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APPLICATION OF AN ELECTROMAGNETIC NUMERICAL MODEL IN ACCURATE MEASUREMENT OF HIGH VELOCITIES

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Abstract. *The velocity of various objects measured within a large number of disciplines and activities. This paper presents the process of designing an accurate method and equipment for the measurement of velocity in one-shot nonlinear processes, which occur only once and are thus characterized by zero repeatability. The measurement methods must therefore enable the recording, saving, and retroactive evaluation of the processes at a pre-defined accuracy; all these operations are performed to facilitate comparison of the recorded event and other similar processes. However, the electromagnetic method described in the paper does not include the disadvantages of known optical methods. We therefore present the design of an inductive sensor equipped with an electronic signal processing system. This design is based on numerical evaluation of the relativistic effect occurring during the application of the electromagnetic principle in sensing the position and velocity of an object J. Van Bladel. The final section of the paper contains a discussion of the measured results. The authors investigate the use of a coupled model of the magnetic field and analyze the motion of a conductive object in this field. The analysis shows that, for an exact evaluation of the influence of all effects, it is necessary to consider the phenomena related to the movement of a system relative to the other one. It is shown that related distinctive effects affect the resultant electromagnetic field distribution already at the relative velocity of* $v_0 = 1m \cdot s^{-1}$.

Keywords: relativistics model, numerical model, FEM, electro-hydro-dynamics, moving objects, projectile

ZASTOSOWANIE ELEKTROMAGNETYCZNEGO MODELU NUMERYCZNEGO W DOKŁADNYCH POMIARACH DUŻYCH PRĘDKOŚCI

Streszczenie. *Pomiar prędkości różnych obiektów ma zastosowanie w wielu dziedzinach i działaniach. W artykule przedstawiono proces projektowania dokładnej metody i urządzenia do pomiaru prędkości w niepowtarzalnych procesach nieliniowych, które występują tylko raz. Metoda pomiaru musi zatem pozwalać na nagrywanie, zapisywanie i wsteczną ocenę procesów przy zadanej dokładności; wszystkie te operacje są wykonywane w celu ułatwienia porównania nagranego zdarzenia i innych podobnych procesów. Opisana w artykule metoda elektromagnetyczna nie zawiera wad znanych sposobów optycznych. Została opisana konstrukcja czujnika indukcyjnego wyposażonego w elektroniczny układ przetwarzania sygnału. Metoda ta jest oparta na numerycznej ocenie wystąpienia efektu relatywistycznego przy stosowania zasady indukcji elektromagnetycznej do wykrywania położenia i prędkości obiektu (J. Van Bladel). Ostatnia część artykułu zawiera omówienie wyników pomiarów. Autorzy badają użycie sprzężonego modelu pola magnetycznego i analizują ruch przewodzącego obiektu w tym polu. Analiza wykazuje, że dla dokładnej oceny wpływu wszystkich oddziaływań, należy wziąć pod uwagę zjawiska związane z ruchem jednego systemu w stosunku do drugiego. Wykazano, że związane z nimi efekty mają wpływ na wynikowy rozkład pola elektromagnetycznego już przy prędkości względnej równej 1m/s.*

Słowa kluczowe: model relatywistyczny, model numeryczny, PEM, elektrohydrodynamika, pocisk

Introduction

The velocity of fast-moving objects is determined by measuring the position and time. Thus, in most measurement procedures, it is important to detect the position of the object at the start and end points of the measured path and to determine the length of the time during which the monitored body travels the given distance. The detection of the projectile is performed using two parallel structures (ports) placed at an exactly defined distance from each other; this distance is referred to as the basis. As the projectile must pass through both ports, the structures have to be positioned in a perfectly parallel manner. The block diagram of this type of measurement system is shown in Fig. 1.

The individual ports record the passage of the projectile and generate the START – STOP pulses, whose function is to activate and deactivate the fast counter (a microcontroller unit) measuring the time at which the projectile passes through the ports. Based on the known dimension of the basis and the passage time of the projectile, we can easily acquire the value of the instantaneous speed of the projectile flight over the path given by the basis of the sensor. However, it is also necessary to consider possible measurement errors Boquan, Li [2].

1. Velocity measurement

The ports measuring the velocity of a fast-moving object are based on either the optical or the electromagnetic principle, and each of these two approaches has been used to fabricate several types of PROTOTYPA [16] ports.

