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THE SYNTHESIS OF MATHEMATICAL MODELS OF NONLINEAR DYNAMIC SYSTEMS USING VOLTERRA INTEGRAL EQUATION

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Abstract. *The problem of creating mathematical models of nonlinear dynamical systems does not have an unambiguous solution and requires the creation of a separate synthesis method for each such object. To develop a method for synthesizing mathematical models of an extensive class of nonlinear dynamical systems with polynomial nonlinearities. The work uses a method based on the solution of the Volterra integral equation in the ideology set forth in Van Trees H.L., according to which the structure of a nonlinear dynamical object present47s a series connection of the linear part, characterizing the inertial properties of the system, and the nonlinear element, given by static characteristic. The difference of the suggested version of the method from the classical one, proposed in the works of Van Trees H.L., is an expansion of their input and output signals into Fourier series and a representation of the inertial part of these systems by their Bode plots, connected into one structure with input and output signals and non-linearity by Volterra integral equation. The algorithm of the proposed method is disclosed by the example of solving the problem of identifying a nonlinear dynamical system which impulse response of the inertial part satisfies the separability requirement, the order of the polynomial nonlinearity is three, and the model of the input signal has the form of a sinusoid "raised" over the time axis on a priori given constant level. A computational experiment was carried out on the example of nonlinear dynamical systems with the third order of the nonlinear characteristic and the first and second orders of the model of the inertial part of these systems with the specified algorithms of their parametric identification. The suggested method allows to synthesis the mathematical model of a nonlinear dynamical system with the polynomial static characteristic to the case when the input signal has an arbitrary number of harmonics, and the model of the inertial part and the nonlinear polynomial function have an arbitrary order.*

Keywords: nonlinear dynamical system, mathematical model, polynomial nonlinearity function, Bode plot, Fourier series, Volterra integral equation

SYNTEZA MATEMATYCZNYCH MODELI NIELINIOWYCH UKŁADÓW DYNAMICZNYCH Z WYKORZYSTANIEM RÓWNANIA CAŁKOWEGO VOLTERRY

Streszczenie. *Problem tworzenia modeli matematycznych nieliniowych układów dynamicznych nie ma jednoznacznego rozwiązania i wymaga stworzenia odrębnej metody syntezy dla każdego takiego obiektu. Celem pracy jest opracowanie metody syntezy modeli matematycznych rozległej klasy nieliniowych układów dynamicznych o wielomianowej nieliniowości. W pracy zastosowano metodę opartą na rozwiązaniu całkowego równania Volterry w ideologii przedstawionej przez Van Trees H.L., zgodnie z którą struktura nieliniowego obiektu dynamicznego przedstawia szeregowe połączenie części liniowej, charakteryzującej własności inercyjne układu, oraz elementu nieliniowego, zadanego charakterystyką statyczną. Różnica proponowanej wersji metody od klasycznej, zaproponowanej w pracach Van Treesa H.L., polega na rozwinięciu sygnałów wejściowych i wyjściowych w szeregi Fouriera oraz przedstawieniu części inercyjnej tych układów za pomocą ich charakterystyk Bodego, połączonych w jedną strukturę z sygnałami wejściowymi i wyjściowymi oraz nieliniowością za pomocą całkowego równania Volterry. Algorytm proponowanej metody ujawniono na przykładzie rozwiązania problemu identyfikacji nieliniowego układu dynamicznego, którego odpowiedź impulsowa części inercyjnej spełnia warunek rozdzielności, rząd nieliniowości wielomianowej jest trzeci, a model sygnału wejściowego ma postać sinusoidy "uniesionej" nad osią czasu na zadanym z góry stałym poziomie. Przeprowadzono eksperyment obliczeniowy na przykładzie nieliniowych układów dynamicznych o trzecim rzędzie charakterystyki nieliniowej oraz pierwszego i drugiego rzędu modelu części inercyjnej tych układów z zadanymi algorytmami ich identyfikacji parametrycznej. Zaproponowana metoda pozwala na syntezę modelu matematycznego nieliniowego układu dynamicznego o wielomianowej charakterystyce statycznej dla przypadku, gdy sygnał wejściowy ma dowolną liczbę harmonicznych, a model części inercyjnej i nieliniowa funkcja wielomianowa mają dowolny rząd.*

