

DETECTION OF AIR GAPS IN COPPER-MINE CEILING BY ELECTRICAL IMPEDANCE TOMOGRAPHY

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Abstract In this paper, we investigate the inverse problem for the electric field so-called copper mine problem. In general, this task assumes detection of all air gaps. Gaps are localised above ceiling in a copper mine. Such task can be considered as application of the electrical impedance tomography. In order to solve forward problem there was used the boundary element method or the finite element method. The inverse problem is based on the level set method. There was considered extension of boundary element method (BEM). For simplicity zero order approximation has been chosen. The BEM has been connected with the infinite boundary elements. Hence, open domain problems with infinite boundary curves can be analysed. For such domain, we have solved the Dirichlet problem for two-dimensional Laplace's equation. The proposed numerical model has been verified.

Keywords: inverse problem, boundary element method, electrical impedance tomography

WYKRYWANIE SZCZELIN POWIETRZNYCH W CHODNIKU KOPALNI MIEDZI ZA POMOCĄ ELEKTRYCZNEJ TOMOGRAFII IMPEDANCYJNEJ

Streszczenie W tym artykule przyjrzymy się problemowi odwrotnemu dla pola elektrycznego na tak zwanym problemie kopalni miedzi. Głównym zadaniem w tym zagadnieniu jest wykrycie szczelin powietrznych, które są zlokalizowane w stropie kopalni. Takie zadanie można rozwiązać za pomocą elektrycznej tomografii impedancyjnej. W celu rozwiązania zagadnienia prostego można użyć metody elementów brzegowych (MEB) lub metody elementów skończonych. Zagadnienie odwrotne zostało oparte na metodzie zbiorów poziomicowych. Dla uproszczenia zostały zastosowane elementy zerowego rzędu. Metoda elementów brzegowych została rozszerzona o elementy nieskończone. Stąd problemy otwarte domeny z nieskończonymi brzegami mogą być analizowane. Dla takiej domeny, musimy rozwiązać zagadnienie Dirichleta dla równania dwuwymiarowy Laplace'a. Zaproponowany model numeryczny został zweryfikowany.

Słowa kluczowe: zagadnienie odwrotne, metoda elementów brzegowych, tomografia impedancyjna

Introduction

In this paper, we propose algorithm based on the combination of the boundary element method (or the finite element method) and the level set method to solve the inverse problem arising from electrical impedance tomography (EIT) [11–14]. The representation of the boundary shape and its evolution during an iterative reconstruction process is achieved by the level set method [2, 5–8, 15]. In our numerical algorithm we have used the gradient technique in order to calculate the velocity. This idea has been applied successfully in the context of inverse problem [3, 12, 16].

We focus our attention on so-called copper mine problem. Generally, this problem involves the detection of all air gaps are located above the roof of the copper mine (see Fig. 1). This task is very important for safety reasons. It could be done EIT.

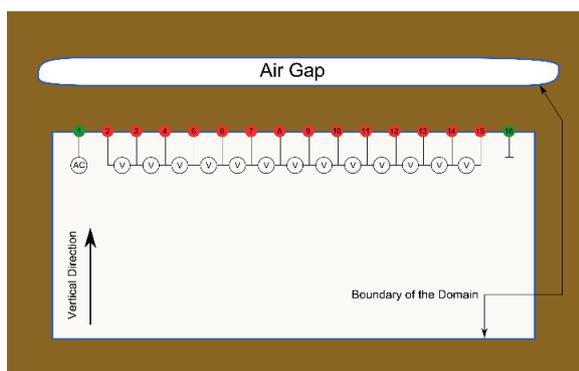


Fig. 1. Cross section of a copper mine, green and red circles denotes current and voltage electrodes, respectively, the boundary of air gap have to be localised as the final solution of the inverse problem

The electrical impedance tomography is very important field of research nowadays. It possesses many applications, for example this technique may be used in medical imaging, geophysics and other scientific areas. However, EIT is not easy to use due to necessity of solving the inverse problem during calculations. In three dimensional cases this requires a lot of computational effort. Using the level set method and the finite element method coupled together is proper way to solve the inverse problem. In particular all gaps in copper-mine ceiling can be localized.

