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HYBRID TECHNIQUES TO SOLVE OPTIMIZATION PROBLEMS IN EIT

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Abstract. *This paper presents the hybrid algorithm for identification the unknown shape of an interface to solve the inverse problem in electrical impedance tomography. The conductivity values in different regions are determined by the finite element method. The numerical algorithm is a combination of the level set method, Gauss-Newton method and the finite element method. The representation of the shape of the boundary and its evolution during an iterative reconstruction process is achieved by the level set function. The cost of the numerical algorithm is enough effective. These algorithms are a relatively new procedure to overcome this problem.*

Keywords: Inverse Problem, Level Set Method, Electrical Impedance Tomography

TECHNIKI HYBRYDOWE DO ROZWIĄZYWANIA ZAGADNIEŃ OPTYMALIZACYJNYCH W ETI

Streszczenie. *W artykule przedstawiono hybrydowy algorytm do identyfikacji nieznanego kształtu interfejsu, w celu rozwiązania zagadnienia odwrotnego w elektrycznej tomografii impedancyjnej. Wartości przewodności w różnych regionach określono za pomocą metody elementów skończonych. Algorytm numeryczny jest kombinacją metody zbiorów poziomicowych oraz metody Gaussa-Newtona i metody elementów skończonych. Odwzorowanie kształtu granicy i jej ewolucję w trakcie iteracyjnego procesu rekonstrukcji osiągnięto poprzez użycie metody zbiorów poziomicowych. Koszt algorytmu numerycznego jest dosyć efektywny. Algorytmy te są relatywnie nowymi rozwiązaniami dla tego typu problemu.*

Słowa kluczowe: zagadnienie odwrotne, metoda zbiorów poziomicowych, tomografia impedancyjna

Introduction

The article focuses on the inverse problem of identifying an unknown object. There were implemented the new algorithms to identify unknown conductivities. Numerical methods were based on the level set method (LSM) and the Gauss-Newton method. Level set methods have been applied very successfully in many areas of the scientific modelling [1, 3-6, 12, 17]. The level set function and the gradient techniques are based on shape and topology optimization in electrical impedance tomography (EIT) [8-11, 16]. The finite element method has been used to solve the forward problem. The cost of the numerical algorithm is enough effective. The proposed solution algorithm is initialized by using topological sensitivity analysis [13]. The Gauss-Newton method has been incorporated with the level set method to investigate shape optimization problems. The coupled algorithm is a relatively new procedure to overcome this problem. There was implemented the new hybrid algorithm to identify unknown values by electrical tomography [2, 3, 14, 15]. The purpose of the presented method is obtaining the better image reconstruction than gradient methods.

1. Optimization

Electrical impedance tomography is known that the inverse problem is nonlinear and highly ill-posed. Level set methods are often discretized on a regular grid that conveniently coincides with the finite element mesh (used for structural analysis). Typical problem in EIT requires the identification of the unknown internal area from near-boundary measurements of the electrical potential. It is assumed that the value of the conductivity is known in sub-regions whose boundaries are unknown. Figure 1 presents the algorithm based on the level set method.

The level set method is the numerical technique which can follow the evolution of interfaces. These interfaces can develop sharp corners, break apart and merge together. The level set function ϕ has the following properties:

$$\begin{gathered}\boldsymbol{\phi}(\mathbf{x},\mathbf{y},t)=0 \text{ for } (\mathrm{x},\mathrm{y})\in\partial\Omega(t)\equiv\Gamma(t),\\ \boldsymbol{\phi}(\mathbf{x},\mathbf{y},t)>0\ for\ (\mathrm{x},\mathrm{y})\in\Omega(t),\\ \boldsymbol{\phi}(\mathbf{x},\mathbf{y},t)<0\ for\ (\mathrm{x},\mathrm{y})\notin\Omega(t)\end{gathered} \tag{1}$$

where t is time.

The level set function ϕ is updated by solving discretized version of the Hamilton-Jacobi equation:

$$\boldsymbol{\phi}^{\mathrm{k+1}}=\boldsymbol{\phi}^{\mathrm{k}}-\mathbf{v}_{\mathrm{n}}^{\mathrm{k}}\left|\boldsymbol{\nabla}\boldsymbol{\phi}^{\mathrm{k}}\right|\Delta\mathrm{t} \tag{2}$$

where **v** is velocity.

Reinitialization is necessary when flat or steep regions complicate the determination of the zero contour.