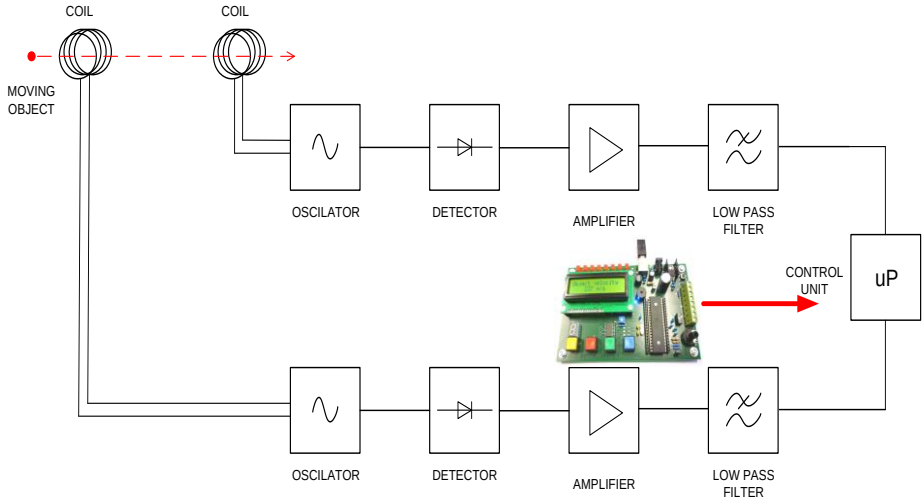


Fig. 1. A block diagram of the system for the measurement of fast-moving objects

Fig. 2. The different port types: a) diode, b) laser, c) electromagnetic

Magnetoelectric methods utilize mainly coils, whose properties are influenced by the passage of the projectile; alternatively, voltage can be induced in these coils. One of the variants of the magnetoelectric methods is a technique where magnetic flux density is used to generate a pulse during the passage of the projectile through the coil. The magnetized projectile is surrounded by a static magnetic field; during the passage of the projectile through the measuring coil, this field will be changing proportionally to the speed of the projectile and depending on the relative motion of the object and the port Fiala P., Szabo Z., Friedl M. [8]. With respect to the fact that the measurement of the instantaneous velocity $\boldsymbol{v}$ of moving objects is converted to the measurement of position, the design of the port was realized using the principle of eddy currents known from inductive position sensors with the effect of the theory of relativity Stratton [17], Haňka [9]. The principle of inductive sensors with a suppressed magnetic field consists in suppressing the magnetic field of the sensor by the magnetic field of the eddy currents. In wide-basis sensors (1.000 m), Fig. 2, it is possible to use known compensation methods to eliminate the effect of the change of the shape of the induced voltage in the sensor; such a change is caused by the velocity of the object entering and leaving the magnetic field. By contrast, short-basis measurement (0.1000 m) is rather more complicated due to the emphasis placed on observing the parameters of the applied method to secure high measurement accuracy and thermal stability. In this type of measurement, the induced voltage of the sensor strongly influences the relative motion of the projectile with respect to the magnetic field of the sensor, as analytically described by J. Van Bladel [19, 20]. The authors of this paper have therefore pursued in detail the effect of the relative motion of the moving object with respect to the sensor (coil) and modelled the sensor system that captures the motion of the projectile. The results of the analysis were used in the process of setting and testing the short-basis sensor for the measurement of the velocity of a moving object (Fig. 3).

Fig. 3. A prototype of the EMG-1 measurement/detection system by PROTOTYPA a.s., PROTOTYPA [16]

2. Motivation of the model

From the first moments of its application Maxwell [15], Dědek, Dědková [3], Stratton [17], Haňka [9], electromagnetic theory has included (in addition to electrodynamics and relativistics) also the static, quasi-static, and non-stationary parts of the description of the electromagnetic field. Sources describing the theoretical part of applied electrodynamics and relativistics include, for example, Haňka [9], Stratton [17], and Kikuchi [13].

At present, the modelling and simulation of technical systems constitutes a substantial part of the total of modelled problems. Among the currently modelled tasks there are various problems related to provinces characterized by the fastest development and application of modelling, namely energetics, mechanical engineering, metallurgy, electrical engineering, space and nuclear technologies Kuneš, Vavroch [14], Ansys [1]. This paper examines the problem of utilizing the coupled hybrid magnetic field model and analyzes the issue of the motion of a conductive object in a magnetic field. Let us consider the example of conductor movement in a stationary or non-stationary magnetic field. This appears to be a trivial problem with a straightforward solution; however, for an exact evaluation of the influence of all effects, the phenomena related to the movement of a system relative to the other one must not be neglected. As shown in Haňka [9], this distinctive effect begins to influence the resultant electromagnetic field distribution already at the relative velocity of $\boldsymbol{v}_0 = 1\ \mathrm{m{\cdot}s^{-1}}$.

3. Description of the Model

The physical model is based on the solution of the reduced Maxwell equations in Heaviside's notation Stratton [17]. The stationary magnetic/electric field can be described as

$$rot\boldsymbol{H} = \boldsymbol{J}\,,\ rot\boldsymbol{E} = 0\,,\ div\boldsymbol{B} = 0\,,\ div\boldsymbol{J} = 0\,, div\boldsymbol{D} = 0\,, \quad (1)$$

where ***H, J, E, D, B*** are the vectors of the magnetic field intensity, current density, electric field intensity, electric flux density, and magnetic flux density, respectively. The material relations are represented by the formula

$$\boldsymbol{B} = \mu_0\mu_\mathrm{r}\,\boldsymbol{H}\,,\ \boldsymbol{J}_\mathrm{v} = \gamma\,\boldsymbol{E}\,,\ \boldsymbol{D} = \varepsilon_0\varepsilon_\mathrm{r}\,\boldsymbol{E}\,, \quad (2)$$

where ε, μ, γ are the permittivity, permeability, and conductivity of the environment. Vector functions of the electric and magnetic fields ***E, B*** are expressed by means of the scalar electric potential ϕ_e and the vector magnetic potential ***A***. For the stationary, quasi-static, quasi-stationary, and non-stationary task, Fig. 4, in the relation for the electric field intensity, the time derivation of the vector magnetic potential is zero:

$$\boldsymbol{E} = -grad\,\phi_{\boldsymbol{e}} - \frac{\partial \boldsymbol{A}}{\partial t}\,,\ \boldsymbol{B} = rot\boldsymbol{A}\,. \quad (3)$$

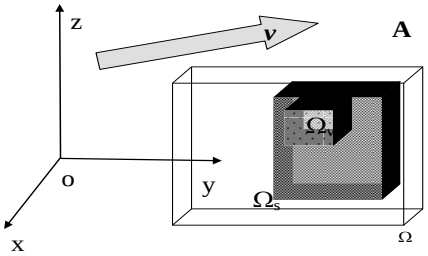


*Fig. 4. A basic model of the **A** system in the Cartesian coordinate system; motion of the area portion at velocity **v***

The resulting current density ***J*** from formula (2) is formed by the exciting current density $\boldsymbol{J}_\mathrm{s} = \rho\boldsymbol{v}$ with the specific density of the electric charge ρ and the current density caused by eddy currents:

$$\boldsymbol{J}_v = \frac{\partial \rho}{\partial t}\mathrm{d}\boldsymbol{l}\,, \quad (4)$$

where d***l*** is the element of length of the trajectory on which the eddy currents close. The motion effect for the instantaneous velocity vector ***v*** is respected in the model by the current density

$$\boldsymbol{J}_\mathrm{m} = \gamma\left(\boldsymbol{v}\times\boldsymbol{B}\right). \quad (5)$$

Then, respecting eddy currents (4), we have $\boldsymbol{J} = \boldsymbol{J}_\mathrm{v} + \boldsymbol{J}_\mathrm{s} + \boldsymbol{J}_\mathrm{m}$. The electromagnetic field distribution is formulated using expressions (1) to (5) in the entire region of the model Ω,

$$rot\boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}\,. \quad (6)$$

For individual parts of the model Ω there holds $\Omega \subset \Omega_v \cup \Omega_s$, where Ω_v is the area with dominant eddy currents which behave according to formula (2), and Ω_s is the area with known distribution of the current density $\boldsymbol{J}_s$. In the assumed model, we have $\Omega_s \subset \Omega_v$. These basic relations were complemented with boundary and initial conditions; using the finite element method, the physical model was then transferred to the mathematical one. The finite element method (FEM) Ansys [1] program system enabled us to perform a complete analysis; the FEM program was modified using suitably selected boundary conditions in order to respect the model characteristics described in formulas (4) to (5), Fiala [7]. According to Haňka [9], in any investigation into electro-dynamic systems it is necessary to respect the relativistic approach to electrodynamics. As described below, the entire problem begins at the moment when the vector of intensities of both the electric and the magnetic fields of the moving system is relative.