Słowa kluczowe: nieliniowy układ dynamiczny, model matematyczny, wielomianowa funkcja nieliniowości, charakterystyki Bodego, szereg Fouriera, równanie całkowe Volterry

Introduction

The purpose of this paper is the synthesis of a wide class of mathematical models of nonlinear dynamic systems with polynomial nonlinearities. And the movement toward this purpose we start with the observation that the problem of constructing mathematical models of nonlinear dynamic systems does not have a unique solution and requires to create a separate synthetic procedure for each object. The majority of these methods, are presented in a review papers [6, 16, 17], which states that the most efficient methods, based on various options for solving Volterra integral equation ideology laid down in the [18], following which, the nonlinear dynamic object structure shall be formed as a serial connection of the line part that characterizes the inertial properties of the system, and nonlinear section, set by nonlinear static characteristic.

One of the authors started to research the possibilities for practical application of the ideas suggested in [18] at the beginning of the 80-th of the last century, suggesting in the work [9] a recovery method for the input signals of the measuring systems with non-linear polynomial function of conversion with the presentation of these systems by frequency characteristics of inertial part as well as using the integral Volterra equation for their binding into one mathematical structure. The modified algorithm of this method was suggested in [8] for the simultaneous solution of the problem of input signals recovery in nonlinear dynamical systems, as well as the problem of synthesis of mathematical models of these systems.

With the decomposition into the Fourier series of the output signal of nonlinear dynamic systems and its presentation by the Fourier series with the unknown coefficients of its input signal, which are stipulated for by the computational algorithm of the suggested method, there appear the complex combinatorial relationships of the Fourier coefficients with the discrete values of the frequency characteristics of inertial part of these systems. The work [12] suggests an algorithm for automatic determination of combinatorial relationships that made this method available for synthesis of mathematical models of signals and systems for practical application.

But despite the automated search for combinatorial links, the suggested algorithm was too complicated and did not find a wide application. This encouraged the authors to find the simplified options for the method, one of which was suggested in [11], a distinctive characteristics of which is the fact that it uses only the first harmonica of the whole Fourier series, which present the input signal of nonlinear dynamic system, and to receive the frequency characteristics of the inertial part of this nonlinear system it uses a special algorithm for processing a segment of the system response to an input signal, described by a stepped unit function.

The work, presented below, we suggest a modified version of the method for synthesis of mathematical models of nonlinear dynamic systems suggested in the [11], the implementation of which does not require an additional experiment on determining the frequency characteristics of inertial nonlinear dynamic system, neither it needs the use of a special algorithm for processing the response of the system to the input signal in the form of a single stepped function.

1. Formulation of the problem

The withdrawal of the design ratio for method of synthesis of mathematical models of nonlinear dynamic systems as the initial conditions of the method, as well as in [8, 9, 11], there had been taken first the condition set in the [18], stating that the nonlinear dynamic system under consideration may be presented as a serial connection of the two structural units, as shown in Fig. 1, where one of the links characterizes the inertial properties and is described by impulse response g(t), and the second, the inertia-free one characterizes the nonlinear properties, connecting its input with the output Z of an ordered polynomial:

$$Z=f_k(y)=v_1 y+v_2 y^2+\cdots+v_k y^k \tag{1}$$

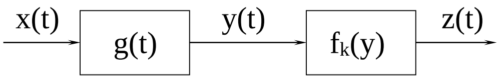

Fig. 1. Block diagram of a nonlinear dynamical system

This precondition, as shown even in the same work [18], allow to bind an input signal $x(t)$ of the system with the output $z(t)$ Volterra integral equation

$$\begin{aligned} z(t)=&\sum_{i=1}^{k} v_i\cdot\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} x(t-\tau_1)\ldots x(t-\tau_i)\times\\ &\times g(\tau_1,\ldots,\tau_i)\,d\tau_1\ldots d\tau_i \end{aligned} \tag{2}$$

