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MODELLING OF VERTICAL SOIL COLLECTORS OF THERMAL PUMPS

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Abstract. *The mathematic model of non-stationary process of heat distribution in soil collector is made. Convection heat transfer in the tube of collector and in depth soil temperature are taken into account. Algorithm of soil collector determination of approximating transmission function collector is created. The results and diagrams of transitional processes and dynamic description of the object are given.*

Keywords: thermal pumps, soil collectors, thermal conductivity

MODELOWANIE GRUNTOWYCH KOLEKTORÓW PIONOWYCH DO POMP CIEPLNYCH

Streszczenie. *Zbudowano matematyczny model niestacjonarnego procesu dystrybucji ciepła w kolektorze gruntowym. Uwzględniono konwekcyjny transfer ciepła w rurze kolektora oraz rozkład temperatury na głębokości gruntu. Stworzony został algorytm aproksymacji dla określenia funkcji transferu ciepła z gruntowego wymiennika ciepła. Pokazano wyniki i wykresy procesów przejściowych oraz opis dynamiki obiektu.*

Słowa kluczowe: *pompy cieplne, kolektory gruntowe, przewodność cieplna*

Introduction

The problem of usage of natural sources of low-potential heat for heat supply of buildings is very popular during the last years. The heat of external air, the air accumulated by soil and surrounding pond is used with the help of heat pumps. The effectiveness of such systems largely depends on the type of peripheral equipment. The vertical heat energy soil collectors are widespread.

Seasonal and daily changes of intensity of solar radiation and the temperature of external air cause temperature fluctuations in the upper layers of the soil (15–20 meters). The change of aggregative state of moisture which is in pores of soil under the operation of geothermal heat pumps of soil tracts which are within the bounds of zone of thermal influence of pipes register of soil heat exchanger as a result of seasonal changes of external climate parameters. These changes happen as a result of operational load on the system of heat collector. So the foundation soil tracks is a complex system which is formed by a great number of solid parts of different shapes and size. The foundation can be both hard and mobile. It depends on the connectability of solid particles. The gaps between the solid particles can be filled with moisture, gas, steam and ice or with one or another simultaneously.

Under long – term operation of geothermal heat pumps different situations can arise. During the heating season the soil temperature near the soil collector can fall down but in summer the soil won't warm thoroughly to the initial temperature. So general falling down of thermal soil capacity takes place. In this case the consumption of energy during the next heating period causes more falling down the soil temperature and its thermal capacity will fall down further.

1. Statement of the problem and its relationship to important scientific and practical tasks

1.1. Analysis of recent research and publications which discuss current issues

Heat transfer processes from soil to heat carrier of soil collector were investigated by many authors. In more cases the pipe is described as a dot heat receiver. The temperature allocation is described by axially symmetric equation of thermal conduction [1, 7]. Both analytical solutions of the task and numerical methods [1, 3], which help investigate space three-dimensional models. They also help to take into account the change of depth soil temperature. In particular the correlation for determining in depth soil temperature during a year and the model of work of vertical soil collector is proposed in this article [1]. Mathematical model of thermal conditions of heat collection of low-potential warm soil taking into account the phase transitions of steam wet soil under long-term operation is also described in the article. The article deals with the determination of thermal power of vertical t soil collectors taking into account the heat accumulation properties of bore wells [6, 9]. Heat transfer processes in the collectors pipe, the in-depth soil temperature and its convection are analysed in the article [10].

1.2. Formulation of the problem

The task of this research is the construction of the algorithm definition of approximating transfer function of soil collector, taking into account convective heat transfer in the heat exchanger tube and its in-depth temperature.

2. Statement of main research data with full justification of scientific results

For the building of the model and setting initial and critical convention U – similar pipe (soil collector) with radius R "straighten" in the right line. The depth of laying the pipe – $l/2$ (Fig. 1).

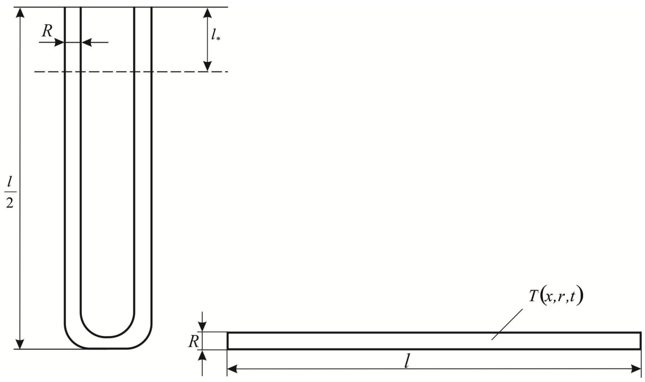

Fig. 1. Soil collector

Differential equation is written down as

$$a\left(\frac{\partial^2 T}{\partial x^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right)-v(x)\frac{\partial T}{\partial x}=\frac{\partial T}{\partial t}, \quad (1)$$

$T(x,r,t)$ – the temperature in the heat collector at the point with coordinate (x, r) at the moment of time t, $a=\frac{\lambda}{\rho c}$ – coefficient of heat capacity carrying fluid, λ – coefficient of heat capacity, ρ, c – density and heat capacity, $v(x)$ – gear of convective transfer.

