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TOPOGRAPHY DEPENDENT VERTICAL WIND DISTRIBUTION ESTIMATION

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Abstract. *We propose a novel technique for estimation of vertical wind distribution. Proposed modification of logarithmic profile relies on introducing topography dependent dispersion parameter calculated based on fractal dimension of topography. Initial results compared against full scale wind measurement show high agreement. Proposed methodology brings promise of precise a priori calculations of wind profile in case of non-flat, nonhomogeneous surface roughness terrain, improving the precision of wind potential estimation for wind energy sector.*

Keywords: surface roughness, fractal dimension, wind vertical distribution, estimation of wind energy potential, topography, micrometeorology

SZACOWANIE PIONOWEGO ROZKŁADU WIATRU NA PODSTAWIE WYMIARU FRAKTALNEGO TERENU

Streszczenie. *W niniejszej pracy prezentujemy nowatorską technikę estymacji pionowego rozkładu wiatru. Proponujemy modyfikację stosowanego obecnie rozkładu logarytmicznego przez związanie parametru rozkładu z wymiarem fraktalnym terenu. Wstępne wyniki porównane z rzeczywistymi odczytami wiatru wskazują dużą zgodność. Proponowana metodologia pozwala na skuteczne włączenie topografii i nierównomiernej szorstkości terenu do estymacji rozkładu, w sposób znaczny zwiększając dokładność estymacji potencjału energetycznego.*

Słowa kluczowe: chropowatość terenu, wymiar fraktalny, pionowy rozkład prędkości wiatru, szacowanie potencjału energetycznego wiatru, topografia, mikrometeorologia

Introduction

Wind energy is nowadays an important part of energy sector in many developed countries (EU, USA, China) and it is still on growth. Although it is a mature technology, a lot of potential can be found in grid, conversion and in basic aerodynamics as well. In opposition to other renewable sources, like solar or biomass, the initial estimation of potential for wind energy is challenging due to geographic-based highly inhomogeneous distribution. Although detailed descriptions of wind climate are existing [2, 6], they are not able to provide enough information for the detailed examination of candidate sites for wind development in microscale [4].

One of the crucial parameters for wind energy potential estimation is wind it its vertical wind velocity distribution being function of terrain – see figure 1.

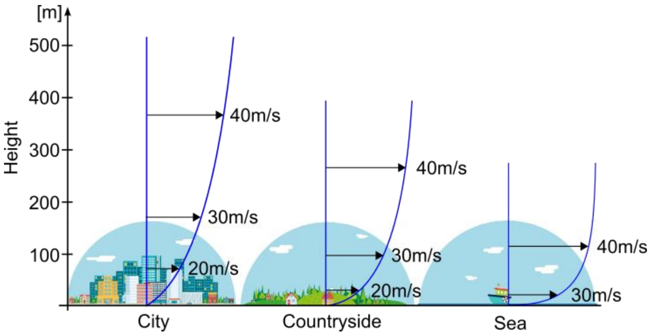

Fig. 1. Wind vertical distribution dependence on terrain characteristics

As visible on Fig. 1, the average velocity grows from zero at ground level – the no-slip boundary condition, a foundation of classical fluid mechanics, to geostrophic value at some altitude. Growth rate depends on the terrain characteristics. As power that can be extracted from wind is proportional to velocity cubed, the knowledge of velocity at desired hub-height or velocity vertical distribution is crucial.

The vertical distribution needed for extrapolation of velocity for desired altitude can be only roughly estimated before the measurement.

Distribution typically a log law, is function of surface type, its parameter, surface roughness z_0, can be found in literature categorized by terrain type. Comparison of z_0 values between sources [1, 3, 5] shows however significant discrepancy, driving towards errors in wind power estimation. Moreover, the available in literature values are valid only for flat homogeneous terrain, what in many cases is highly idealized.

