

Analysis of viscoelastic behaviour in asphalt pavement through four-point beam bending tests

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Abstract: This study, conducted in accordance with ASTM T321-14 standards, offers crucial insights into the behaviour of asphalt materials subjected to cyclic loading. For proper maintenance and pavement design, it is essential to understand the material response under different loading conditions. This study focuses on the four-point beam bending test to investigate the viscoelastic behaviour of asphalt pavement. The four-point beam bending test is a useful method for determining the material's ability to withstand cyclic loading and deformation, which the material experiences during field traffic conditions. The experimental setup involves subjecting asphalt samples to cyclic loading using a four-point bending apparatus. The imposed load causes the specimen to experience bending strains, representing the actual loading conditions that pavements endure. The data gathered during testing include stress, strain, and deformation properties under various loading conditions. The stress-strain response demonstrates the material's resilience to fatigue, with a gradual decrease in stiffness beyond 10,300 cycles. Fatigue failure criteria include a 50% reduction in initial stiffness for strain-controlled fatigue tests and cracking in stress-controlled tests. The dynamic modulus in a compressive-type, repeated load test follows a three-phase pattern, highlighting the impact of temperature and binder characterization methods on sample performance. The

results provide information about the material's resilience to rutting and fatigue cracking, the most significant distresses indicated in asphalt pavements. The findings from this study contribute to an in-depth understanding of the viscoelastic behaviour of asphalt pavement and can aid in the development of improved design guidelines and maintenance strategies characterizing the material response to cyclic loading. Engineers and researchers can make better decisions on the durability and performance of asphalt pavements, resulting in more cost-effective and sustainable road infrastructure.

Keywords: asphalt mixture, deformation properties, four-point beam, fatigue cracking, dynamic modulus

1. Introduction

The fatigue performance, deformation resistance, and deformation properties of asphalt mixtures significantly impact the operational efficiency of asphalt pavements. During construction, the asphalt binder binds the aggregate particles in the pavement's surface layer. Therefore, material characteristics must be considered from the time of construction to the point of disposal or recycling [1, 2].

Asphalt concrete comprises a mixture of aggregates, asphalt, and air voids. Throughout its service life, asphalt pavement contends with traffic, environmental factors, and inconsistencies in construction [3]. Based on these factors, the pavement may exhibit various behaviours. The characteristics of the asphalt binder significantly affect the mechanical properties of an asphalt mixture [4]. Altering the percentage of fine aggregate in asphaltic concrete mixes substantially impacts the rheological qualities of the resulting materials [5]. Mineral fillers significantly affect HMA performance. One major issue in highway pavement construction is the scarcity of crusher dust from aggregate crushing [6]. Today, it is generally recognized that asphalt concrete responds differently depending on temperature rate, time, and exhibits non-linear behaviour under various loading circumstances [7]. One of the main causes of premature failure is reflective cracking or the propagation of existing cracks in the newly constructed pavement [8]. Recycling asphalt pavement also involves creating a bituminous mixture from conventional materials. The milling process, known as RAP, often produces an old and degraded pavement. If successful, RAP will produce a material with qualities similar to those of a conventional asphalt mixture [9]. The primary distress affecting asphalt concrete pavements is low temperature cracking, which considerably reduces the durability and service life. The bitumen's viscoelastic characteristics mainly govern the viscoelastic response of asphalt concrete [10]. On the other hand, the asphalt concrete used in pavements is a heterogeneous mixture composed of air spaces, coarse aggregate ($d > 2.36$ mm), and asphalt mortar [11]. An in-depth knowledge of the damage mechanisms in asphalt pavements is essential due to the inelastic mechanical behaviour of the asphalt concrete and complex microstructural characteristics [12]. Asphalt mixtures fall within the category of quasi-brittle materials, where the fracture mechanism is typically considered a complex phenomenon that occurs in a zone prior to the crack tip, usually referred to as the fracture process zone (FPZ) [13]. The critical region for load-induced cracking is typically thought to be the bottom of the asphalt mixture layer, where repetitive tensile strains cause cracks to begin and eventually spread. Determining fracture characteristics is critical for reducing crack development in asphalt pavements due to the considerable cost and time involved [14]. Understanding the fundamental material parameters is necessary to determine whether the induced pavement response is severe enough to cause failure under one or more load applications [15].