As it is necessary to determine transfer functions, so initial conditions are zero:

$$T(x,r,t)\big|_{t=0}=0,\ T(0,r,t)=0. \quad (2)$$

Final conditions are written down as:

$$\left.\frac{\partial T(x,r,t)}{\partial r}\right|_{r=0}=0,\ \left.\frac{\partial T(x,r,t)}{\partial x}\right|_{x=l}=0,$$

$$\left.\frac{\partial T(x,r,t)}{\partial r}\right|_{r=R}=\alpha\left(T(x,R,t)-T_{gr}(x,t)\right), \tag{3}$$

$T_{gr}(x,t)$ – soil temperature, α – coefficient of heat emission from soil to the wall of heat exchanger.

In surface sphere $0<x<l_*$ soil temperature $T_{gr}(x,t)$ depends on seasonal and daily variations of outward air. Below l_* soil temperature rises evenly an average for 3 degrees every 100 metres, but the influence of outward air is practically lacking. Soil temperature is determined as:

$$T_{gr}(x,t)=\begin{cases}T^*_{gr}(x,t), 0<x<l_*, l-l_*<x<l,\\ T^*_{gr}(x)+x\dfrac{a_{zp}(t)}{100}, l_*<x<l-l_*.\end{cases} \tag{4}$$

$a_{zp}(t)$ – coefficient of heat capacity of soil which is not a constant quantity during a long-term operation and at present it can be determined only experimentally, $T^*_{gr}(x,t)$ – temperature of virgin soil for definite depth and day of the year, which was calculated as a known dependence [4]:

$$T^*_{gr}(x,t)=T_g-A_s e^{-3{,}16\cdot 10^{-4\cdot x\sqrt{\frac{\pi}{365 a_{gr}(t)}}}}\cdot$$

$$\cdot\cos\left(\frac{2\pi}{365}\left(d-d_0-1.834\cdot 10^{-2}\cdot x\sqrt{\frac{365}{\pi a_{gr}(t)}}\right)\right), \tag{5}$$

T_g – the temperature on the soil surface for concrete period of the year, A_s – annual amplitude of fluctuation in temperature on the soil surface, d – ordinal number of the day of the year, d_0 – phase constant, which is equal to 171 days.

We substitute continued changeable x and r for their discrete analogue [1, 5] in differential equation (1) and boundary data (3). For this we discretise changeable x and r on the intervals $0\le x\le l$, $0\le r\le r_0$ by mesh with dimensionless steps: $h_x=2/(m-1)$, $h_r=1/(n-1)$, m and n – the number of nodes in mesh in radial direction and along axis x (Fig. 2).

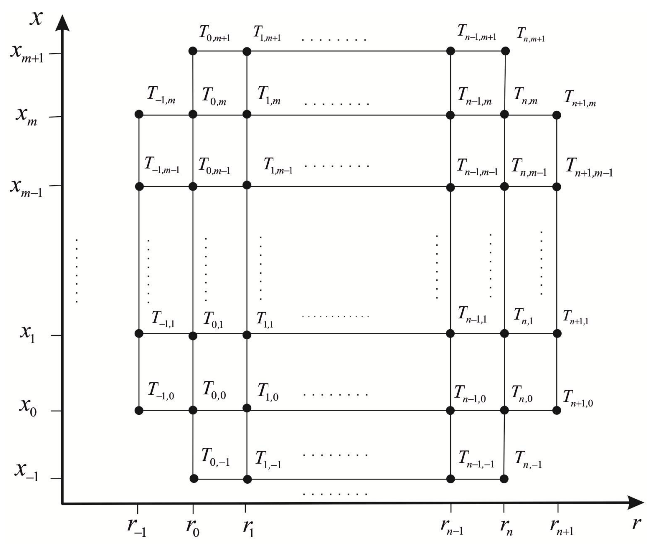

Fig. 2. Resulting – differential graticule

Value x and r in nodes of mesh:

$$x_i=h_x i,\ r_j=h_r j, \tag{6}$$

$i=0,...,m,\ \ j=0,...,n,\ \ i,j\in N$.