4. Relativity effect in the model

In the example provided by Haňka [9], the current density in the A system is $\boldsymbol{J} = \rho \boldsymbol{v}$; for the mutually moving A-A´ systems, the density is expressed in the component form

$$\boldsymbol{J}_{\square} = J'_x \boldsymbol{u}_x + J'_y \boldsymbol{u}_y + J'_z \boldsymbol{u}_z + \mathrm{j}c\rho' \boldsymbol{u}_t . \tag{7}$$

The continuity equation can be formulated as

$$div\, \boldsymbol{J}_{\square} = 0 . \tag{8}$$

After applying the Lorentz transformation to obtain a simple representation of the moving systems in the direction of *x*, the current density (7) can be written in the form

$$\boldsymbol{J}_{W\square} = \underbrace{\frac{J'_x + \mathrm{j}\frac{v_x}{c} J'_t}{\sqrt{1-\left(\frac{\|v\|}{c}\right)^2}} \boldsymbol{u}_x}_{\mathrm{C}} + J'_y \boldsymbol{u}_y + J'_z \boldsymbol{u}_z + \mathrm{j} \underbrace{\frac{c\rho' - \frac{v_x}{c} J'_x}{\sqrt{1-\left(\frac{\|v\|}{c}\right)^2}} \boldsymbol{u}_t}_{\mathrm{D}} . \tag{9}$$

From formula (7), for the instantaneous velocity of the moving system A´ in relation to observer A (Fig. 1) and at *v*<<*c*, the component part C is negligible. A simplified form of the above relations (9) can then be written as

$$J_{W\square} = \underbrace{\frac{J'_x}{\sqrt{1-\left(\frac{\|v\|}{c}\right)^2}} \boldsymbol{u}_x}_{\mathrm{C}} + J'_y \boldsymbol{u}_y + J'_z \boldsymbol{u}_z + \mathrm{j} \underbrace{\frac{c\rho' - \frac{v_x}{c} J'_x}{\sqrt{1-\left(\frac{\|v\|}{c}\right)^2}} \boldsymbol{u}_t}_{\mathrm{D}} . \tag{10}$$

This relation, or component C from expression (10), is well-known from electromagnetodynamics in respecting the ratio of velocities *v*/*c*. However, the second part, or component D in expression (9), is nonnegligible and has a relative character with respect to the systems A-A´ and component D in expression (10). The first part of the relation (C) from formula (10) is based on electrodynamics at the conventional current of v<<c. The second relation, D, (10) is – for the magnitude of the electric charge density ρ'=0 – unknown in electrodynamics and is typical of the relativistic description of the behaviour of two mutually moving systems. Then we can easily formulate

$$\rho = \frac{v}{c^2} J_x \tag{11}$$

In the non-dynamic system, the model is shown in relations (1) and (2). In order to eliminate possible errors, it is suitable to include in relation (7) the term which respects Faraday´s law of induction:

$$rot\, \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} + rot\left(\boldsymbol{v} \times \boldsymbol{B}\right),$$

$$rot\, \boldsymbol{H} = \boldsymbol{J}_s + \boldsymbol{J}_m + \frac{\partial \boldsymbol{D}}{\partial t} + rot\left(\boldsymbol{v} \times \boldsymbol{D}\right), \tag{12}$$

where ρ is the volume density of the electric charge. The complete Maxwell equations are covariant in all the systems; it is thus not important to specify the system within which the observer moves, because the described relations always hold true. After the derivation of the four-vector and respecting the Lorentz transformation for the moving electric charge with the density ρ from the viewpoint of two individual systems relatively moving at the velocity of $\boldsymbol{v}$ (A and A´), the source current density is written in the form

$$\boldsymbol{J}_{s\square} = \rho \frac{\partial \boldsymbol{s}}{\partial t} + \mathrm{j}c\rho \boldsymbol{u}_t , \tag{13}$$

where $\boldsymbol{s}$ is the position vector of a material point in the coordinate system. Then the interface between the dielectric with the electric permittivity ε and the conductive material having the conductivity γ (12) is written as

$$rot\, \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} + rot\left(\boldsymbol{v} \times \boldsymbol{B}\right) - \frac{1}{\gamma} rot\left(\rho \boldsymbol{v} + \mathrm{j}c\rho \boldsymbol{u}_t\right)$$

$$rot\, \boldsymbol{H} = \rho \boldsymbol{v} + \gamma\left(\boldsymbol{v} \times \boldsymbol{B}\right) + \mathrm{j}c\rho \boldsymbol{u}_t + \frac{\partial \boldsymbol{D}}{\partial t} + rot\left(\boldsymbol{v} \times \boldsymbol{D}\right) \tag{14}$$

where j is the symbol of the imaginary component of the quantity complex shape, *c* is the velocity module of light in the vacuum, $\boldsymbol{s}$ is the position vector of the relative system and the system, Fig. 4, of the moving electric charge in axis x of the Cartesian coordinate system with the electric charge density ρ. For the continuity equation (1) there holds

$$div\left(\boldsymbol{J}_{\square}\right) = 0 . \tag{15}$$

For simplification, if we assume the motion of one reference system as ()´ (for example, in the *x* axis of the Cartesian coordinate system), Fig.5, the four-vector of the current density can be written – after application of the Lorentz transformation – in the (invariant) form (9), and the electric charge volume density in the reference system is, after transformation, written as

$$\rho = \frac{\rho' - \frac{v_x}{c^2} J'_x}{\sqrt{1-\left(\frac{\|\boldsymbol{v}\|}{c}\right)^2}} , \quad \rho' = \frac{\rho - \frac{v_x}{c^2} J_x}{\sqrt{1-\left(\frac{\|\boldsymbol{v}\|}{c}\right)^2}} . \tag{16}$$