Simple flexure tests are the most commonly used technique for collecting information on fatigue life. This loading method is highly effective for simulating fatigue damage at the base of the asphalt layer. Improved servo-hydraulic and computer-controlled pneumatic loading systems have led to an increase in the application of three-point bend (3PB) and four-point bend (4PB) setups with rectangular beam specimens. The four-point beam bending test, often referred to as the flexural beam fatigue test, is probably the most widely used fatigue test in the United States [16]. A beam supported by two loading points and two supports is subjected to a load, and the midpoint deflection is measured. The results of the test reflect the material's strength, stiffness, and resistance to bending stresses. Fatigue cracking and fracture cracking are the two most common types of cracking that impact the life of asphalt pavement [15]. The four-point beam bending test and its applications in several fields have been the subject of many research investigations conducted in recent years. The test accurately measured the flexural strength and stiffness of the mixtures, which could potentially be used to optimize the design of asphalt pavements [17].

An experimental study conducted by [18] evaluated the applicability of the four-point beam bending test for assessing the stiffness of asphalt mixtures. The findings suggest that the test was reliable and could provide accurate stiffness measurements. However, the results were sensitive to specimen preparation and loading conditions. The application of the four-point beam bending test for evaluating the fatigue performance of asphalt mixtures was also investigated by [18]. The test results were useful in predicting the fatigue life of asphalt mixtures, although they were susceptible to temperature and loading frequency.

The stiffness and fatigue performance of recycled asphalt pavement were evaluated using the four-point beam bending test [19]. The findings suggest that the test was successful in forecasting the stiffness and fatigue performance of recycled asphalt pavement. Furthermore, the test findings were not significantly impacted by the type of loading mechanism used. The test techniques provide the basic criteria that are significant in the design and operation of pavement construction materials. Materials can be categorized into three groups based on how they deform: solid, viscous, and plastic. The Boltzmann theory of linear viscoelastic materials, which considers the consequences of repeated stress, describes the behaviour of materials used in road construction over short- or long-term periods. Most materials used in road construction fall into the category of linear viscoelastic matter, which is mechanically modelled in the Boltzmann theory by a system of springs and dampers arranged either in series or parallel, capable of experiencing reversible or variable permanent deformations [4]. The percentage of air voids is high when initially constructed, which is why deformation due to applied load and compaction can be severe [8]. Repeated load tests under uniaxial pressure are one method used in laboratories to evaluate the deformation behaviour of asphalt concrete. According to [20], the behaviour of asphaltic materials under a constant axial load is typically divided into three stages. During the first two stages, the energy accumulated in the mixture due to load repetition results in the growth and initiation of microcracks [21]. Highly accurate mathematical and computer-based techniques have been used to determine the boundaries of the second and third stages and the flow number (FN). The FN test can be described by higher regression coefficients from these methods.

Numerous investigations have demonstrated that applying repeated loads causes microdamage in the sample. By weakening the bonds between the bitumen and the aggregate, this microdamage is produced along with a loss of the material's strength and modulus. Finally, in the third stage, shear flow in the sample occurs because of the connectivity of microcracks leading to the development of macrocracks. Pavement deterioration has always been primarily caused by cracking in asphalt pavements [22].

The modelling of HMA's viscoplastic behaviour was the subject of the study [1]; their conclusions indicate that the viscoplastic models lack the calibration of cyclic loading.

The review of previous literature suggests that the most common technique for evaluating the fatigue life of asphalt pavement is flexure testing, which effectively simulates the fatigue damage that develops at the base of the asphalt layers due to the accumulation of induced stresses. Additionally, under compressive concrete loading cycles with intermittent rest time, significant variation is observed in the viscoplastic characteristics of asphalt concrete. Asphalt pavements experience premature cracking and rutting throughout their service life due to high temperatures and uncontrolled axle loads, requiring regular maintenance which places an extra burden on the nation's economy. Determining the relevant factors for the mix design of asphalt concrete is a pressing need to improve the required characteristics of asphalt pavement. The objective of this research is to evaluate how asphalt responds to repeated load cycles under various temperatures and loading conditions using a four-point beam bending test.