We also add assistive nodes with numbers $(i=-1, j=0,...,n)$, $(i=m+1, j=0,...,n)$, $(i=0,..., m, j=-1)$, $(i=0,..., m, j=n+1)$, which are beyond the calculated sphere and they will be used for calculating derivatives.

Derivatives for a variable x and r approximate with central difference:

$$\left.\frac{\partial T}{\partial r}\right|_{i,j}=\frac{T_{i,j+1}-T_{i,j-1}}{2h_r}+O(h_r^2),\ \left.\frac{\partial^2 T}{\partial r^2}\right|_{i,j}=\frac{T_{i,j+1}-2T_{i,j}+T_{i,j-1}}{h_r^2}+O(h_r^2),$$

$$\left.\frac{\partial T}{\partial x}\right|_{i,j}=\frac{T_{i+1,j}-T_{i-1,j}}{2h_x}+O(h_x^2),\ \left.\frac{\partial^2 T}{\partial x^2}\right|_{i,j}=\frac{T_{i+1,j}-2T_{i,j}+T_{i-1,j}}{h_x^2}+O(h_x^2).$$

This calculated scheme is permanent in any correlation of steps h_x, h_r [5].

As a result the equation will be

$$\left(\frac{a}{h_x^2}-\frac{v(h_x i)}{2h_x}\right)T_{i+1,j}-2a\left(\frac{1}{h_x^2}-\frac{1}{h_r^2}\right)T_{i,j}+\left(\frac{a}{h_x^2}+\frac{v(h_x i)}{2h_x}\right)T_{i-1,j}+$$

$$+\frac{a}{h_r^2}\left(1+\frac{1}{2j}\right)T_{i,j+1}+\frac{a}{h_r^2}\left(1-\frac{1}{2j}\right)T_{i,j-1}=\frac{\partial T_{i,j}}{\partial t}. \tag{7}$$

We do transformation of Laplace (under zero initial condition) in the (7) and start transmitting functions, taking for entrance – the temperature at the entrance to the heat exchanger. We get the following from equation(6):

$$\left(\frac{a}{h_x^2}-\frac{v(h_x i)}{2h_x}\right)W_{i+1,j}(p)-2a\left(\frac{1}{h_x^2}-\frac{1}{h_r^2}\right)W_{i,j}(p)+\left(\frac{a}{h_x^2}+\frac{v(h_x i)}{2h_x}\right)W_{i-1,j}(p)+$$

$$+\frac{a}{h_r^2}\left(1+\frac{1}{2j}\right)W_{i,j+1}(p)+\frac{a}{h_r^2}\left(1-\frac{1}{2j}\right)W_{i,j-1}(p)=0. \tag{8}$$

As it is known [2], transmitting function of the object can be written down as

$$W_{ij}=\frac{B_{i,j}(p)}{A(p)}, \tag{9}$$

$B_{i,j}(p)$, $A(p)$ – characteristic polynomial of the object.

So equation (8) is

$$-\left(\frac{a}{h_x^2}+\frac{v(h_x i)}{2h_x}\right)B_{i-1,j}(p)+2a\left(\frac{1}{h_x^2}-\frac{1}{h_r^2}\right)B_{i,j}(p)-\left(\frac{a}{h_x^2}-\frac{v(h_x i)}{2h_x}\right)B_{i+1,j}(p)-$$

$$-\frac{a}{h_r^2}\left(1+\frac{1}{2j}\right)B_{i,j-1}(p)-\frac{a}{h_r^2}\left(1+\frac{1}{2j}\right)B_{i,j+1}(p)=0. \tag{10}$$

For solving the equation (10) we use tridiagonal matrix algorithm, using polynomial from p. As a result we have recurrence correlations, which allowed us to determine with using of $B_{i,j}(p)$. According to (9) we get transmitting function, which we searched. This algorithm is implemented in Matlab programme with usage of m-files.

We give the results of numerical calculations, taking into account the tube of heat exchanger with radius $r_0=0.03$ m and length – 100 m is filled with water solution of ethylene glycol. Its properties are coefficient of thermal conductivity $\lambda=0.4$ W/m·s, density $\rho=1060$ kg/m^3, heat capacity $c=3.31$ kJ/kg.

Rational transferring characteristics of heat exchanger for virgin soil in January for different types of soil are shown in the third. Different coefficients of heat transfer are correspondent to them: 1) $\alpha_1=10–15$ W/m^2 – dry sandy soil, 2) $\alpha_2=20–25$ W/m^2 – dry clayey soil, 3) $\alpha_3=30–35$ W/m^2 – soil with subsoil waters (Fig. 3). Speed of transferring of heat transfer in the tube of heat exchanger is $v(x)=1$ m/s. Temperature is increasing of 1°C at the entrance of heat exchanger.