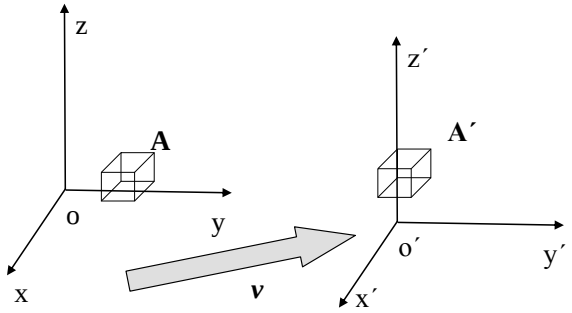


*Fig. 5. The relation of two moving systems with the objects **A** and **A´** (moving mutually at velocity **v**)*

Applying Maxwell's equation for the presence of electric charge in the modeled area, Fig. 2, we have

$$\rho = div\, \boldsymbol{D} . \tag{17}$$

The expression for the current density $\boldsymbol{J}$ can be written in the form for the interface between the dielectric and the conductive layer, Fig. 6.

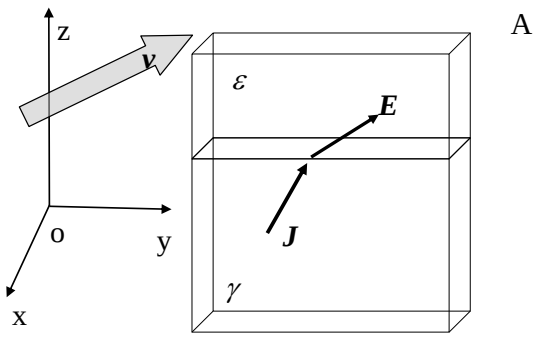


Fig. 6. The approximation of conditions on the dielectric/metal interface for the relativistic approach to the model

Then for the unabbreviated formula, respecting the motion at velocity v in all directions, the current density with respect to the moving system A-A´ can be written as

$$\boldsymbol{J}_{\square} = \frac{J'_x - v_x div\left(\frac{\varepsilon'}{\gamma'}\boldsymbol{J}'_{\Delta}\right)}{\sqrt{1-\left(\frac{\|\boldsymbol{v}\|}{c}\right)^2}}\boldsymbol{u}_x + \frac{J'_y - v_y div\left(\frac{\varepsilon'}{\gamma'}\boldsymbol{J}'_{\Delta}\right)}{\sqrt{1-\left(\frac{\|\boldsymbol{v}\|}{c}\right)^2}}\boldsymbol{u}_y + \frac{J'_z - v_z div\left(\frac{\varepsilon'}{\gamma'}\boldsymbol{J}'_{\Delta}\right)}{\sqrt{1-\left(\frac{\|\boldsymbol{v}\|}{c}\right)^2}}\boldsymbol{u}_z + j\frac{c\,div\left(\frac{\varepsilon'}{\gamma'}\boldsymbol{J}'_{\Delta}\right) - \frac{v_x}{c}J'_x - \frac{v_y}{c}J'_y - \frac{v_z}{c}J'_z}{\sqrt{1-\left(\frac{\|\boldsymbol{v}\|}{c}\right)^2}}\boldsymbol{u}_t \quad (18)$$

The expression for the current density in the area of conductive material with conductivity γ can be written from formula (10) in the form assuming zero value of the electric charge ρ in the electrically conductive material (metal)

$$\boldsymbol{J}_{\square} = \frac{J'_x}{\sqrt{1-\left(\frac{\|\boldsymbol{v}\|}{c}\right)^2}}\boldsymbol{u}_x + \frac{J'_y}{\sqrt{1-\left(\frac{\|\boldsymbol{v}\|}{c}\right)^2}}\boldsymbol{u}_y + \frac{J'_z}{\sqrt{1-\left(\frac{\|\boldsymbol{v}\|}{c}\right)^2}}\boldsymbol{u}_z + j\frac{-\frac{v_x}{c}J'_x - \frac{v_y}{c}J'_y - \frac{v_z}{c}J'_z}{\sqrt{1-\left(\frac{\|\boldsymbol{v}\|}{c}\right)^2}}\boldsymbol{u}_t \quad (19)$$

$$\rho = \frac{\rho' - \frac{\boldsymbol{v}}{c^2}\boldsymbol{J}'_{\Delta}}{\sqrt{1-\left(\frac{\|\boldsymbol{v}\|}{c}\right)^2}}$$

$$\boldsymbol{J}_w = \frac{\boldsymbol{J}'_{\Delta} + j\nu\rho'}{\sqrt{1-\left(\frac{\|\boldsymbol{v}\|}{c}\right)^2}} \quad (20)$$

For the relationship within one system A the relations are known and written in the form

$$rot\,\boldsymbol{H}_{\Delta} = \boldsymbol{J}_{\Delta} + \frac{\partial \boldsymbol{D}_{\Delta}}{\partial t} + rot\left(\boldsymbol{v}_{\Delta}\times\boldsymbol{D}_{\Delta}\right)$$

$$rot\,\boldsymbol{E}_{\Delta} = -\frac{\partial \boldsymbol{B}_{\Delta}}{\partial t} + rot\left(\boldsymbol{v}_{\Delta}\times\boldsymbol{B}_{\Delta}\right), \quad (21)$$

and the expression of the current density $\boldsymbol{J}_{\square}$ within two mutually moving systems A-A´ is

$$\boldsymbol{J}_{\square} = \begin{pmatrix} 0 & \frac{\partial H_3}{\partial x_2} & \frac{\partial H_3}{\partial x_3} & -jc\frac{\partial D_1}{\partial x_4} \\ -\frac{\partial H_3}{\partial x_1} & 0 & \frac{\partial H_1}{\partial x_3} & -jc\frac{\partial D_2}{\partial x_4} \\ \frac{\partial H_2}{\partial x_1} & -\frac{\partial H_1}{\partial x_2} & 0 & -jc\frac{\partial D_3}{\partial x_4} \\ jc\frac{\partial D_1}{\partial x_1} & jc\frac{\partial D_2}{\partial x_2} & jc\frac{\partial D_3}{\partial x_3} & 0 \end{pmatrix}. \quad (22)$$

5. Demonstration of the relativity effect on a simple model

It has been shown by Haňka [9] that a simple model can be used to demonstrate the critical velocity at which the relativistic effect described in the current density $\boldsymbol{J}_{\square}$, relation (18), will appear. The model, whose scheme is presented in Fig. 4, is a magnetic circuit composed of pole extensions and an exciting coil. A very long ("infinite") conductive strip of finite thickness passes through an air gap in the magnetic circuit; with respect to the circuit, this strip moves at an instantaneous velocity with the component in the x-axis of the Cartesian coordinate system, v_x = 1 m/s.