2. Experimental program

2.1. Materials selection

During the initial stage, a reconnaissance study was conducted to gather asphalt samples from an appropriate location as per [23]. These samples were obtained from the Pir Qala Bypass Road following its construction to assess the characteristics of the field-collected materials. The entire mixture was then transported to the UET Taxila laboratory, where the asphalt samples were pre-heated before the preparation of the required samples. The binder content remained consistent at 4.2% across all mixtures. An additive significantly affected the characteristics of the asphalt binder. Laboratory tests, including the Marshall test, were conducted to identify the optimum bitumen content (OBC) for the hot mix [24]. The Marshall mix-design technique was used to create hot mix asphalt mixes, with increasing proportions of recycled aggregates replacing coarse virgin aggregates (0%, 25%, 50%, and 75%). The produced mixtures were compared to a control sample [25]. It was observed that an increase in the proportion of recycled aggregates decreased the stability of the mixes. Additionally, the amount of air voids and mineral aggregate-filled cavities increased. A regularly used performance grading (PG) system, storage stability test, and various conventional and rheological tests were carried out to optimise the dosage of the STF into the bitumen (investigating the influence of 2%, 4%, and 6% of STF on the bitumen) and define the related features [26]. The results demonstrate that applying 4% STF by weight of the binder (an asphalt binder with a penetration grade) enhances high-temperature viscoelasticity and bitumen performance grading. The chosen asphalt mixture is Grade-B, which was carefully prepared in an Asphalt Plant following a suitable mix design. The parameters obtained are shown in Table 1. These findings can be used to optimise designs, improve material selection, and ensure the structural integrity and safety of the beam in real-world applications.

Table 1. Various bitumen test results

S. No	Test Type	Values	NHA Specification
1	Ductility Test	101.1 cm	100 cm Min
2	Softening Point	51.0°C	49-56°C
3	Penetration Test	67.6 mm	60-70 mm

2.2. Distribution of particle size

Table 2 presents a tabular representation of the asphalt concrete's particle size distribution. This data was obtained from the Fine Home Asphalt Batching Plant.

Table 2. Distribution of particle sizes from field-collected asphalt (Fine Home Asphalt Plant)

Sieve No	Particle Size (mm)	Weight Retained Cumulative	Cumulative percent Retained	Percent Finer	Percent Finer	Specificati on NHA-B
3\4	19	NIL	NIL	100	100	100
1\2	12.5	236	20.6	84.3	184.3	75-90
3\8	9.5	337	32.9	75	259.3	60-80
No. 4	4.75	513	44.7	55.3	314.6	40-60
No. 8	2.36	876	76.3	23.7	338.3	20-40
No. 50	0.3	1086	94.6	5.4	343.7	5--15
No. 200	0.075	1113	97	3	346.7	3--8

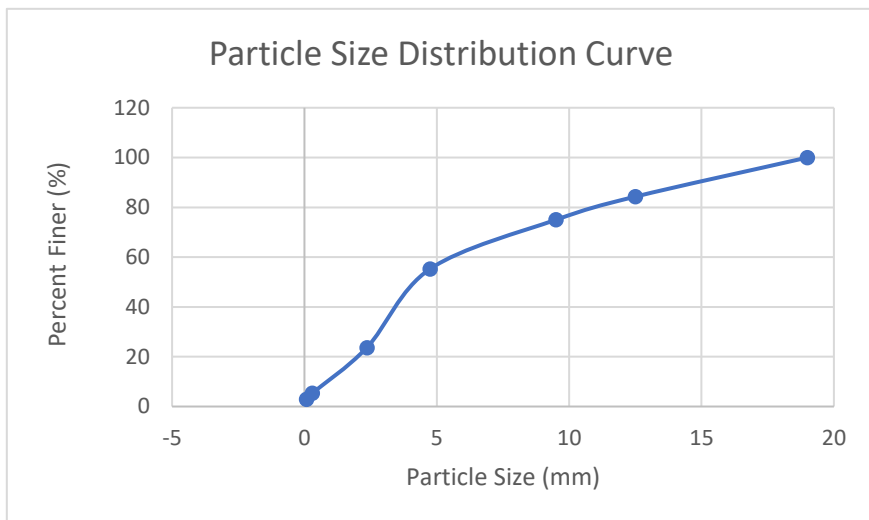


Fig. 1. Distribution of particle sizes curve for field-collected asphalt

2.3. Sample preparation

To create a 450 x 450 x 75 mm slab, a batch of aggregate weighing 16,000g was prepared. The slab was then placed beneath a roller compactor for compaction. The roller compactor underwent a total of 30 cycles of repeated loading, following a specific pattern: 10 cycles were applied at Pressure 1, followed by 10 cycles at Pressure 2, and finally, 5 cycles each at Pressure 3 and Pressure 4. Once the compaction process was completed and the sample had cooled down, the slab was cut into 5 beams, each measuring 450 mm in length and 75 mm in height. The preparation of the sample is depicted in Fig. 2(a, b, c).