1. Boundary element method

Physical phenomena are described usually by sets of differential equations. Numerical techniques give us opportunity to find approximate solutions of differential equations which cannot be solved by means of analytical ones. Among various numerical tools let us concentrate our attention on the boundary element method. BEM can be effectively employed on condition that partial differential equation can be transformed to integral form. Additionally, the Green's function has to be calculated. The explicit form of this one is desired. Often BEM can be easy coupled with other numerical methods or even analytical ones [1, 4]. Application of infinite boundary elements (IBEs) in BEM is rather less common because of difficulties with accurate numerical calculations of integrals with infinite limits of integration. In this paper we propose the generalisation of classical BEM with constant element interpolation for field function and its normal derivative by coupling with IBEs. For simplicity we consider Laplace's equation in two spatial dimensions.

Exterior domain with open boundary curve cannot be completely discretised by standard boundary elements with finite length. Hence, in the numerical model the IBEs should be introduced. So far several kinds of such boundary elements have been researched [9, 10, 17]. In case of IBEs one have to select interpolation functions which carry out appropriate conditions. In this paper we propose utilisation of the interpolation functions with exponential decay. This approach assumes that along IBEs the solution of differential equation and its normal derivative tend exponentially to zero. Speed of decay is described by one positive parameter.

It is clear that in realistic technical problems domains are usually finite. However, some engineering phenomena can be considered as unbounded domain problems and they are effectively solved using infinite elements.

Section 3 is the heart of this work and contains explanation of the theoretical model. The third section is devoted to numerical results and conclusions.

Let us consider the Laplace's equation in two-dimensional Cartesian coordinate system:

$$\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} = 0 \quad (1)$$

where $(x, y) \in \Omega$. We assume that Ω is homogeneous open set in general. Additionally electrical potential (u) or its normal derivative (q) is known for all boundary points. The differential problem defined in described manner may be regarded as forward problem for the electric field. Starting point for our research is typical for BEM integral equation, where the boundary curve is divided into N elements:

$$c(\vec{r}_i)u(\vec{r}_i) + \sum_{j=1}^N \int_{\Gamma_j} u(\vec{r}) q^*(\vec{r}, \vec{r}_i) d\gamma_j = \sum_{j=1}^N \int_{\Gamma_j} q(\vec{r}) u^*(\vec{r}, \vec{r}_i) d\gamma_j \quad (2)$$

Only three values of function c are possible for constant boundary elements. If a given point belongs to boundary of domain Ω , then the value equals 0.5. The value of function c equals 1, when a given point lies inside of Ω and equals 0 in other cases. The Green's function u^* may be obtained by solving the fundamental equation and is given by the following formula:

$$u^*(\vec{r}, \vec{r}_i) = \frac{1}{2\pi} \ln \left(\frac{A}{|\vec{r} - \vec{r}_i|} \right) \quad (3)$$

where A is a positive constant. Function q^* represents derivative of the Green's function in normal direction appointed by unit vector $\vec{n}(\vec{r})$. After elementary calculations we get:

$$q^*(\vec{r}, \vec{r}_i) = -\frac{1}{2\pi} \frac{\vec{n}(\vec{r}) \cdot (\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^2} \quad (4)$$

The vector formula for j -th constant boundary element is given by:

$$q^*(\vec{r}, \vec{r}_i) = -\frac{1}{2\pi} \frac{\vec{n}(\vec{r}) \cdot (\vec{r} - \vec{r}_j)}{|\vec{r} - \vec{r}_j|^2} \quad (5)$$

In order to extend our theoretical formalism into cases of open domain problems with infinite boundary curves, we have to introduce some modifications. Those modifications are attributable to infinite elements, which must be added to the set containing all finite boundary elements. If both quantities u and q are constant along given IBE, then some integrals appearing in BEM will be divergent. This difficulty disappears if we assume that quantities u and q tend asymptotically to zero.