A question, how to include the topography in wind analysis, incorporating the effects of nonhomogeneous terrain, changes in surface roughness, complicated, non-flat topography is a challenge and alive topic not only in wind energy but in general micrometorology and pollution dispersion studies as well [1].

1. Purpose

The aim of our current work is therefore to propose a tool for implementing the site surrounding topography influence on vertical wind distribution allowing its (wind vertical distribution) precise a priori calculations.

Our idea relies on describing the topography level of complication by its fractal dimension and to integrate it with classical logarithmic relation for vertical wind distribution.

Finally we will show positive validation of proposed methodology by comparison of our results with real wind data gathered at the site of interest.

2. Methodology

2.1. Fractal dimension

We are accustomed to topological or geometric definition of dimension – we know that line is one-dimensional, plane is two-dimensional and our space is three-dimensional. The intuitive definition of dimension is the minimal number of coordinates required to describe investigated shape (e.g. space, cube, torus etc.).

We can imagine a lot of shapes. Some of them are smooth e.g. ice rink or glass, another are rough e.g. mountains or clouds. To distinguish, to what extent one shape is more complex than another, some measure is needed. This class of measure is the fractal dimension, which in current work will be used for description of site surrounding topography.

Fractal dimension is a correspondence, which to any shape assigns a real number. In case of a square, the fractal dimension is equal to topological dimension i.e. 2, but more complicated shapes are described by not obvious and not straightly intuitive, non-integer values.

A fractal dimension can be applied to every set, not only in "our space", but generally in any Euclidean spaces $\mathbb{R}^n, n \in \mathbb{N}$, as well as in all metric spaces (although in current work we focus on objects in our, three-dimensional Euclidean space– on the terrain surrounding the wind measurement station).

Briefly, the fractal dimension measures, how complicated is examined form. It characterizes complexity of an object by the rate of volume growth, when the measures are more and more precise. The basic principle for calculation of fractal dimension is that volume and precision are related by power law. Hence, based on these two parameters, it is possible to calculate the fractal dimension.

As fractal and fractal dimension are actually a capacious term, hard to be defined formally from mathematical point of view, the diversity and multitude of definitions exist in literature (at least ten to our knowledge). From this set we decided to use the Minkowski dimension, for the reason that its definition allows for counting fractal dimension of an arbitrary shape. That feature is crucial for our purpose – the natural terrain under investigation is of unknown shape – surely, it is not simple to describe it with mathematic formula (see: section 2.3).

2.2. Calculation of fractal dimension – data extraction

In first step, a topography of place of interest, the surroundings of wind measurement station has to be digitized or extracted from available database.

One of advantages and strong points of presented method is that for most places of potential interest, the GIS (geographic information system) data are publicly available.

In presented case we used geospatial data from Norwegian Mapping Authority via website http://www.norgeskart.no.

Firstly, a grid was created and heights were located in that grid. To achieve grid with resolution of 1 m, determination of this distance on map was necessary. One meter between latitudes (geographic coordinates) is always constant on Earth, so exact values of latitudes were learned. Next, the distances between longitudes were calculated on north and south margin of considered area (because it is various on Earth). Differences on extremes are negligible, hence the distance giving 1 m between longitudes was the central value assumed. In this manner the mesh grid was created (see table 1).

Table 1. The creation of heights scheme matrix

	longitude 1	longitude 2	…	longitude n
latitude 1	height 11	height 12	…	height 1n
latitude 2	height 21	height 22	…	height 2n
...	...	...	…	...
latitude m	height m1	height m2	…	height mn

