>$d_{C11}$</td>
<td>0.493</td>
</tr>
<tr>
<td>$d_{B11}$</td>
<td>0.475</td>
</tr>
<tr>
<td>$d_{A11}$</td>
<td>0.505</td>
</tr>
<tr>
<td>$d_{D12}$</td>
<td>0.458</td>
</tr>
<tr>
<td>$d_{C12}$</td>
<td>0.640</td>
</tr>
<tr>
<td>$d_{B12}$</td>
<td>0.391</td>
</tr>
<tr>
<td>$d_{A12}$</td>
<td>0.505</td>
</tr>
<tr>
<td>$d_{D21}$</td>
<td>0.317</td>
</tr>
<tr>
<td>$d_{C21}$</td>
<td>0.287</td>
</tr>
<tr>
<td>$d_{B21}$</td>
<td>0.474</td>
</tr>
<tr>
<td>$d_{A21}$</td>
<td>0.443</td>
</tr>
<tr>
<td>$d_{D22}$</td>
<td>0.312</td>
</tr>
<tr>
<td>$d_{C22}$</td>
<td>0.465</td>
</tr>
<tr>
<td>$d_{B22}$</td>
<td>0.610</td>
</tr>
<tr>
<td>$d_{A22}$</td>
<td>0.493</td>
</tr>
</tbody>
</table>

The coordinates of the vertices belonging to $\Omega_{d2} (B_{vd2})$ are provided in the Tab. 9.

Table 9. The coordinates of the vertices $A_{ij}$, $B_{ij}$, $C_{ij}$, and $D_{ij}$ (for $i,j = 1,2$) belonging to the second derivative structure $\Omega_{d2}$. Source: own study

<table>
<thead>
<tr>
<th>Vertex</th>
<th>x-coordinate [mm]</th>
<th>y-coordinate [mm]</th>
<th>z-coordinate [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{11}$</td>
<td>-5,067.5</td>
<td>-4,932.5</td>
<td>24,662.4</td>
</tr>
<tr>
<td>$B_{11}$</td>
<td>-5,375.4</td>
<td>4,624.6</td>
<td>23,122.8</td>
</tr>
<tr>
<td>$C_{11}$</td>
<td>4,953.3</td>
<td>5,046.7</td>
<td>25,233.4</td>
</tr>
<tr>
<td>$D_{11}$</td>
<td>5,261.2</td>
<td>-4,738.8</td>
<td>23,693.8</td>
</tr>
<tr>
<td>$A_{12}$</td>
<td>-5,067.5</td>
<td>4,932.5</td>
<td>24,662.4</td>
</tr>
<tr>
<td>$B_{12}$</td>
<td>-3,426.3</td>
<td>18,928.1</td>
<td>28,371.6</td>
</tr>
<tr>
<td>$C_{12}$</td>
<td>3,620.7</td>
<td>18,368.4</td>
<td>27,532.7</td>
</tr>
<tr>
<td>$D_{12}$</td>
<td>4,645.4</td>
<td>5,354.6</td>
<td>26,773.0</td>
</tr>
<tr>
<td>$A_{21}$</td>
<td>5,569.2</td>
<td>-4,430.8</td>
<td>22,154.2</td>
</tr>
<tr>
<td>$B_{21}$</td>
<td>5,261.2</td>
<td>4,738.8</td>
<td>23,693.8</td>
</tr>
<tr>
<td>$C_{21}$</td>
<td>20,532.4</td>
<td>2,869.0</td>
<td>19,223.0</td>
</tr>
<tr>
<td>$D_{21}$</td>
<td>19,673.4</td>
<td>-3,167.4</td>
<td>20,510.6</td>
</tr>
<tr>
<td>$A_{22}$</td>
<td>4,953.3</td>
<td>5,046.7</td>
<td>25,233.4</td>
</tr>
<tr>
<td>$B_{22}$</td>
<td>3,923.5</td>
<td>17,496.5</td>
<td>26,225.8</td>
</tr>
<tr>
<td>$C_{22}$</td>
<td>15,414.2</td>
<td>13,379.4</td>
<td>23,716.5</td>
</tr>
<tr>
<td>$D_{22}$</td>
<td>19,660.5</td>
<td>3,171.9</td>
<td>20,530.0</td>
</tr>
</tbody>
</table>
6. Discussion