The key step in our generalisation is proper choice of interpolation of u and q along IBE. If the boundary element is finite and we have three values for each quantity (three nodes), then the interpolation can be defined in the following way:

$$u(\xi) = \sum_{\alpha=f,m,l} N_\alpha(\xi) u_\alpha$$

$$q(\xi) = \sum_{\alpha=f,m,l} N_\alpha(\xi) q_\alpha \quad (6)$$

where, $\xi \in (-1, 1)$, $N_\alpha(\xi)$ are so-called interpolation functions:

$$N_f(\xi) = 0.5\xi(\xi - 1)$$

$$N_m(\xi) = 1 - \xi^2$$

$$N_l(\xi) = 0.5\xi(\xi + 1) \quad (7)$$

One should notice that the function $N_\alpha(\xi)$ takes value 1 for his "own" node and it takes value 0 for all other nodes. First of all, in order to expand the theory on IBEs we should multiply equations (7) by appropriate exponential factors. Every factor not have to modify the value of the interpolation function for her "own" node and it have to tends to zero for asymptotical values of the parameter ξ . For subsequent considerations let us assume $\xi \in (-1, +\infty)$. This conditions are satisfy when:

$$M_f(\xi) = 0.5\xi(\xi - 1)e^{-\frac{(1+\xi)}{\xi_e}}$$

$$M_m(\xi) = (1 - \xi^2)e^{-\frac{\xi}{\xi_e}}$$

$$M_l(\xi) = 0.5\xi(\xi + 1)e^{\frac{(1-\xi)}{\xi_e}} \quad (8)$$

Positive and dimensionless parameter ξ_e is responsible for speed of decay of electrical potential and its normal derivative along IBEs. Since for constant elements we have $u_f = u_l = u_m$ and $q_f = q_l = q_m$, so after changing $N_\alpha(\xi)$ to $M_\alpha(\xi)$ formulas (6) becomes:

$$u(\xi) = u_m S_\infty(\xi)$$

$$q(\xi) = q_m S_\infty(\xi) \quad (9)$$

where:

$$S_\infty(\xi) = \sum_{\alpha=f,m,l} M_\alpha(\xi) \quad (10)$$

is the sum of the interpolation functions providing exponential decay. Generally, the sum (10) can be expressed as follows:

$$S_\infty(s_0\xi) = \left[2(s_0\xi)^2 \sinh^2 \left(\frac{1}{2\xi_e} \right) + s_0\xi \sinh \left(\frac{1}{\xi_e} \right) + 1 \right] e^{-\frac{s_0\xi}{\xi_e}} \quad (11)$$

In above expression we assume that $s_0 = +1$ when the set of admissible values for parameter representing the IBE takes form $\Gamma_\xi = \langle -1, +\infty \rangle$, and $s_0 = -1$ when $\Gamma_\xi = \langle -\infty, +1 \rangle$. Let us notice from formula (11) that in case of discussed elements the sum of interpolation functions is not equal one. The graph of the function $S_\infty(\xi)$ for several selected parameters ξ_e is shown in Figure 2. This figure shows unequivocal oscillations for small values of ξ_e . Therefore, during numerical calculations the condition $\xi_e \geq 2$ should be satisfied. One can notice that the amplitude of unphysical oscillations decreases when ξ_e is larger and larger.