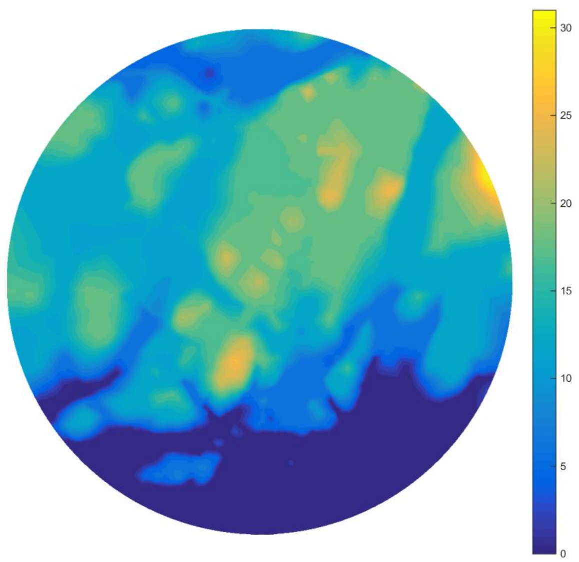

Fig. 2. Hypsometric map of area of interest built from acquired GIS data

For sake of keeping the high resolution of 1m and finite computing time we decided to focus on the area of 500 m radius from the measurement tower in the first step or research.

Data needed to create height map was provided by script written in Ruby programming language. Script connected to the web application programming interface, aggregated and deserialized JSON (JavaScript Object Notation) data. This set was allocated in adequate place in table 1. In next step, acquired data were processed in MATLAB for visualization (see map on Fig. 2) and further calculations of fractal dimension.

2.3. Calculation of fractal dimension – box counting method

In current step based on pre-processed data the fractal dimension can be calculated.

If matrix of heights is prepared, it is possible to create map of terrain (Fig. 1). Here we can start with research of fractal dimension.

The definition of Minkowski dimension is given as:

$$d = \lim_{\varepsilon \to 0} \frac{\log(num(C_\varepsilon))}{\log\frac{1}{\varepsilon}}, \quad (1)$$

where ε is size of cube C_ε, and $num(C_\varepsilon)$ is minimal number of cubes C_ε, which are necessary to cover of considered shape.

Unfortunately, due to limitation of available data to 1 m resolution, we cannot, accordingly to definition, obtain convergence of cube sizes to zero, nevertheless cut off at 1 m will not influence the results in considered case. Fractal dimension can be found by approximation of volume on number relation and was done by built in MATLAB linear least square (LLSQ) method, exhaustive in satisfied extent.

To develop the calculations reliability, the improvement of a standard procedure of fractal dimension calculation by box counting has been applied, which schematically is presented on figure 3.

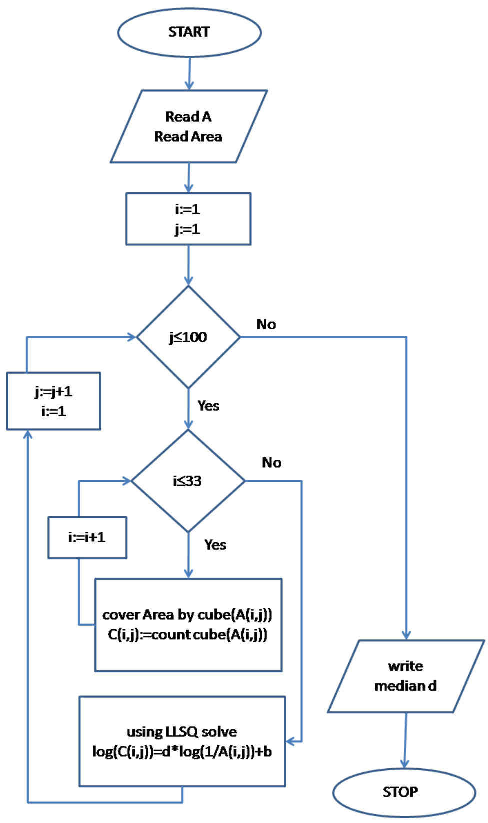

Fig. 3. Flowchart representing the fractal dimension calculation

This recipe is repeated n-times. We have decided on n equals 33. To achieve more comprehensive results, the procedure was realized on 100 subsets of A (A is the matrix including 100 sequences of cube sizes) with 33 elements in each subset. 30 first elements are random integers from 1 to 120, the last three are always 250, 1 and 2 (to provide start and end of segment).