Fig. 2. a) Compacted the asphalt using compactor, b) asphalt slab after compaction, c) beam samples after saw cutting to uniform size

2.4. Storage of sample prior to testing

When cutting the asphalt slab with a saw, extra care must be taken to protect the mix and shape of the material. The slab is neatly cut into five identical pieces. Water is used to reduce cutting friction. Prior to testing, the beams are stored on a flat platform in a refrigerator at a temperature of 0.5°C .

2.5. Test setup and procedure

The test system apparatus comprises three primary parts: 1) monitoring and data gathering, 2) environmental chamber, 3) loading device.

2.6. Universal servo-pneumatic testing machine

Advanced collection and control techniques, high-precision servo-pneumatic valves, low-friction actuators, and servo-hydraulic systems can be used to demonstrate the effectiveness of various servo-hydraulic systems. The actuators provide test samples with accurate, digitally generated waveforms, generating repeatable stress changes that replicate the stress fluctuations experienced by the pavement with passing vehicles. Compressive and tensile forces can be applied simultaneously due to the dual actions of the actuators. In Fig. 3(a, b), the servo-pneumatic UTM is shown.



Fig. 3. a,b): Servo-pneumatic universal testing (UET Taxila Lab)

2.7. Data collection and monitoring unit

The data collection and monitoring unit should be capable of measuring the beam and controlling the loading conditions. Due to the regulation of loading conditions, the sample should be able to endure repeated cycles of loading at constant rates and displacements. Throughout the loading cycle, the following information must be recorded at user-specified intervals: 1) Load cycles, 2) Temperature, 3) Maximum tensile strain, 4) Maximum tensile stress, 5) Phase angle, 6) Beam displacement, 7) Stiffness.

2.8. Loading pattern

The loading pattern typically involves applying two concentrated loads at specific positions along the beam. The stepwise loading pattern is illustrated in Fig. 4(a, b, c). Initially, the chamber is set to the desired temperature, and the beam is placed horizontally on two support rollers or anvils. Measure and mark the positions where the concentrated loads will be applied. These positions are usually specified as fractions of the beam span; then mark the first load position, denoted as "a," at $0.1L$ from the left support (where L is the total span of the beam) and the second load position, denoted as "b," at $0.4L$ from the left support. The centre span length between clamps is maintained at 117.5mm, while the loading temperature is set to 20°C [27]. A computerized data recording system was used to record the load and the mid-span deflection during the flexural tests.



Fig. 4. a): Adjusting the temperature (UET Taxila), b): adjusting the rotation clamps, c): sinusoidal wave loading pattern

3. Experimental program

Flexure testing is a common method for determining the fatigue life of compacted hot mix asphalt (HMA) subjected to repeated cyclic loads (AASHTO 2005). This test provides information about the relationship between the applied load and the associated deflection, which can be plotted to observe the beam's behaviour under flexural loading. The load-deflection curve can indicate the stiffness, ductility, and strength of the material. Several methods are used to investigate the penetration point, softening point, and stability of the field-collected asphalt. It also aids in identifying the beam's elastic and plastic deformation areas. The permanent deformation is represented by the plastic region, while the reversible deformation is indicated by the elastic region. The test can provide information about the material's yield point, ultimate strength, and post-yield behaviour. Additionally, the test helps to determine the mode of failure of the beam, which could be a fracture, significant deflection, or another failure mechanism. Observing the failure mechanism allows for an analysis of the

beam's structural integrity and the material's suitability for the intended application. Overall, the results and overview of a 4-point beam bend test provide valuable insights into the tested material's mechanical behaviour, strength, and performance.