On this stage of consideration it is possible to write down the new version of integral equation (2). Collecting previous modifications and making discretization we obtain:

$$c(\vec{r}_i)u(\vec{r}_i) + \sum_{j=2}^{N-1} u_j \int_{\xi=-1}^{\xi=+1} q^*(\vec{r}_j(\xi), \vec{r}_i) d\gamma_j + u_1 \int_{\xi \rightarrow -\infty}^{\xi=+1} S_\infty(-\xi) q^*(\vec{r}_1(\xi), \vec{r}_i) d\gamma_1 + u_N \int_{\xi=-1}^{\xi \rightarrow +\infty} S_\infty(\xi) q^*(\vec{r}_N(\xi), \vec{r}_i) d\gamma_N = \sum_{j=2}^{N-1} q_j \int_{\xi=-1}^{\xi=+1} u^*(\vec{r}_j(\xi), \vec{r}_i) d\gamma_j + q_1 \int_{\xi \rightarrow -\infty}^{\xi=+1} S_\infty(-\xi) u^*(\vec{r}_1(\xi), \vec{r}_i) d\gamma_1 + q_N \int_{\xi=-1}^{\xi \rightarrow +\infty} S_\infty(\xi) u^*(\vec{r}_N(\xi), \vec{r}_i) d\gamma_N \quad (12)$$

It is natural that the first and the last element of the boundary are infinite. The other $N - 2$ boundary elements have finite length. Equation (12) represents model in which area Ω is described by one open curve only. However, generalisation of the formula (12) is easy and may be done by adding appropriate terms.

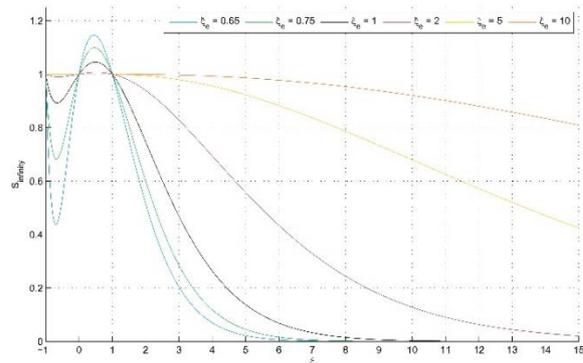


Fig. 2. The sum of the interpolation functions for IBEs defined by equation (11) (where $s_0 = +1$) versus the curve parameter ξ for selected coefficients ξ_e

2. Electrical Impedance Tomography

Efficient algorithms for solving forward and inverse problem in electrical impedance tomography have to be developed in order to use this approach for practical tasks. Moreover, it is necessary to improve performance of selected numerical methods. Typical problem in EIT requires identification of unknown internal area from near-boundary measurements of the electrical potential. It is assumed that the value of the conductivity is known in subdomains whose boundaries are unknown. Geometrical structure of the problem researched in this paper is shown in Figure 7. Boundary element method is well known and effective numerical technique used to solve partial differential equations. Infinite boundary elements give us possibility to solve equations with boundaries described by open curves. Proposed model consists of several numerical methods. The optimisation algorithm, which minimalizes the objective function is shown in Figure 3.

