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CONSTRUCTION METHOD OF OPTIMAL CONTROL SYSTEM OF A GROUP OF UNMANNED AERIAL VEHICLES

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Abstract. *In the following work the authors implement mathematical representation of a control system of complex dynamic system. An example of such system is a group of unmanned aerial vehicles. The sufficiency of controlled object mathematical representation is implemented, using the system approach, which in turn describes system elements, taking into account all the relations between them.*

Keywords: controlled object, control systems, Laplace transformation (LT), unmanned aerial vehicle (the UAV)

METODY TWORZENIA OPTYMALNEGO SYSTEMU KONTROLI GRUPĄ BEZZAŁOGOWYCH STATKÓW POWIETRZNYCH

Streszczenie. *W prezentowanej pracy autorzy wdrażają matematyczny opis systemu kontroli złożonego systemu dynamicznego. Przykładem takiego systemu jest grupa bezzałogowych samolotów. Zaimplementowano odpowiedni opis matematyczny kontrolowanego obiektu stosując podejście systemowe, które opisuje wszystkie elementy systemu uwzględniając wszystkie relacje między nimi.*

Słowa kluczowe: obiekt kontrolowany, system sterowania, transformata Laplace'a, bezzałogowe statki powietrzne

Introduction

Characteristic features of team flight of unmanned aerial vehicles (the UAV) with consideration for the possibility of failure of data channel as the controlled object [1-3]: the lack of full mathematical representation of occurring changes; random nature of processes and non-stationariness of parameters allow considering it as a complex system.

It is noted that adequate general approach to solving control tasks related to the complex object functioning is an application of system approach.

1. Problem analysis

The main principle of systematic control task solving is decomposing complex systems into conventional "small" elements and synthesis of control with the condition of consideration of all relations between elements.

Implementing systematic approach assumes adequate mathematical representation of controlled object, which allows to describe elements of the system and take into account all the relations between them.

In the theory of automatic control systems two basic types of mathematical representation of objects and control systems are being used. The first type lies in representation of processes in the frequency domain, in the "space of signals", when all the elements features are determined by the transfer functions. This type of representation is focused on solving tasks of stabilization, when the program trajectory is a priori known; the object allows linearization with a small deviation from it and the necessary parameters of the transition process have to be ensured. Despite its widespread use, the disadvantage of such representation lies in the fact that it does not allow taking into account an integrated use of all available resources in a closed autonomous dynamic system and solving current tasks of automatic control with object functions.

For the synthesis of optimal control "in large" when it is necessary to simultaneously determine the best program trajectory and implement stabilization on it, the second method of mathematical representation, namely the state of space method is used. Under the terms of this method, the mathematical system model, which reflects the characteristics and existing restrictions, should be represented in the state of space - a metric domain, each element of which fully determines the state of the considered system.

Such representation allows using both classic (various methods of variational calculus), and modern methods of optimization (principle of maximum, generalized work method, method of analytical design of optimal regulators). Therefore, in order to conduct a research, the mathematical model, where the state vector includes the phase coordinate of the system in state of space, has been used. The possibility of applying the principle of separation for closed dynamical systems, presented in the space of phase states, is of importance.

2. Problem solving

With respect to controlled coordinate, generalized controlled object, for short hereinafter referred to as object, can be described by equation of the form:

$$y^{(n)} + \sum_{i=0}^{n-1} a_i(t) \cdot y^{(i)} = \sum_{j=0}^{m} d_j(t) \cdot x^{(i)}, \qquad (1)$$

where x is a controlling action; y is an output coordinate; $a_i(t)$, $d_j(t)$ are coordinates variable in time, or in operator form

$$A(p,t)Y(p) = D(p,t)X(p), \qquad (2)$$

where $A(p,t), D(p,t)$ are linear differential operators.

Note that the equation of the form (1) or (2) can describe the motion of many objects, including aerial vehicles. A linear mathematical model of the object motion relative to the estimated trajectory is correct only under certain restrictions imposed on object signals and coordinates, range and rate of change of its coefficients. The said restrictions can be described in the form of inequations

$$B_k[p, g, y, x, a_i(t), d_j(t), t] \le 0,$$

where B_k is some operators, g is incoming signal, p is a parameter of Laplace transformation, t is current time.