This process is described by following partial differential equation [6]:

$$\frac{\partial\boldsymbol{\phi}}{\partial t}+S(\boldsymbol{\phi})(|\nabla\boldsymbol{\phi}|-1)=0 \tag{3}$$

where $S(\boldsymbol{\phi})=\frac{\boldsymbol{\phi}}{\sqrt{\boldsymbol{\phi}^2+\varepsilon^2}}, |\varepsilon|\ll 1$

Differential equation (3) is solved until a steady state is achieved.

A Gauss-Newton method is deployed to the regularized tangential movement problem. The electrodes move tangentially to the domain at each iteration and so do not in general lie on the boundary of the domain after each iteration. These are thus projected back onto the fixed domain by computing the nearest boundary simplex of the finite element mesh.

The Gauss-Newton algorithm is following form [3]:

$$\begin{gathered}\boldsymbol{\gamma}^{k+1}=\boldsymbol{\gamma}^{k}+\alpha\left(\mathbf{J}_t^k\mathbf{J}^{\boldsymbol{k}}+\lambda\mathbf{W}\right)^{-1}\cdot\\ \cdot\left(\mathbf{J}_t^k\mathbf{W}\left(\mathbf{u}_{\boldsymbol{m}}-\mathbf{h}(\boldsymbol{\gamma}^k)\right)-\lambda\mathbf{W}\boldsymbol{\gamma}^k\right)\end{gathered} \tag{4}$$

where $\boldsymbol{\gamma}$ denotes the vector of the unknown conductivity, α is the step size, λ is a regularization parameter, $\mathbf{J}^{\mathrm{k}}$is the Jacobian matrix, $\mathbf{W}$ is a regularization matrix and $\mathbf{h}(\boldsymbol{\gamma}^k)$ is the observation model derived from a finite elements model.

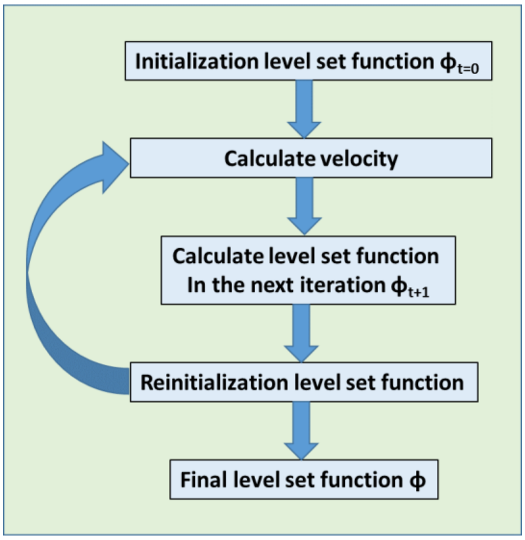

Fig. 1. The level set method- algorithm

The evolution of the level set function presents in Figure 2. Level sets are shown in Figure 3. The reinitialization of the level set function is presented in Figure 4. The numerical algorithm requires to solve the forward problem and the adjoint equation, these solutions present Figure 5 and 6.