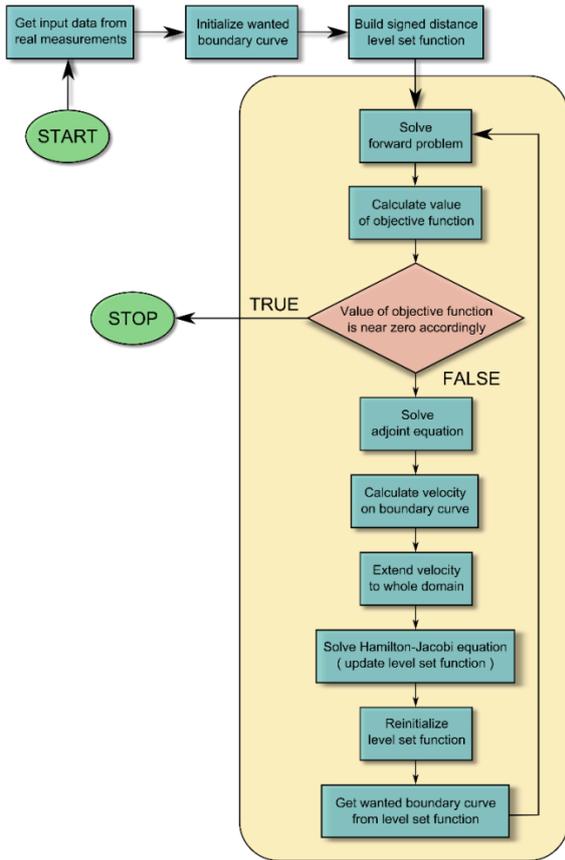


Fig. 3. Block diagram of the optimisation algorithm

3. Numerical results

The definite integrals present in equation (12) have been calculated by means of Gauss-Legendre quadrature, Gauss-Laguerre quadrature and Gauss quadrature with logarithmic weight function. Some diagonal integrals have been calculated analytically. The appropriate method has been chosen according to existence of singularity and type of domain of integration (finiteness or infiniteness). In numerical experiments we set $A = 10^{15}m$ and $\xi_e = 50$. Let us introduce open domain, where unit of length is the meter. The sketch of geometrical structure is given in Figure 4. One can notice that our geometry contains four IBEs. Whole boundary of the domain Ω is divided into forty elements ($N = 40$). Exact boundary conditions are expressed as:

$$u(x, y) = \begin{cases} -5V & \text{for } x = 0 \\ +5V & \text{for } x = m \end{cases} \quad (13)$$

In case of Laplace's equation (1), Dirichlet problem (13) has analytical solution. Normal derivative q is trivial to obtain and it takes following form on the boundary of the domain Ω :

$$q(x, y) = \begin{cases} -10Vm^{-1} & \text{for } x = 0 \\ +10Vm^{-1} & \text{for } x = m \end{cases} \quad (14)$$

However, one should remember that electrical potential tends exponentially to zero along IBEs. Therefore, parameter ξ_e should be large enough in order to create good approximation of conditions (13). Fig. 5a shows electrical potential for all nodes.

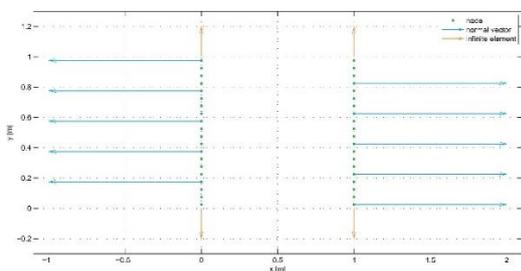


Fig. 4. The boundary problem: unit normal vectors are always outward pointing, positions of infinite elements are indicated

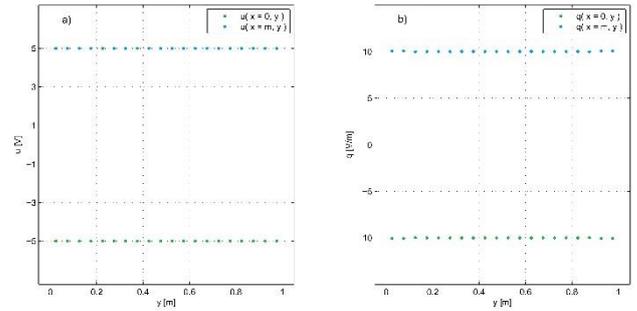


Fig. 5. The left graph a) presents the boundary conditions imposed on Laplace's equation, whereas the right graph b) shows the numerical solution

The solution of considered problem is given in Fig. 5b. From Fig. 6 we can see that percent errors are less than 0.7%. Good agreement of the numerical result with exact solution (14) is demonstrated. As we expected, reflective symmetry exists in Fig. 6. Additionally, percent errors are the same for each straight line ($x = 0$; $x = m$). The numerical results show that our theoretical formalism gives appropriate approach to forward problem described by open boundary curves.