Heat exchanger with less conductivity has better results because raising of heat – transfer through the cold layers of soil the part of heat is given back to environment.

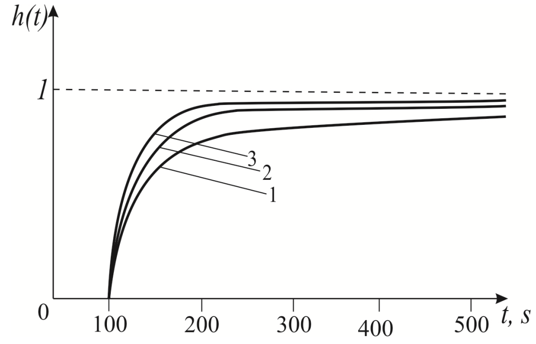

Fig. 3. Transferring characteristics h(t) of vertical heat exchanger for soil with coefficient of heat emission: 1) α_1 = 10–15 W/m², 2) α_2 = 20–25 W/m², 3) α_3 = 30–35 W/m²

In the fourth picture the diagrams of transferring characteristics of heat-exchanger which is situated in dry clayey soil are given (Fig. 4). Coefficient of heat emission is $\alpha = 25$ W/m² in different months of the year (in the middle of the month): 1 – December, 2 – January, 3 – February, 4 – March.

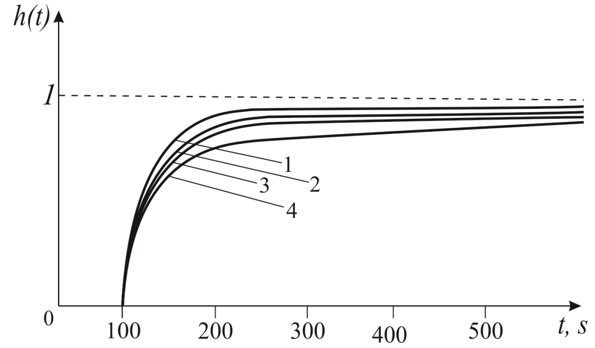

Fig. 4. Transferring characteristics h(t) of heat exchanger which is situated in the soil , coefficient of heat emission is α = 25 W/m² in different months of the year: 1 – December 2 – January, 3 – February, 4 – March

The slowest increasing of the temperature is in March, because working fluid emits the part of heat in surface layers of soil before the coming out on the surface. They are already chilled in winter months of the year.

Approximating transferring function of heat exchanger is obtained by the same way. The speed of transferring of heat carrier in the tube is taken as the entrance parameter. In the fifth picture characteristics for virgin soil in January for α_2 = 20–25 W/m² (Fig. 5).

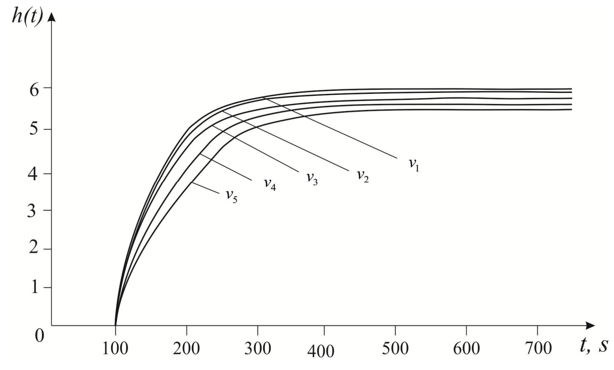

Fig. 5. Transferring characteristics h(t) of collector for virgin soil in January for α_2 = 20–25 W/m² under different increasing of collector speed at the entrance in the tube of collector

In this case (Fig. 5) at the entrance heat carrier with temperature $T(0,r,t)=0$ under different increasing of heat carrier speed at the entrance of heat carrier tube: $v_1(x)$ from 0 to 0.2 m/s, $v_2(x)$ from 0 to 0.3 m/s, $v_3(x)$ from 0 to 0.4 m/s, $v_4(x)$ from 0 to 0.5 m/s, $v_5(x)$ from 0 to 1.0 m/s.

As is seen from the modelled start mode of heat carrier, bigger coefficient of intensification and smaller constancy of time are for $v_2(x)$.

3. Conclusions

Mathematical model is built in this article. It gives us the opportunity to get approximating transferring function of vertical soil collector taking into account the convective transferring in the tube of collector and soil depth temperature. As the result of numerous calculations, soils with the content of water permit us to get more increasing of thermal energy in compare with dry sandy and clayey soil. It is also got a result that more increasing of thermal energy is for condition of the collector speed $0.2 < v(x) < 0.4$ m/s.

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