It is evident from both Haňka [9] and the example according to Fig. 7 that a significant contribution to the effect can be assigned only to the normal component of the electric field intensity E_n at the boundary of conductor/air in the magnetic circuit air gap. For the first approximation, the component is perpendicular to the velocity vector $\boldsymbol{v}_x$ of the moving conductive strip. The resulting form of the flow lines is shaped by, among other aspects, the electric charge density ρ in formula (9). Even though at first sight the electric charge density ρ may appear as a very small value, it will cause excitation of the electric intensity and current density, which in superposition with the exciting value of the electric intensity and current density will create a deformed magnetic field. However, there remains the question of the value of the instantaneous velocity $\boldsymbol{v}$ at which the effect will manifest itself.

The problem can be described as follows:

We chose J = 1 A/mm^2 (formal notation: $J = 1.10^6$ A·m^{-2}) with the electrical conductivity of γ = 100 MS m^{-1}; the electric field intensity is $E = J/\gamma$, E = 0.01 V·m^{-1}. Let us assume a plane unlimited problem and thus the electric field intensity function depending on the distance from the plate E_x=$k.x$. Then, for simplicity, we can write the above formula (17) as

$$\rho = div\left(\varepsilon E\right), \text{ then } div\left(E\right) = \frac{\rho}{\varepsilon},\ div\left(E\right) \cong \frac{\mathrm{d}E_x}{\mathrm{d}x} \quad (23)$$

By comparison of relations (23), we have

$$\frac{\mathrm{d}E_x}{\mathrm{d}x} = \frac{\rho}{\varepsilon} \quad (24)$$

and, therefore, for the approximation $\varepsilon = 8.8\cdot10^{-12}$ F·m^{-1} the electric charge volume density is in the distance of $x = 10^{-1}$ m

$$\rho = \varepsilon\frac{\mathrm{d}E_x}{\mathrm{d}x},\ \rho = 8.8\cdot10^{-12}\frac{10^{-2}}{10^{-1}}\ \text{C.m}^{-3}. \quad (25)$$

From expression (16) we can obtain the formula for the value of instantaneous velocity in the moving systems A-A´ respecting the relativistic effect of the system electrodynamics:

$\rho = \frac{\rho' - \frac{v_x}{c^2}J'_x}{\sqrt{1-\left(\frac{\|\boldsymbol{v}\|}{c}\right)^2}}$, which for the assumed problem and the low value of the instantaneous velocity module from relation (11) is

$$v_x = \frac{\rho' c^2}{J'}. \quad (26)$$

For the above-quoted problem,

$$v_x = \frac{\rho' c^2}{J'} = \frac{8.8\cdot10^{-11}\left(3\cdot10^8\right)^2}{1\cdot10^6} \cong 0.79\ \text{m·s}^{-1}.$$

Then, with respect to formula (26), for the non-moving system A, in which the instantaneous velocity value v will not influence the relativistic effect of the moving systems A-A´, we can calculate a different value v according to (4):

$$v = \frac{10^6}{8.8\cdot10^{-13}} \cong °1.13\cdot10^{19}\ \text{m·s}^{-1}. \quad (27)$$

Thus, in relations (26) and (27) can be traced the difference of results with comparable values of the electric field intensity and at the instantaneous motion velocity value of the moving strip. There follows an example of the representation of changes in the electromagnetic field distribution for the given task, Fig. 7; for simplicity, the velocity of the moving strip is assumed to be v_x = 1 m·s^{-1}.

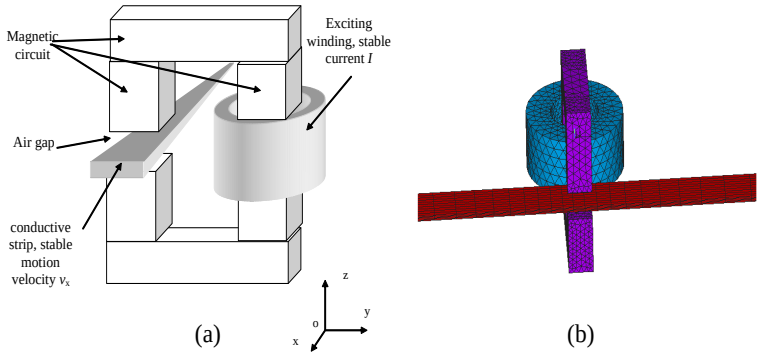
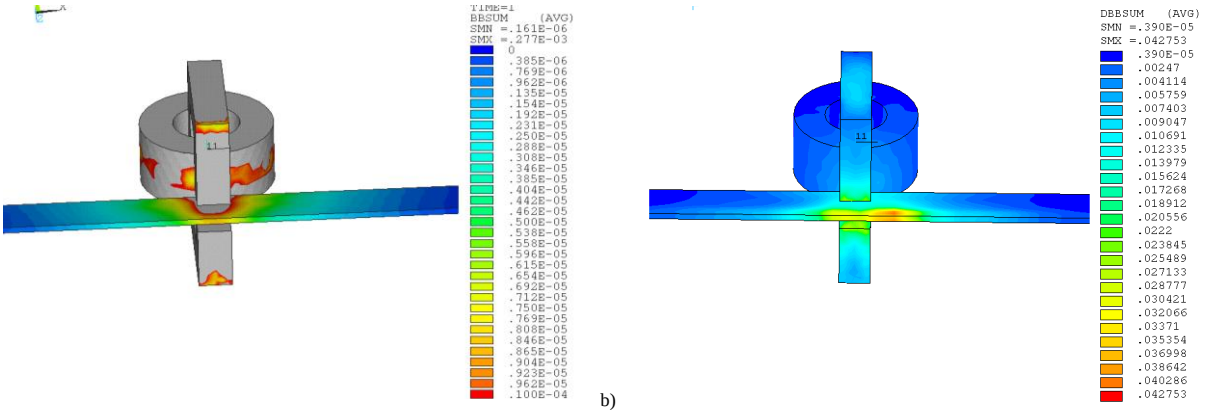