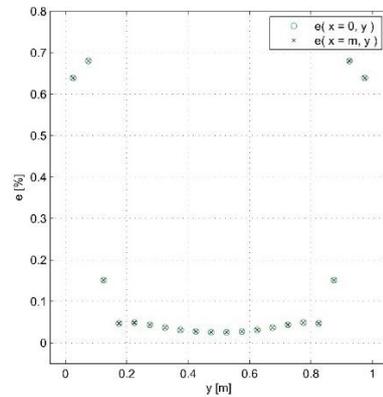


Fig. 6. The percent error obtained for the numerical solution of the Dirichlet problem

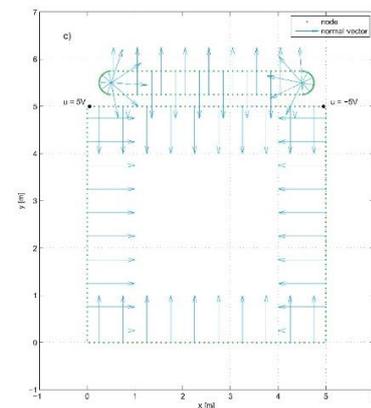


Fig. 7. Boundary problem: normal vectors and nodes on boundary elements are indicated

The definition of the boundary problem is shown in Figure 7. Normal vectors and nodes on boundary elements are indicated here. Figure 8 presents the image reconstruction of the copper-pine ceiling using BEM. The picture shows different object and the reconstructed image. The original object is noted by the blue line and the final figure is red. Figure 9 presents the image reconstruction in EIT obtained through the finite element method. The final contour represents the zero value of the level set function. The process of reconstruction is good, because the region borders are located nearly the object edges.

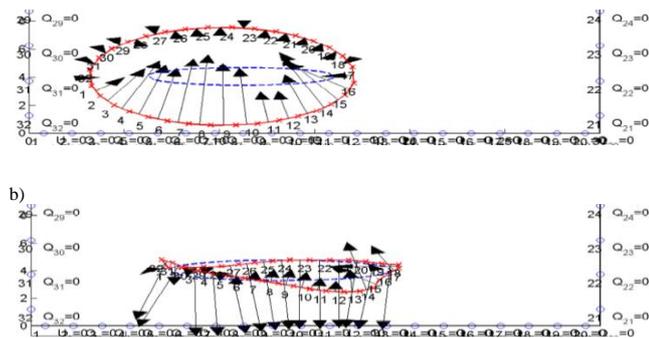


Fig. 8. Reconstruction by BEM and the level set method for the square area of the ceiling: a) the zero level set function and the searched object. b) the final reconstruction

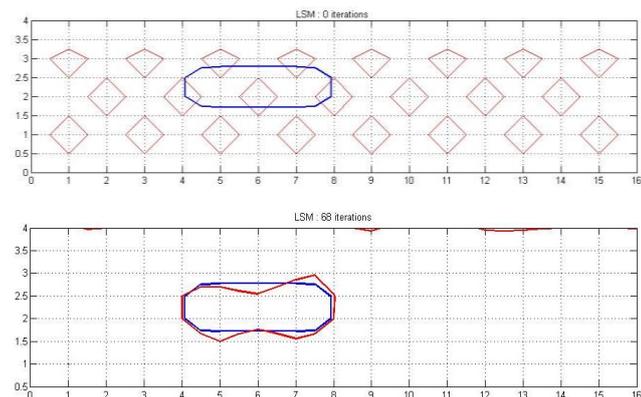


Fig. 9. Reconstruction by the finite element method and the level set method for the square area of the ceiling: a) the zero level set function and the searched object, b) the final reconstruction

4. Conclusion

In this paper, there was presented the method to solve the inverse problem for the electric field so-called copper mine problem. The level set idea is the good tool to the topological changes of the interface and gives the good quality reconstruction of unknown areas with one or many objects. All gaps in copper-mine ceiling were properly localized. This problem was motivated by electrical impedance tomography. Gaps were localised above ceiling in a copper mine. The applications were depended on a specially built model. There were used the boundary element method or the finite element method with the level set method to solve this problem. The level set function techniques were shown to be useful in this system. The boundary (finite) element method with the level set method gave the successfully results to identify the unknown properties of the object.

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