3.1. Flexural stiffness

According to ASTM T321-14, the experimental testing data are plotted against the number of cycles to calculate the flexural stiffness at a specific cycle number. Figure 5 shows the relationship between stiffness and loading cycles, which reflects the material's ability to withstand applied loads and maintain its shape under bending stress. In this fatigue 4-point beam bend test, cyclic bending loads are applied at two points along the specimen's length, with deflection measured at two additional points. The test continues until either the specimen fails or a predetermined number of cycles is reached. Throughout the fatigue test, the material undergoes alternating stresses and strains, transitioning between tension and compression due to bending. Calculate the product of the load cycle and the flexural stiffness ($S \times N$) for each load cycle for which data were gathered. Using appropriate smoothing techniques—adding cubic splines and sixth-order polynomials to $S \times N$ curves—to remove any irregularities in the data collection. According to ASTM T321-14, the data line is well-fitted with an R^2 value of 0.9155. The research findings indicate that the analysed sample material exhibits low flexural stiffness; this characteristic is reflected in the material's pronounced deflections and deformations under cyclic loads, resulting in the formation of localized stress concentrations and an accelerated rate of fatigue damage.

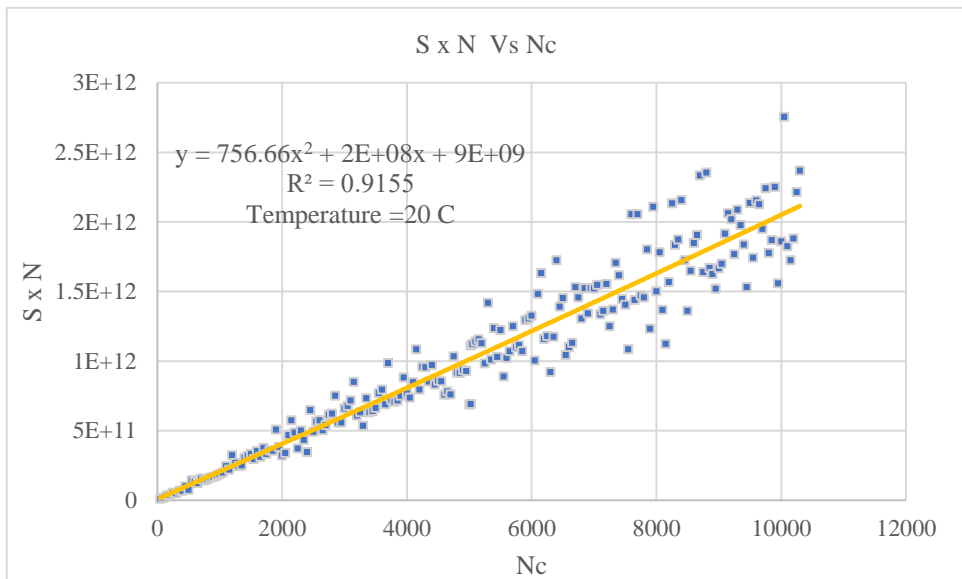


Fig. 5. Typical $S \times N$ vs Number of Cycles graph under fatigue loading

3.2. Stiffness modulus

The stress-strain response of pavement under traffic loads is significantly influenced by the stiffness modulus of bituminous materials, reflecting their ability to resist deformation and maintain overall stiffness. Figure 6 illustrates a gradual decrease in stiffness, with no

abrupt decline observed beyond 10,300 cycles, where stiffness is initially set at 100%. The R^2 value of 0.547 indicates a moderate level of data fitting. A thorough understanding and accurate estimation of the stiffness modulus are vital for designing resilient pavements, optimizing asphalt mixtures, and assessing susceptibility to cracking and rutting. The fatigue failure criteria for strain-controlled fatigue tests include a 50% loss in initial stiffness, and for stress-controlled fatigue testing, the beam specimens must fracture. Ongoing research and advancements in testing techniques and modelling approaches continue to enhance our understanding of the stiffness modulus, contributing to more reliable and efficient pavement designs capable of withstanding challenges posed by heavy traffic loads and environmental conditions.