Fig. 7. A simple geometry for the verification of the relativistic approach of electrodynamics (a) and its numerical model (b)



a) b)

Fig. 8. The analysis of magnetic flux density: A) quasi-stationary B) distribution of the superposed magnetic flux density Bs vector module (with the relativistic effect)

In the numerical analysis, the simple model in Fig. 7 is used to demonstrate the influence of the relativistic approach on the mathematical model (18), (19) and to compare the results obtained by analyzing the non-relativistic (1) – (6) and relativistic (18), (19) models (Fig. 8). The physical model is based on the solution of the reduced Maxwell equations [17], Fiala [5]. The resulting current density $\boldsymbol{J}$ is formed by the exciting current density $\boldsymbol{J}_s = \rho \boldsymbol{v}$ with the specific density of the electric charge ρ and the current density caused by the eddy currents $\boldsymbol{J}_v$. The motion effect for the instantaneous velocity vector $\boldsymbol{v}$ is respected in the model by the current density $\boldsymbol{J}_m$. The finite element method (FEM) program was modified using suitably selected boundary conditions in order to respect the model characteristics described in Fiala [5, 7]. The entire problem begins at the moment when the vector of intensities of both the electric and the magnetic fields of the moving system is relative**.** The complete Maxwell equations are covariant in all the systems; therefore, it is not important to specify the system within which the observer moves as the described relations always hold true. After the derivation of the four-vector and respecting the Lorentz transformation, the current density is written in the form (21).

There is no problem to obtain, by means of reversing the sign of velocity, transformed quantities of the opposite system (which is moving) for the above-expressed transformed quantities. Let us have a simple geometric task, Fig. 7. The model example consists of a conductive strip in the air gap which is a part of the magnetic circuit. The distribution of the magnetic field in the system is evaluated for a different case of the strip movement and for the non-relativistic and relativistic approaches.

Numerical analysis of the above-mentioned example provided by the author of [9] is realized for the assumption of linear material environment by means of the superposition method. Generally, the first step consists in an elementary analysis of the magnetic field without the moving strip, formula (1), the second step embodies an analysis of the magnetic field with a steady motion of the conductive strip and the effect of eddy currents, and the third action involves the evaluation of current density in the moving strip with electrodynamic and relativistic effects according to relations (18) to (22). These analyses were performed using the program providing FEM analysis Ansys [1] (program in the APDL language for ANSYS).

The resulting distribution of the magnetic field and the real component module of the magnetic induction Bre vector are shown in Fig. 8, naturally, it is possible to perform the analysis also for the imaginary part of the current density component $\boldsymbol{J}_{ima}$**.**

6. Example of the relativity effect on the experimental model

The numerical model and the basic geometry of the ballistic projectile velocity measurement are shown in Fig. 9. There are electric coils for the excitation of the basic magnetic flux density *B*, Fig. 10. The secondary magnetic flux density on the surface of the projectile is dependent on the effect of the velocity of the projectile. The final magnetic flux density *B* depends on the effects of eddy currents and the theory of relativity. Then the final vector magnetic flux density is measured with classic sensors, Fig.11a. Thus, we can demonstrate the real and imaginary components of the current density module *J*, Fig. 11b, and indicate the results of the relativistic contribution of the magnetic flux density module, B_{re}, B_{im}, Fig. 12.

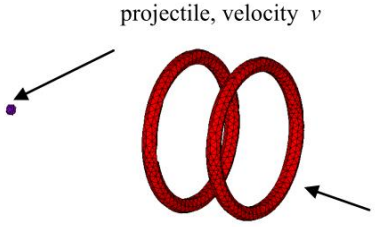
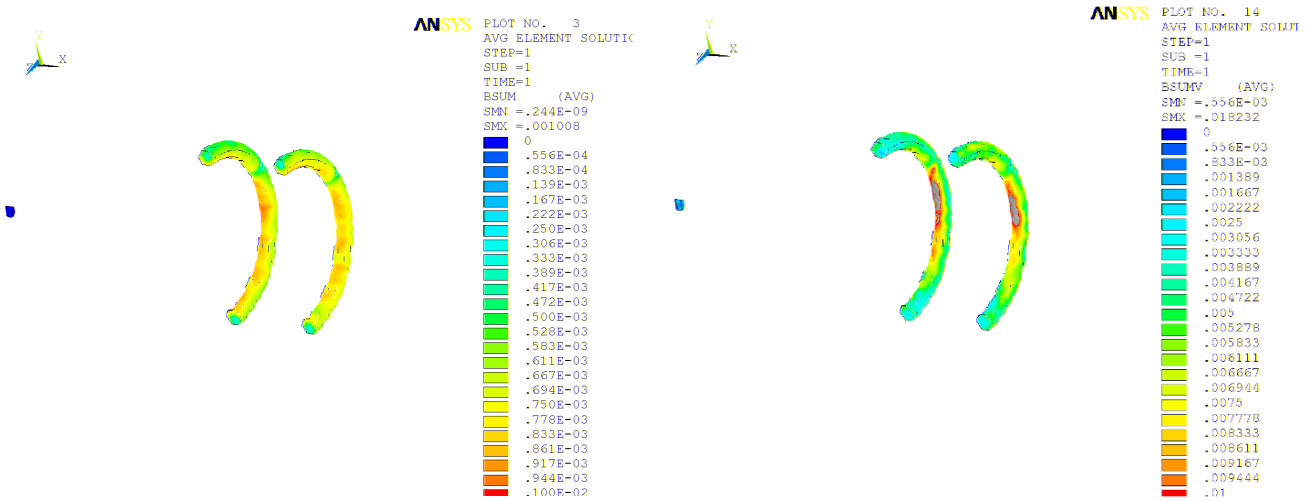
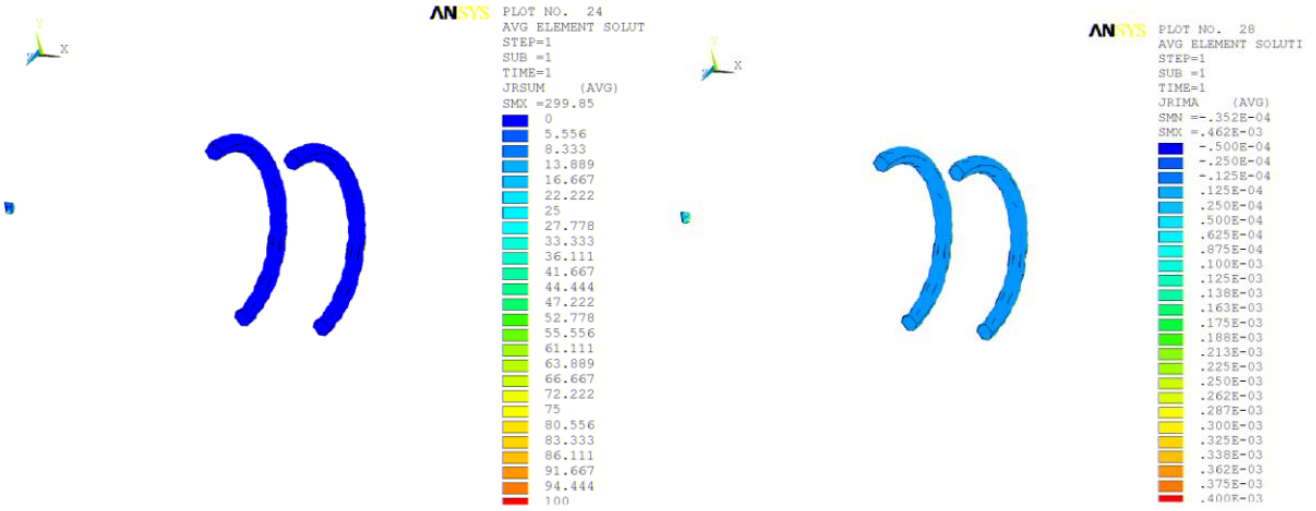