Taking into account the laws of control, the equation of primary system takes on the appearance on

$$y^{(n)} + \sum_{i=0}^{n-1}[a_i(t) + C_i(t)]y^{(i)} = \sum_{j=0}^{m}[d_j(t) + Cx_j(t)]x^{(j)}. \qquad (3)$$

Put the case of rebuilt coefficients $C_i(t)$ and $Cx_j(t)$ in the form of sum of two components

$$\begin{cases} C_i(t) = \overline{C}_i(t) + \Delta C_i(t); \\ Cx_j(t) = \overline{C}x_j(t) + \Delta Cx_j(t), \end{cases} \tag{4}$$

where $\overline{C}_i(t), \overline{C}x_j(t)$ are invariables, $\Delta C_i(t), \Delta Cx_j(t)$ are rebuilt components. By introducing notation

$$\begin{cases} \mathrm{a_i = a_i(t) + \overline{C}_i;} \\ \mathrm{d_j = d_j(t) +, \overline{C}x_j,} \end{cases}$$

to the formulas (3) and (4), it is possible to set down

$$y^{(n)} + \sum_{i=0}^{n-1}[a_i(t) + \Delta C_i(t)]y^{(i)} = \sum_{j=0}^{m}[d_j(t) + \Delta Cx_j(t)]x^{(j)}. \tag{5}$$

Changes of equation coefficients a_i and d_j, caused by changes in the parameters of the object, will be balanced out by the relative changes in coefficients $\Delta C_i(t), \Delta Cx_j(t)$ to values, defined by the model. The equation of a master model can be represented as follows

$$y_м{}^{(n)} + \sum_{i=0}^{n-1} b_i y_м{}^{(i)} = \sum_{j=0}^{m} d_{jм} x^{(j)}, \tag{6}$$

where $b_i, d_{jм}$ are the model equation coefficients, independent of time.

If you select the desired values of coefficients equal to model coefficients, and their additional components, the formula (3) shall be reconstructed into the form

$$\begin{aligned} y^{(n)} + \sum_{i=0}^{n-1}\left[b_i(t) + \Delta a_i(t) + \Delta C_i(t)\right]y^{(i)} = \\ = \sum_{j=0}^{m}\left[d_{jм}(t) + d_j(t) + \Delta Cx_i(t)\right]x^{(j)} \end{aligned} \tag{7}$$

Deviation of main system output, and the model $\varepsilon = y - \mathrm{y}_м$ shall be calculated, using the equations (6) and (7):

$$\begin{aligned} \varepsilon^{(n)} + \sum_{i=0}^{n-1} b_i \varepsilon^{(i)} = \sum_{j=0}^{m}\left[\Delta d_j(t) + \Delta Cx_j(t)\right]x^{(j)} - \\ - \sum_{i=0}^{n-1}\left[\Delta a_i(t) + \Delta C_i(t)\right]y^{(i)} \end{aligned}. \tag{8}$$

By grouping members of equation (8), we deduce

$$\begin{aligned} \varepsilon^{(n)} + \sum_{i=0}^{n-1} b_i \varepsilon^{(i)} = \left[\sum_{j=0}^{m} \Delta d_j(t) x^{(j)} - \sum_{i=0}^{n-1} \Delta a_i(t) y^{(i)}\right] + \\ + \left[\sum_{j=0}^{m} \Delta Cx_j(t) x^{(j)} - \sum_{i=0}^{n-1} \Delta C_i(t) y^{(i)}\right]; \end{aligned} \tag{9}$$

$$\varepsilon^{(n)} + \sum_{i=0}^{n-1} b_i \varepsilon^{(i)} = \mathrm{F} + \mathrm{U}, \tag{10}$$

where $\mathrm{F} = \left[\sum_{j=0}^{m} \Delta d_j(t) x^{(j)} - \sum_{i=0}^{n-1} \Delta a_i(t) y^{(i)}\right]$ is an equivalent disturbance, affecting the system and which causes an error; $\mathrm{U} = \left[\sum_{j=0}^{m} \Delta Cx_j(t) x^{(j)} - \sum_{i=0}^{n-1} \Delta C_i(t) y^{(i)}\right]$ – is an equivalent impact of control device.

In order to simplify the results, by introducing notation $\varepsilon^{(i)} = x_{i+1}(i = 0,1,\ldots,n)$, an error equation (10) can be represented in matrix form

$$\dot{X} = AX + U,$$

where

$$\mathbf{X} = \begin{Vmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_n \end{Vmatrix}, \quad \mathbf{U} = \begin{Vmatrix} 0 \\ 0 \\ . \\ . \\ . \\ U_0 \end{Vmatrix}$$

$$\mathbf{A} = \begin{Vmatrix} 0 & 1 & 0 & . & . & . & 0 \\ 0 & 0 & 1 & . & . & . & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ -b_0 & -b_1 & -b_2 & & & & -b_{n-1} \end{Vmatrix}$$

A task of adaptive functional and stable complex synthesis can be reduced in this case to the choice of such control, whereby the equalization of equivalent disturbance F is taking place.

The principle of distribution, which is fundamental for the synthesis of optimal linear systems, can also be applied by the synthesis of optimal functional and stable cotrol of the UAV's team flight.