Such a procedure resulted in two sequences of number in each subset of A: $(num(C_n))_{n=1}^{33}$and $(a_n)_{n=1}^{33}$. Final function $\log(num(C_n))$ = F($\log(^1/_{a_n})$) can be presented on log-log plot. The choice of base of logarithm is free as the consequence of logarithm properties – we should only decide on number from segment $(0, +\infty)\backslash\{1\}$.The slope of this plot found by Linear Least Squares method is the searched fractal dimension of terrain.

Obtaining fractal dimension in this way, we know the consequences of different cubes size. To avoid controversy on correctness and arbitrariness of the chosen sizes, we repeated determination of numbers for 100 different sets. Therefore we obtain 100 fractal dimensions of one and the same shape, calculated on another sets of cubes sizes. From this set we select the central value (median) and this result serves us for further calculations.

2.4. Validation

To test our results we decided to compare vertical velocity distribution extrapolated using our formula with measurement data taken from wind measurement station through 12 months of 2010 year (courtesy of prof. Lars Seatran, Norwegian University of Science and Technology, Trondheim, Norway).

The measuring 100 m height mast is located in Skipheia region, on the western part of Froya island, 150 km west of Trondheim, Norway, on Mid-Norway coast, 63.666 N, 8.343 E, roughly 20 m above sea level, see figure 4. The 6 pairs of sonic anemometers (Gill Wind Observer), with sampling frequency of 1 Hz, measure horizontal wind vector at heights: 10, 16, 25, 40, 70 and 100 m.

Fig. 4. Skipheia wind measurement station location

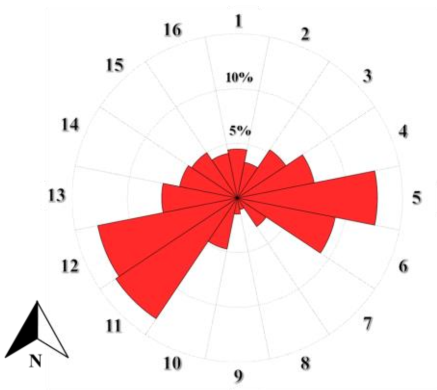

Fig. 5. Skipheia wind rose during neutral conditions

The basic wind climate characteristic of the site can be described as a typical coastal, with average surface roughness of $z_0 = 0.00308$ m and mean wind velocity at 100 m of 8.31 m/s. Wind rose (for neutral stratification) is presented on figure 5.

Data for comparison were filtered to include only neutral stratification (logarithmic profile is valid only for neutral conditions). Stability was calculated based on 10 min intervals (only the samples with 100% data recovery were chosen) by bulk Richardson number and Monin–Obukhov length.

2.5. Surface roughness

Nowadays, the logarithmic law is typically used for calculation of vertical wind profiles [3, 5], that can be presented in form given by equation (2).

$$U(z_2) = U(z_1)\frac{\log\left(\frac{z_2}{z_0}\right)}{\log\left(\frac{z_1}{z_0}\right)}, \tag{2}$$

where ε is size of cube C_ε, and $num(C_\varepsilon)$ is minimal number of cubes C_ε, which are necessary to cover of considered shape.

The term "surface roughness" can be misleading – the logarithmic law is derived from earth momentum equation and z_0, the surface roughness is an integration constant, not a measurable feature of "terrain surface".

Of course knowing the average wind velocities at least two altitudes the distribution parameter can be fitted, but it is not known before the measurement. The only estimate is a tabularized literature value.

That is the most common situation – the wind velocity is known/measured at given height and has to be extrapolated to desired altitude. In such cases standards [3, 5] and books [1] propose application of z_0 from tables. In such tables the terrain description and value z_0, various in different sources can be found. However, the biggest problem is how to apply tables for non-homogenous areas. Standard [5] allows to 10% variation of terrain type, but it is a rare situation.

3. Findings

Finally the regression analysis was performed to find the relation between surface described by fractal dimension and the vertical wind velocity dispersion parameter, the surface roughness.