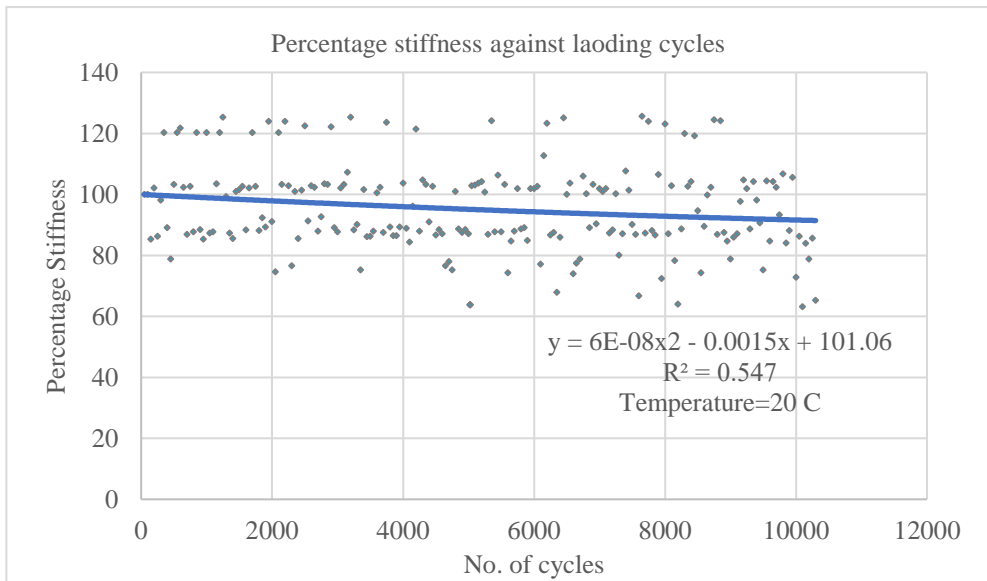


Fig. 6. Typical percentage stiffness vs number of cycles graph under repeated loading

3.3. Fatigue graph

The primary causes of fatigue cracking and rutting are the tensile strain at the base of the asphalt layer and the constant deformations that accumulate on the road surface as a result of repeated wheel load applications. Although predicting the life of asphalt pavements and the depth of ruts can be challenging, the projected fatigue can be computed by plotting the number of cycles against strain. Figure 7 illustrates the relationship between variables, with the x-axis representing log micro-strain, typically plotted on a logarithmic scale. The y-axis represents the log cycle experienced by the asphalt specimen, measured in millimetres as per ASTM D 7460. As the test progresses, the strain on the specimen increases with each subsequent load cycle. This increase in strain illustrates the development of fatigue damage and deformation. The graph shows different trends depending on the behaviour of the asphalt beam under repeated loading. The R^2 value of 0.8607 indicates a good fit of the line to the data. Initially, there may be a linear or non-linear region, indicating elastic behaviour where the strain increases proportionally to the number of load cycles. This region is followed by a distinct curvature, representing the transition from elastic to plastic deformation.

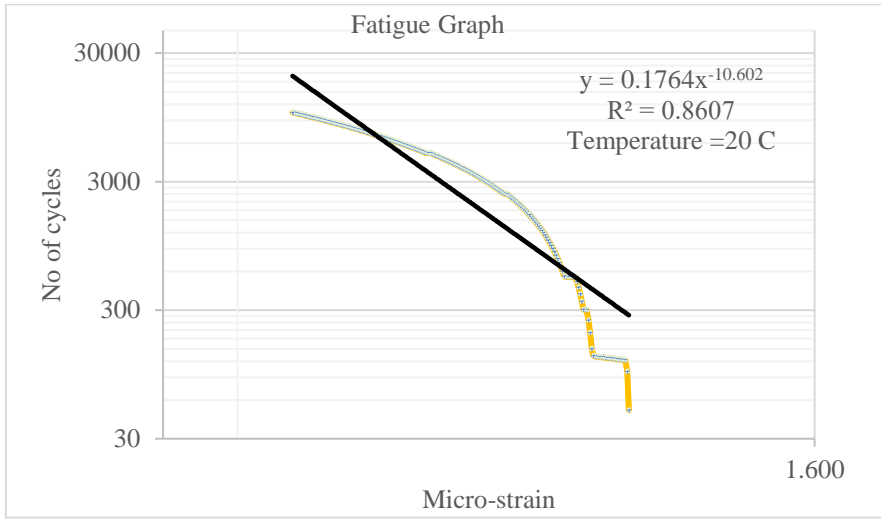


Fig. 7. Typical fatigue graph under repeated loading

3.4. Dynamic modulus of asphalt under varying frequencies

The dynamic modulus serves as an indicator of the stiffness of the asphalt material in a compressive-type repeated load test. The dynamic modulus can be calculated by studying the relationship between stress and strain under cyclic loading, as shown in Fig. 8.