The specificity of the roof structures $\Omega$, created with the help of the innovative method, is related to the fact that the individual shell segments $\Omega_j$ are distributed on the basis of reference surfaces $\omega_i$ with negative Gaussian curvature. To construct a complex building structure having this property the points of intersection of the reference surface $\omega_i$ with all side edges of the reference network $\Gamma$ have to lie between the vertices of $\Gamma$. For example, the point $S_{C11}$ lies on the side edge $c_{11}$ between the vertices $W_{CD11}$ and $W_{BC11}$, Fig. 4.a. Therefore, the value of the division coefficient of the above pair of vertices by $S_{C11}$ must belong to the range $(0, 1)$. The vertex $C_{11}$ of the mesh $B_{v11}$ must lie also on the same straight line $c_{11}$. The value of the division coefficient of $\{W_{CD11}, W_{BC11}\}$ by $C_{11}$ must also belong to $(0, 1)$. The position of $C_{11}$ in relation to $S_{C11}$ is dependent on the difference between the values of the adopted division coefficients.

The characteristic feature of the created structures $\Omega$ and $\Gamma$ is that the vertices $A_{ij}$, $B_{ij}$, $C_{ij}$ and $D_{ij}$ of the subsequent meshes $B_{vij}$ of $B_v$, lie alternately above and below the arbitrary reference surface $\omega_i$. This action is aimed at achieving specific properties of the ribbed roof structures $\Omega$ in accordance with the properties of the reference surface $\omega_i$. Additionally, the size of folding of $\Omega$ depends mainly on the differences between the values of the division coefficients corresponding to all vertices of $B_v$ and points of $\omega_i$.

For the examined continuous roof structures $\Omega$, the positions of the examined tetrads of vertices, e.g. $C_{11}$, $D_{12}$, $A_{22}$ and $B_{21}$ of four adjacent meshes, e.g. $B_v{11}$, $B_{v12}$, $B_v{22}$ and $B_{v21}$, are identical. Therefore, their division coefficients are assumed to be equal. This is the property specific to some group of structures $\Omega(\Sigma)$ called basic. The structure presented in Fig. 6 in the previous section belongs to this group.

It should be summarized that the vertices $A_{ij}$, $B_{ij}$, $C_{ij}$ and $D_{ij}$ of the subsequent meshes $B_{vij}$ of $B_v$ alternate above and below $\omega_i$. Therefore, the division coefficients assigned to these vertices are divided into two different groups. The values of the division coefficients belonging to one group are greater than the values of the division coefficients corresponding to the points of $\omega_i$. The values of the division coefficients belonging to other group are smaller than the values of the division coefficients corresponding to these points of $\omega_i$. However, these values are not significantly different from those related to $\omega_i$. Since the above-mentioned vertices of the four adjacent meshes $B_{vij}$, $B_{vij+1}$, $B_{vij+1}$ and $B_{vij+1}$ have one common vertex $C_{ij}$, the respective division coefficients have to take equal values.

If a discontinuous structure is to be created, it is most convenient to build it as a derivative configuration of the base continuous structure. Each discontinuous structure is characterized by the fact that at least one of four common vertices $C_{ij}$, $B_{ij+1}$, $D_{ij+1}$ and $A_{i+1j+1}$ of four adjacent meshes $B_{vij}$, $B_{vij+1}$, $B_{vij+1}$ and $B_{vij+1}$ is shifted along the respective side edge $c_{ij}$ with respect to three remaining vertices. An analysis was carried out to explore the possibility of creating different groups of similar discontinuous structures derived from the base continuous structure $\Omega_c(\Sigma_c)$ presented in Fig. 6. The form $\Sigma_c$ was chosen because it is characterized by an attractive general form and specific properties that allow for rational design of its structure. To create a few derivative structures $\Sigma_d$ and $\Omega_d$, an analysis was conducted to determine different mutual locations of the above-mentioned tetrads of vertices located on the same side edge of $\Gamma$, which was previously created for $\Sigma_c$. A comprehensive description of the detailed results of this analysis goes beyond the scope of this work.
This article presents the description of two different derivative structures, which have particularly attractive forms and represent two different groups of discontinuous structures, as the result of this analysis. The first of these structures is shown in Fig. 7. Its characteristic feature is that two specific pairs of identical vertices of four adjacent meshes of $B$, are distinguished on each side edge of $\Gamma$. One pair consists of two identical vertices belonging to two adjacent meshes located diagonally in $B$. The second pair also consists of two identical vertices, but they belong to two other adjacent meshes located in a different diagonal strip of $B$. These two strips also run diagonally to the first central mesh. Thus, each mesh $B_{ij}$ has two opposite vertices in common with the adjacent meshes of the first diagonal strip, and its other two vertices are shared with the corresponding vertices of other two adjacent meshes belonging to the second diagonal strip.