Fig. 9. The electrodynamic magnetic field analysis: a model of the measurement of velocity in a simple ballistic projectile



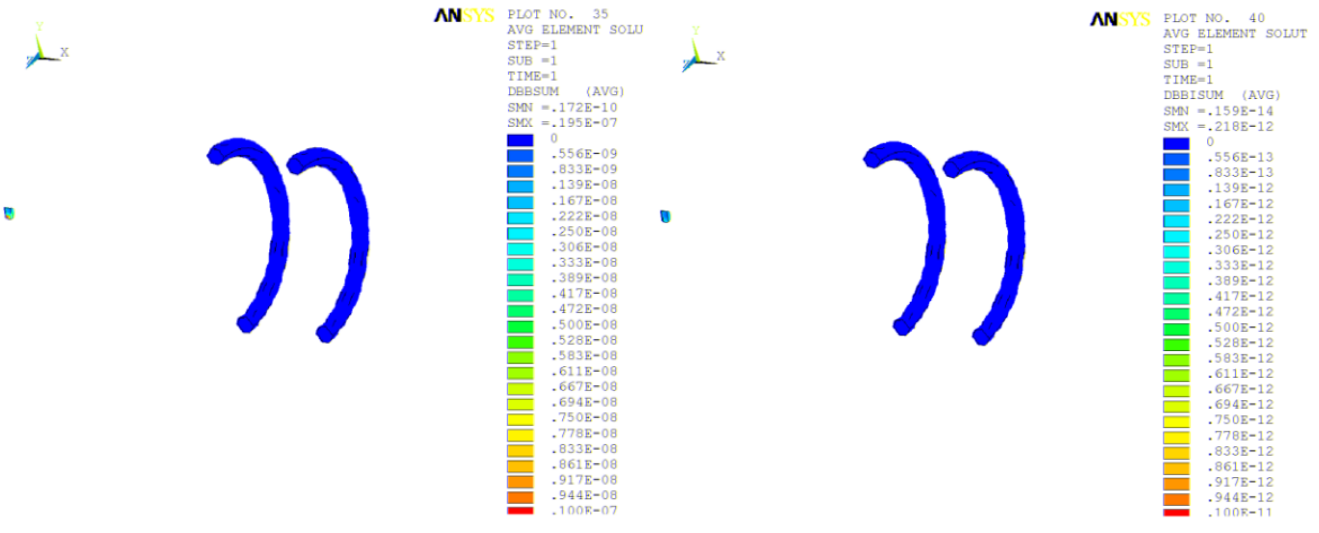
a b

Fig. 10. The electrodynamic magnetic field analysis:a) the basic magnetic flux density vector module B distribution, b) the final magnetic flux density vector module B distribution



a b

Fig. 11. The electrodynamic magnetic field analysis: a) the real component current density J_{re} vector module distribution, b) the imaginary component current density J_{im} vector module distribution



a b

Fig. 12. The electrodynamic magnetic field analysis: a) the real component of the magnetic flux density B_{re}, b) the imaginary component of the magnetic flux density B_{im}

7. Results of the experimental measurement

Using the fabricated prototype, we completed the research by performing a large number of test measurements in both the laboratories of the DTEEE, FEEC BUT (Fig. 13) and the facilities managed by the PROTOTYPA company. The signals obtained from one and two sensors are presented in Fig. 14.

The measured data were digitized and processed via the MATLAB program, in which we conducted the numerical postprocessing and evaluated the envelopes of the signals from the individual sensors, Fig. 15.

The theoretical models were numerically processed and evaluated in partial experiments performed using the finite element method. A connection (and effects too) was proved between the inductance L of the sensor coil and the position of the moving metal object. Based on the related analyses, we designed and fabricated a functional sample of the inductive sensor (Figs. 3, 16, 17).