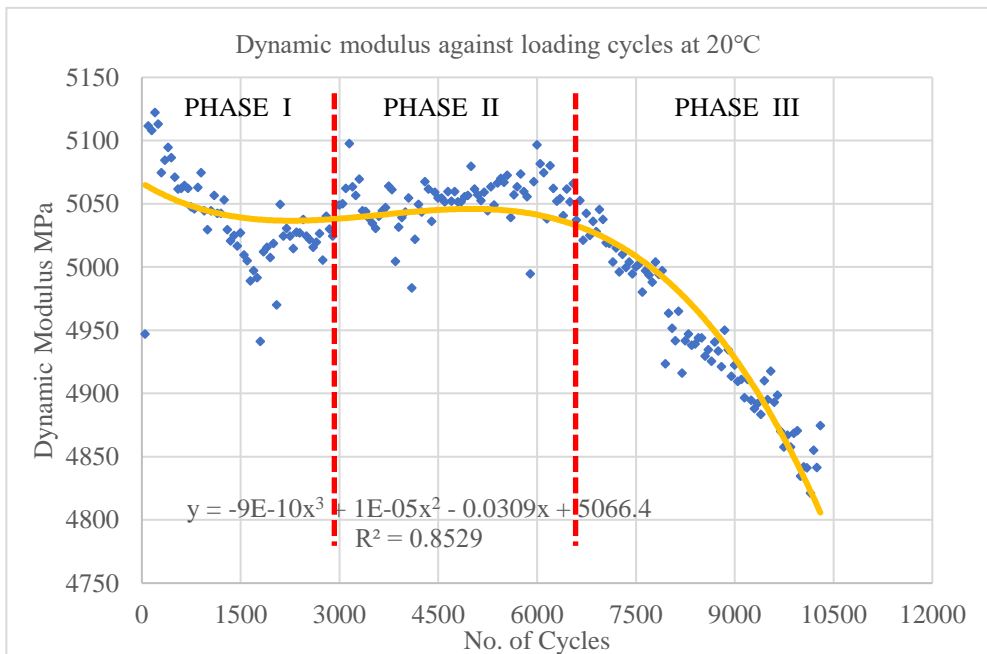


Fig. 8. Typical dynamic modulus vs loading cycle plot in fatigue test

It is important to note that frequency affects the dynamic modulus. The best-fit trend line is produced using a third-order polynomial curve due to the inconsistent behaviour of the data and the presence of two humps. Phase I exhibits an abrupt decrease in dynamic modulus with repeated load cycles. As the number of load repetitions increases in the second stage, the dynamic modulus decreases linearly, indicating a consistent pattern of fatigue damage accumulation. However, the formation of microcracks in the third stage leads to a significant decrease in the stiffness modulus. Results demonstrate that the performance of the investigated samples varies depending on the temperature and binder characterization method. As can be seen from the graph, there is no abrupt change observed during Phase I and Phase II, but at Phase III, the material completely loses its resistance to applied loading, and failure is said to be initiated at approximately 10,500 cycles. As shown in Fig. 8, the R^2 value is 0.8529, indicating a good fit of the line to the data.

4. Conclusions

In summary, this study was conducted in accordance with ASTM T321-14 standards, which provide critical insights into asphalt material behaviour under cyclic loading. The observed low flexural stiffness, as evidenced by noticeable deformations, localized stress concentrations, and accelerated fatigue damage, determines the material's response to repeated traffic loads. The evaluation of fatigue life using laboratory beam specimens, which conform to ASTM T321-14 standards, exhibits significant reductions in stiffness and occurrences of cracking under both strain-controlled and stress-controlled conditions. The resilience to fatigue, characterized by a gradual decrease in stiffness beyond 10,300 cycles, highlights the material's ability to withstand cyclic loading. Ongoing research and testing advancements contribute to a deeper understanding of the stiffness modulus, facilitating the development of more robust pavement designs. The observed strain increase with each load cycle provides a useful tool for predicting pavement fatigue life, while dynamic modulus analysis in compressive-type repeated load tests follows a three-phase pattern, emphasizing the effect of temperature and binder characterization on sample performance. Overall, this study provides useful insights into optimizing asphalt pavements under cyclic loading conditions.

5. Recommendations

In fatigue tests, it is crucial to strictly adhere to the test method to obtain accurate results, as minor deviations from the specifications can significantly impact the outcomes. Testing frame rigidity, specimen alignment, transducer calibration, input and output signal format, data collection accuracy, interpretation, and testing frame rigidity must all be carefully reviewed. It is recommended that the functional fatigue test be used to determine the pavement's life expectancy based on operational performance. The 4PB beam test should be conducted at 30°C and 40°C to calibrate the results at 20°C. It is also suggested that the experimental results be numerically simulated to calibrate and validate the data.

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