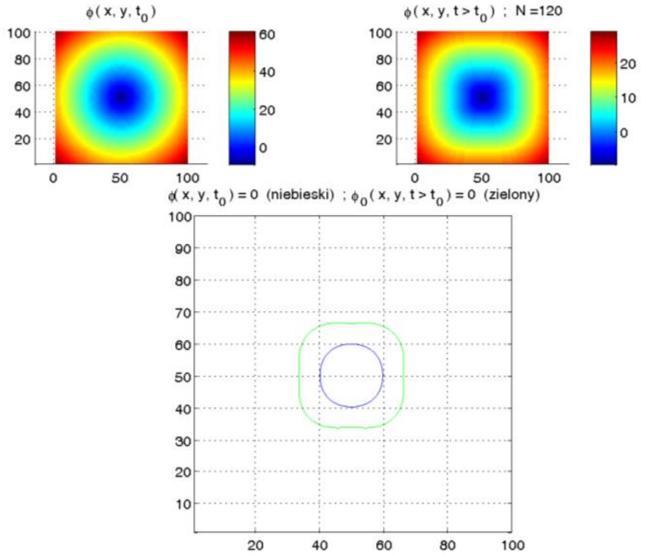

Fig. 2. The level set function – evolution

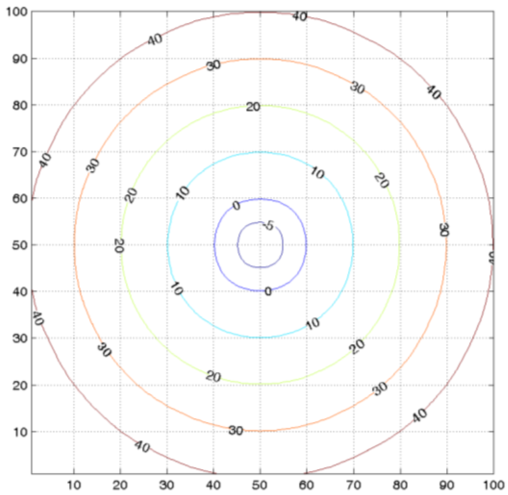

Fig. 3. Level sets

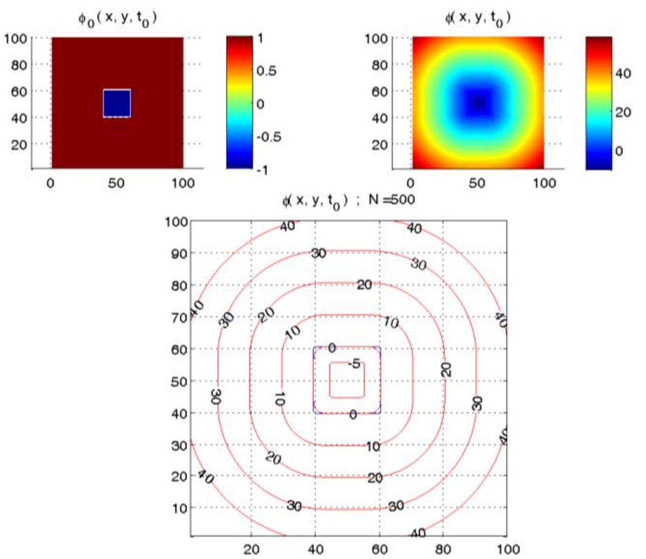

Fig. 4. Level set function -- reinitialization

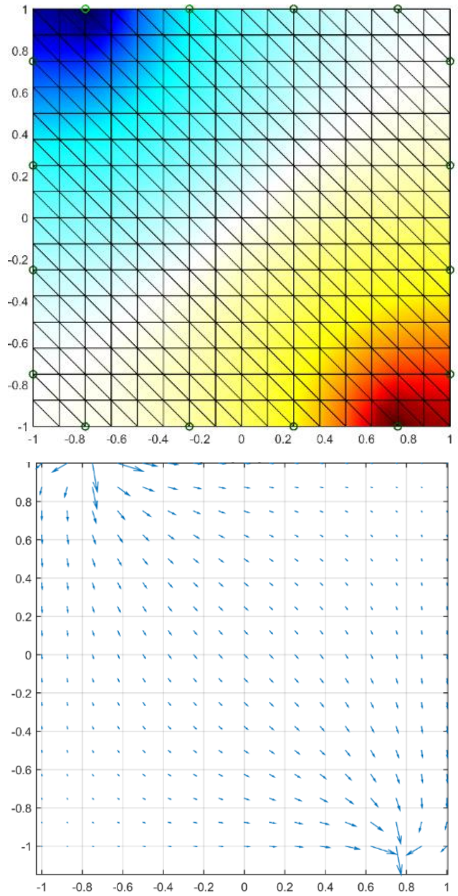

Fig.5. The forward problem

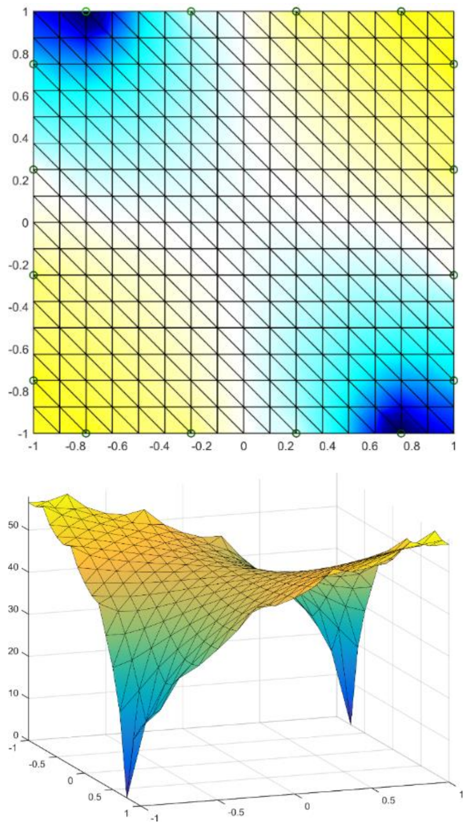

Fig. 6. The adjoint problem

2. Image reconstruction

In examples reported below, several numerical models with different discretization elements are presented. Additionally, there was shown different geometries of the conductivity distributions. The conductivity of searched objects is known. The representation of the boundary shape and its evolution during an iterative reconstruction process is achieved by the level set method and the Gauss-Newton method coupled together. In forward problem, which is given