Among the three control channels of the UAV's team flights: the distance, interval and exceedance, the distance control channel, due to the high response rate, as well as significant and asymmetrical constraints on the control signals during acceleration and braking action of aerial vehicles, is distinguished by technical implementation complexity. In this regard, the argumentation for applying the principle of adaptation in the systems of the UAV group piloting and all subsequent researches will be conducted through the example of distance control channel between the UAV.

In general case, distance control between the UAV must be coordinated, i.e. distance change should be made through simultaneous effect on diving-rudder and engine thrust. In order to implement such control, the signals, proportional to pitch attitude angle, flight altitudes and flight-speed should be given to channels of rudder and thrust.

In this case the motion equation can be introduced as:

$$\left.\begin{aligned} \delta_P = q_{11}(P)d + q_{12}(P)V + q_{13}(P)h - y_1; \\ \delta_в = q_{21}(P)d + q_{22}(P)V + q_{23}(P)h - y_2, \end{aligned}\right\}$$

where $q_{ik}(P)$ $(i = 1,2; k = 1,2,3)$ are the transfer functions of engine thrust and diving-rudder control systems; *d*, V, *h* are current values of distance, pitch attitude angle and altitude respectively; y_i is the signals by trajectory program.

In case when the flight altitude and angular motions of aerial vehicles are stabilized by the quick-responding autopilots, the angular motions have no effect on the motions of aerial vehicle mass center, and to control the distance you can use automatic machine, which affects the engine thrust; the equation of this automatic machine will be [6]:

$$\delta_P = q_{11}(P)d - y_1$$

Distance stabilization loop is relatively low frequency. As a result, there is no need to consider small parameters of this loop. In particular, for the engine the simplest transfer function of inertia links can be used:

$$W_{DW}(P) = \frac{K_{DW}}{T_{DW}P + 1},$$

where K_{DW}, T_{DW} are the augmentation ratio and time constant of aircraft engine respectively.

Thus, the analysis of the dynamic properties and characteristics of the UAV's team flight while providing functional and stable control with consideration for the possibility of data channel, shows that it is possible to divide the UAV's team flight control into three independent channels: the distance, exceedance and interval control between the UAV.

Furthermore, the distance control channel has been analyzed. This allows limiting the dimension of the following model and the complexity of obtained algorithms.

The failures of sub-systems of closed-loop regulating system in theory can be represented by different models [4-8]. Additive models, where the failure is represented by a vector of accidental variations in phase coordinates of a system $\gamma(t)$ of the form

$$\begin{cases} \dot{X}(t) = A(X,t)\cdot X(t) + B(X,t)\cdot U(t) + \xi(t) + \gamma(t); \\ Y(t) = H(t)\cdot X(t) + \eta(t) + \gamma(t), \end{cases} \tag{11}$$

are inconvenient to describe the data channel failure, since they do not allow taking into account the characteristic, specific features of the considered failures (in this case a model of the form (11) can not take into account a variation in quantity of a data update time T_D).

Models of the form

$$\begin{cases} \dot{X}(t) = A^s(X,t)\cdot X(t) + B^w(X,t)\cdot U(t) + \xi(t); \\ Y(t) = H^f(t)\cdot X(t) + \eta(t); \end{cases}$$

which take into account the failure by structure change of the dynamic matrix of a system $A^s(X,t)$, control matrix $B^w(X,t)$ and measurements matrix $H^f(t)$, are more suitable to the physical nature of the processes in the data channel.

Since the data channel failures mean an increase, over the permissible duration, in data update time between base components and the actual UAV T_D, $H_i(t)$ can be described as:

$$H_i(t) = \begin{cases} H_0 \ at \ T_D \le T_D^{\lim}; \\ H_1 \ at \ T_D^{nop} > T_D > T_D^{\lim}. \end{cases}$$

In addition, this model can not take into account the causes of failures (because of enemy counterglow or dysfunction of communication technology) and a priori unknown statistical characteristics of failures.

The principle of distribution, applied for decomposing the controlled object into sealed channels, can be used in order to divide control tasks into two sub-tasks: the formation of program trajectory and stabilization on it. Moreover, by distance stabilization the most important is minimizing the errors of program trajectory exercise. Therefore, the weight coefficients of the first component of the performance function should be significantly greater than the weight coefficients of the second component. Thus, the minimization of performance function of the form

$$I(X(t), U(t)/H_i(t)) = M\left(\int_{t_0}^{t_k} X^T(t)\cdot\beta\cdot X(t)dt\right);$$

i.e. without the component, corresponding to the expenses on control, has been carried out in the following work.

3. Conclusions

Methodical basis for construction of the optimal functional and stable control of the group of the UAV is to use the system approach and the principle of decomposition as the theory of complex technical systems construction.

The structure of optimal functional and stable control of the group of the UAV with consideration for the possibility of data channel failure must include, in addition to the controlled object (the group of the UAV) and a trilateration measuring system, a relative model based on the extrapolation of the relative position in the group and optimal regulator, which implements the algorithm of optimal control.

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