It should be added that the vertices belonging to the first diagonal strip, as discussed above, lie below the reference surface. However, the vertices belonging to the second diagonal strip lie above this surface. The position of the vertices of these two groups is generated using the respective different values of the division coefficients. Depending on the differences in the values of the division coefficients associated with these quadruples of all $B_i$’s vertices, the structure $\Omega$ can be more or less folded, resulting in the larger or smaller flat discontinuous areas for the window openings. In the case of the first discontinuous derivative structure (Fig. 6), these openings are triangular. In the second case, (Fig. 8), the openings are square. The proportions between the areas of flat elevation walls, flat roof windows and shell roof sectors, as well as the lengths of all edges of $\Sigma$ and $\Omega$, determine the attractiveness and rationality of the structures shaped by appropriately selected values of the division coefficients. However, a detailed discussion of these architectural issues is beyond the scope of the study.

Other characteristic features of the complex forms created with the algorithm of this method are: (1) different inclination of the adjacent facade walls to the vertical, and (2) various inclination of the facade walls with their bases shifted to the inside or outside of each considered building form, depending on the direction of the axis $v_{ij}$ or $u_{ij}$ of $\Gamma$.

7. Conclusions

The novel method for modelling unconventional building forms with complex folded elevation walls, inclined to the vertical and roofed with complex transformed shell structures composed of many shell units arranged on regular surfaces characterized by the negative Gaussian curvature, is presented. The algorithm of this method enables the determination of innovative basic ribbed continuous and derivative discontinuous shell roof structures. The method complements the existing methods related to shaping polyhedral free forms roofed with structures composed of many shells arranged on regular surfaces with the positive Gaussian curvature.

The procedure of this method is presented using three specific examples of parametric building forms, controlled with specific sets of division coefficients that express the proportions between the distances of the characteristic vertices of their multi-plane elevation walls and multi-shell roofs, as well as novel auxiliary polyhedral and polygonal networks. The ranges of variability for these coefficients are also discussed to create several specific types of novel building forms. Furthermore, the main relationships governing the positions of all vertices of the polyhedral network $\Gamma$, eaves network $B_i$ (shell structure $\Omega$) and the values assigned to these vertices as partition coefficients are presented.
The characteristic feature of the presented method is that the vertices of the designed shell roof units are generated using division coefficients that take values from the range (0, 1) in relation to the vertices of the reference polyhedral networks $\Gamma$. This limitation results in the fact that the innovative ribbed roof structures are limited to those that are composed of transformed roof shells arranged in conformity with the selected saddle surface, while the facades are limited to polyhedral structures. Additionally, all individual cells of $\Gamma$ are tetrahedrons, and all shell roof units are hyperbolic-parabolic segments limited by spatial quadrangles.

The algorithm of this method, as presented in these examples, allows for further exploration of innovative and visually appealing types of the unconventional complex building forms, roofed with multiple complete transformed shells assembled into complex ribbed shell structures based on reference surfaces with almost any Gaussian curvature. Further research will focus on optimizing the forms and positions of these ribbed roof structures in relation to the smooth reference surfaces. Additionally, research has already begun on development of a parametric method for shaping structural systems intended for these unconventional building forms.

References

Unconventional building forms roofed with innovative structures arranged on regular surfaces...