by Laplace's equation, we have used the finite element method. Figure 7 presents the model with the one object in the corner: (a) the initial model, (b) the reconstructed by Gauss-Newton method. Figure 8 shows the model with 1 object in the center: (a) the initial model, (b) the reconstructed by Gauss-Newton method, (c) zero level set, (d) the reconstructed by the level set method. Figure 9 presents the objective function for the model in Figure 8. Figure 10 shows the geometrical model with 5 objects.

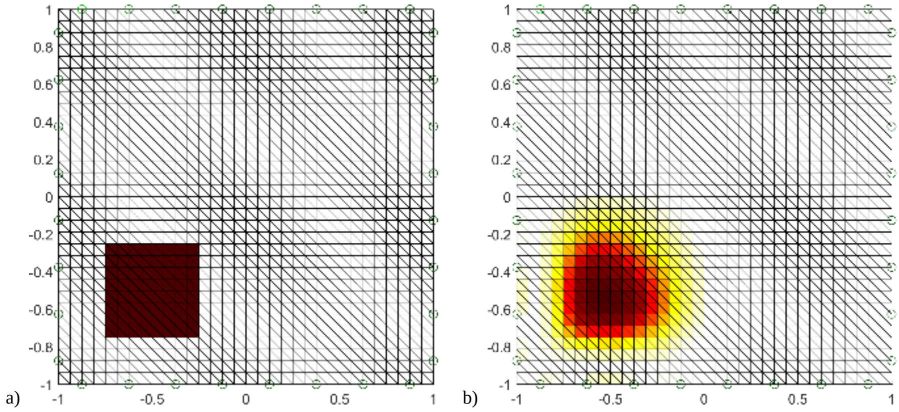

Fig. 7. The model with the one object: a) the initial model, b) the reconstructed by Gauss-Newton method

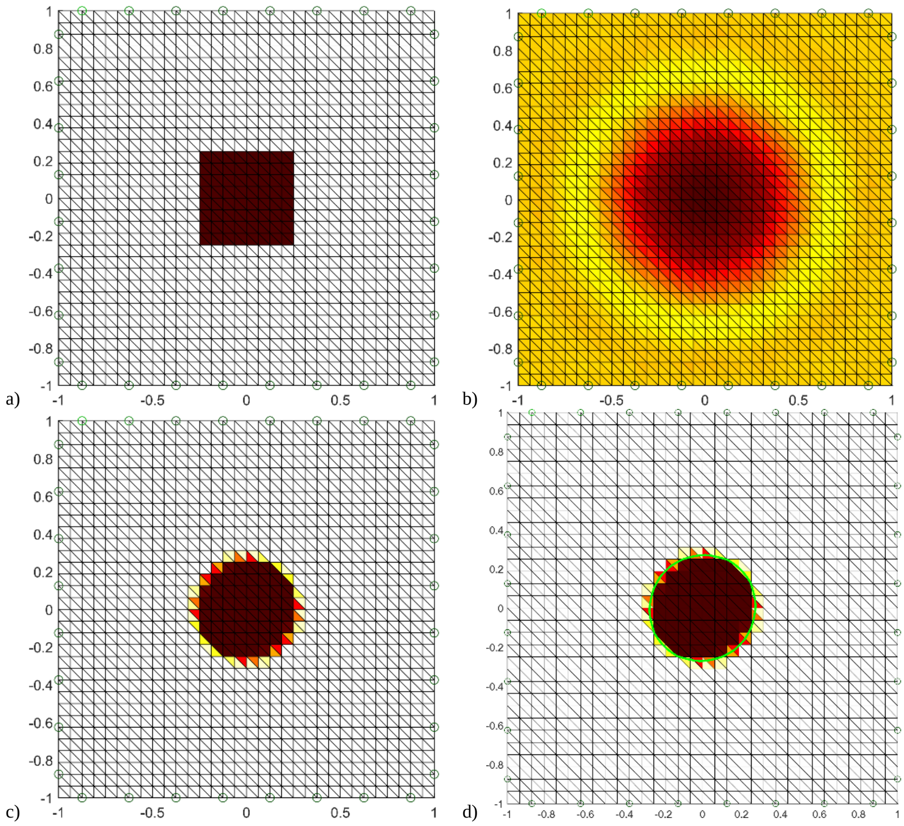

Fig. 8. The model with 1 object: a) the initial model, b) the reconstructed by Gauss-Newton method, c) zero level set, d) the reconstructed by the level set method