To ensure that solution is not a site specific, we based the calculations on 16 direction sectors, each separately representing different surface type. This has given us the 16 datasets of z_0 and fractal dimension.

After preliminary selection, we have decided to describe the regression with custom modified exponential model in form given by equation 3:

$$U(z_2) = U(z_1)\frac{\log\left(\frac{z_2}{\alpha\cdot d\cdot\exp(\beta\cdot d)}\right)}{\log\left(\frac{z_1}{\alpha\cdot d\cdot\exp(\beta\cdot d)}\right)} \tag{3}$$

where:$\alpha = 7.663 \cdot 10^{-11}$, $\beta = 7.98$, d is the fractal dimension of terrain, z_2 – investigated height, z_1 – reference height, $U(z_1)$ – wind speed at height z_1 and $U(z_2)$ – wind speed at height z_2.

Using MATLAB curve fitting tools, the surface roughness was associated with fractal dimension of terrain. The goodness of fit was found more than satisfying, R squared and adjusted R squared were equal 0.82. It is the noticeable improvement of predictions compared to calculations of velocity distribution based on literature available surface roughness (e.g. ESDU and Eurocodes – see figure 6).

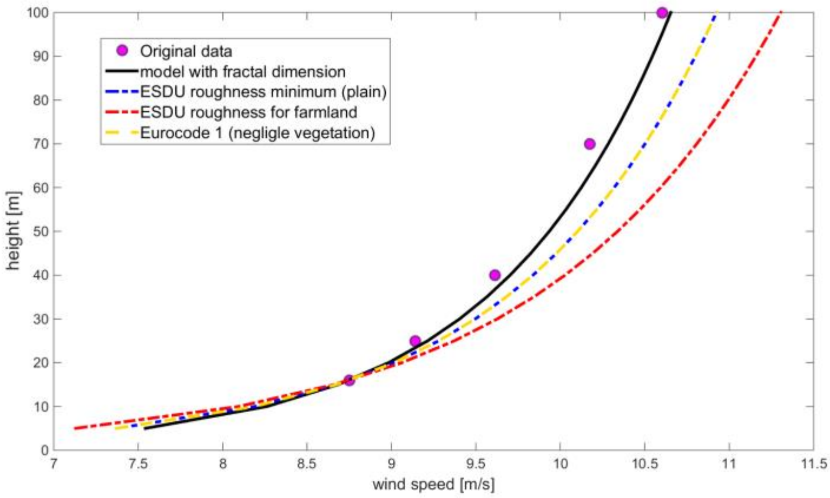

Fig. 6. Comparison of vertical velocity distributions: calculated, proposed in literature and the real, measured one

4. Research limitation

Although the proposed methodology was developed based on 16 direction sector representing different terrain types, we believe it should be tested on different site for rigorous validation.

We are also aware of the limits of our findings due to limited area investigated (500 m radius).

However our formulation was positively tested against atmospheric wind data in six different altitudes (0–100 m) in neutral atmospheric stratification. Our further research will aim for investigation listed limitations improving proposed methodology.

5. Practical implications

Our findings have important practical implications. Presented methodology leads to better estimation of wind vertical profile, one of the basic wind parameter needed not only for estimation of wind potential in energy sector, but also in meteorology and dispersion studies.

Most important our technique relies on topographical data, bringing the promise of straightforward including the terrain shape in basic wind parameters estimation. Additionally, the proposed methodology can be applied easily in cities (urban wind) or in other complicated terrain - fractal dimension is a universal measure and it is applicable to every type of terrain.

6. Originality

According to our best knowledge, there are no reports in literature describing presented issues. Application of fractal dimension as a surface characteristic measure is not a new topic in metallurgy and geology, fractal dimension was used as a characteristic time series feature in wind engineering as well, but as far as we know the relation between fractal dimension and vertical wind velocity was not presented yet.

Acknowledgement

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