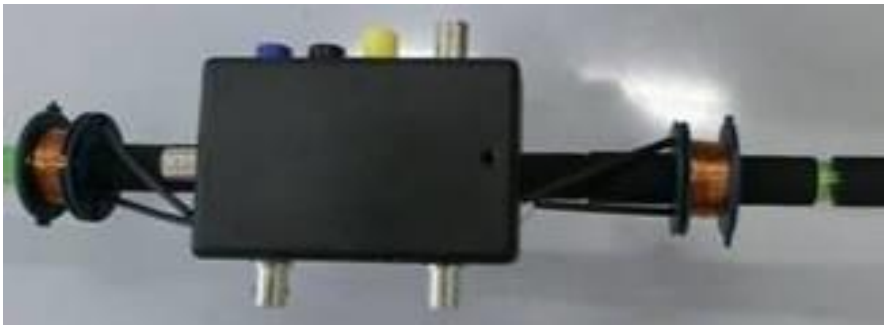


Fig. 13. The measurement system with two ports and electronics

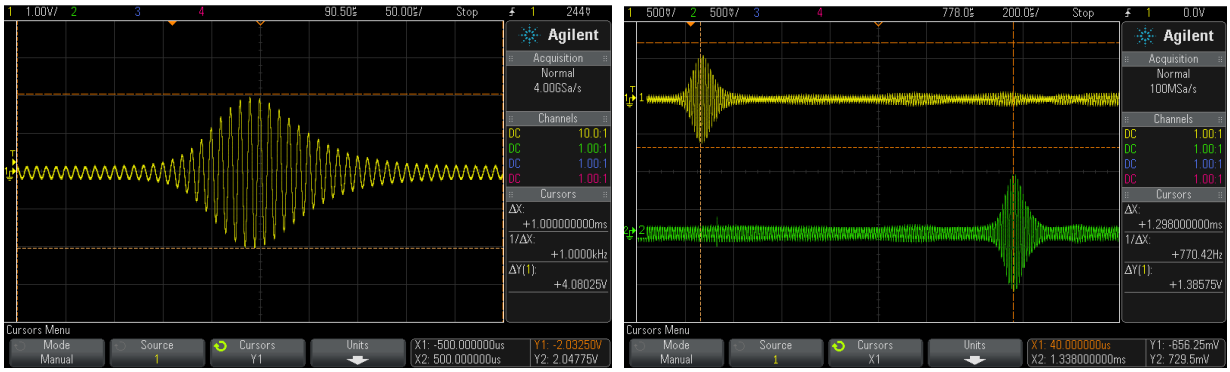


Fig. 14. The measured signals: a) the signal acquired from one sensor, b) the signal obtained from two sensors

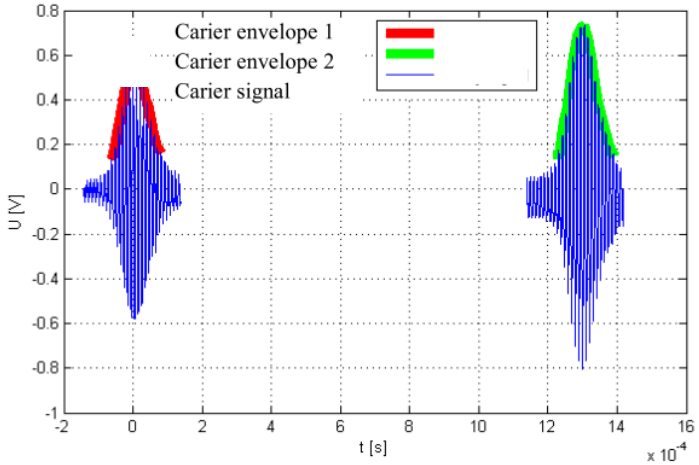


Fig. 15. An example of signal processing in MATLAB

Fig. 16. The final measurement head EMG-1 from PROTOTYPA a.s., PROTOTYPA [16]

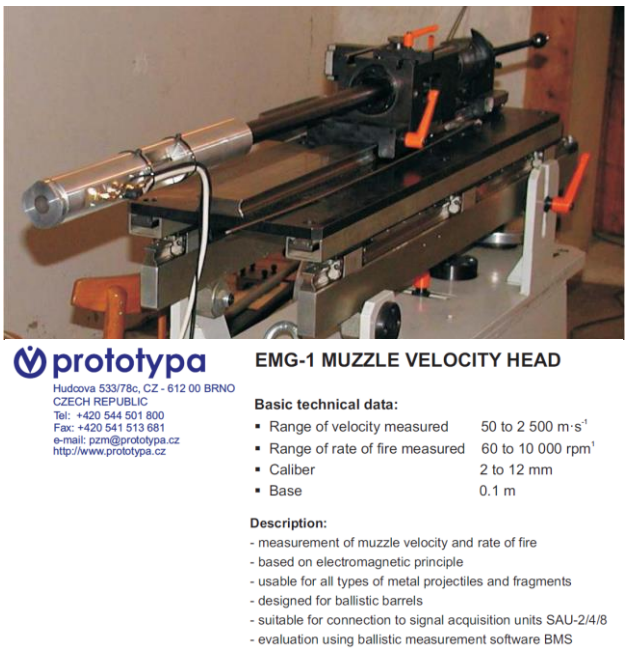


Fig. 17. The final measurement system EMG-1 from PROTOTYPA a.s., PROTOTYPA [16]

8. Conclusion

The model shows the distinction by order of the individual phenomena. It is evident that the relativistic electro-magneto-dynamic principle is indispensable and introduces substantial changes into the original non-dynamic conception of this type of simple problem analysis.

The effect is apparent of motion on the magnetic field imbalance. A higher value of an elementary magnetic field combines with a decrease in the relative magnitude of the electro-magneto-dynamic field influence.

Formulas (18) to (21) ought to be considered and respected in the process of designing dynamic and electromechanical systems such as rotary electrical machines; in the same manner, the relations should be utilized in electromagnetic and electromechanical systems with moving elements designed for velocity in the order of 1 m/s, for example in the topics of papers Jha P., Raj G., Upadhyaya A. K. [12], Holmes J., Ishimaru A. [10], Yarim C., Daybelge U., Sofyali A. [21]. It can be assumed that, due to the disrespect of results following from this interpretation of the electromagnetic field of dynamic systems, inaccuracies occur within the modeling and simulation that supports the actual design and realization of the systems EMG-1 from PROTOTYPA a.s., PROTOTYPA [16].

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