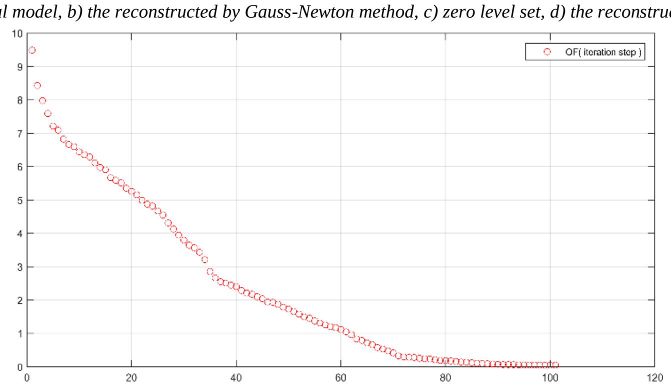

Fig. 9. The objective function for the model in Figure 8

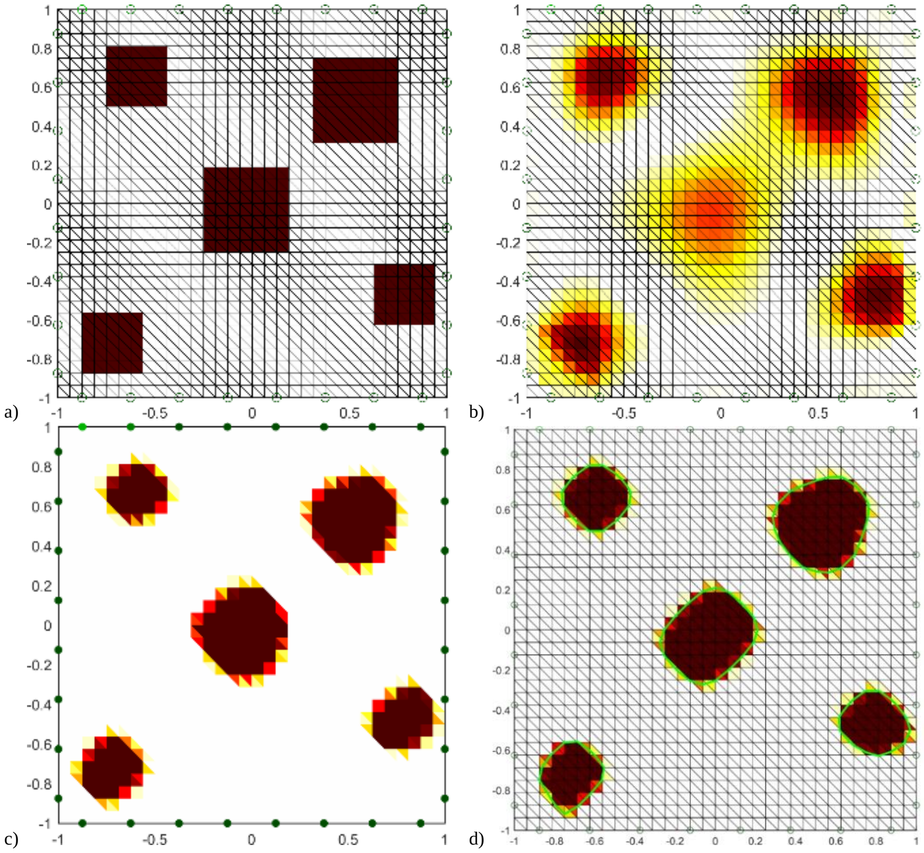

Fig. 10. The model with 5 objects: a) the initial model, b) the reconstructed by Gauss-Newton method, c) zero level sets, d) the reconstructed by the level set method

3. Conclusion

In this paper there was presented the method of approximation of material coefficient. The applications of the level set function, the Gauss-Newton method and the finite element method for the electrical impedance tomography were shown here. They are iterative algorithms, where multiple boundary shape changes smoothly and the searched object is detected. The level set idea is the good tool to the topological changes of the interface. The method of Gauss-Newton gives the best quality of the reconstruction of multiple objects and the hybrid algorithm is faster than the level set model.

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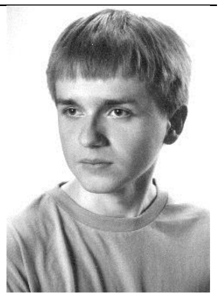

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