The other initial precondition is the possibility for presenting signals $x(t)$, $z(t)$ by the truncated Fourier series –

$$x(t)=\sum_{n=-N}^{N} c_n\cdot e^{jn\omega_1 t} \tag{3}$$

$$z(t)=\sum_{n=-M}^{M} q_n\cdot e^{jn\omega_1 t} \tag{4}$$

where

$$\omega_1=\frac{2\pi}{T} \tag{5}$$

is the frequency of the first harmonica, T is its period, and

$$c_n=\frac{1}{T}\int_0^T x(t)\cdot e^{-jn\omega_1 t}\,dt \tag{6}$$

$$q_n=\frac{1}{T}\int_0^T z(t)\cdot e^{-jn\omega_1 t}\,dt \tag{7}$$

are the Fourier coefficients of function $x(t)$ and $z(t)$.

As a result, we obtain algorithms for the synthesis of functions $g(t)$, $f_k(t)$.

As we have already noted, the problem of constructing mathematical models of nonlinear dynamic systems does not have an unambiguous solution and requires the creation of a separate synthesis method for each such object. The majority of these methods, are presented in a review papers [6, 16, 17], which states that the most efficient methods, based on various options for solving Volterra integral equation ideology laid down in the [17], following which, the nonlinear dynamic object structure shall be formed as a serial connection of the line part that characterizes the inertial properties of the system, and nonlinear section, set by nonlinear static characteristic.

Primarily we must mention those methods, described in the works of [1, 2].

And the works to be mentioned after the publications of [6, 16, 17], are those issued by [3–5], in which the mathematical models of nonlinear dynamic systems are given in the form of transfer functions, algebraically structured in skew polynomials, as well as the works of [13–15], in which mathematical models of nonlinear dynamic systems set by transfer functions with coefficients which depend on the frequency and amplitude of the input periodic signals, shall be really reduced to the class of cybernetic models since their identification is carried out by iterative numerical methods in powerful software environment MATLAB.

2. Theoretical research

To receive the calculated ratios, suitable for qualitative analysis, we specify the number of members N of the series (3) and the value of the polynomial index k (1).

Let $N = 1$, a $k = 3$, i.e., let

$$x(t)=\sum_{n=-1}^{1} c_n\cdot e^{jn\omega_1 t} \tag{8}$$

$$Z=f_3(y)=\nu_1 y+\nu_2 y^2+\nu_3 y^3 \tag{9}$$

Substituting expressions (8) and (9) in the integral equation (2), we receive:

$$\begin{aligned} z(t)=&\,\nu_1\cdot\int_{-\infty}^{\infty}\sum_{n=-1}^{1} c_n\cdot e^{jn\omega_1(t-\tau_1)}\cdot g(\tau_1)\,d\tau_1+\\ &+\nu_2\cdot\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sum_{n=-1}^{1} c_n\cdot e^{jn\omega_1(t-\tau_1)}\times\\ &\times\sum_{n=-1}^{1} c_n\cdot e^{jn\omega_1(t-\tau_2)}\times g(\tau_1,\tau_2)\,d\tau_1\,d\tau_2+\\ &+\nu_3\cdot\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sum_{n=-1}^{1} c_n\cdot e^{jn\omega_1(t-\tau_1)}\times\\ &\times\sum_{n=-1}^{1} c_n\cdot e^{jn\omega_1(t-\tau_2)}\times\sum_{n=-1}^{1} c_n\cdot e^{jn\omega_1(t-\tau_3)}\times\\ &\times g(\tau_1,\tau_2,\tau_3)\,d\tau_1\,d\tau_2\,d\tau_3 \end{aligned} \tag{10}$$

Let the impulse response $g(t)$, which, as is generally accepted, presents the response of the type

$$g(t)=\begin{cases} g(t) & \text{at } t\ge 0\\ 0 & \text{at } t<0 \end{cases} \tag{11}$$

of this dynamic system to the input signal as a Dirac delta function where

$$\delta(t)=\begin{cases} \infty & \text{at } t=0\\ 0 & \text{at } t\ne 0 \end{cases}\quad \int_{-\infty}^{\infty}\delta(t)dt=1 \tag{12}$$

is separable, that is, let the condition come true for it

$$g(\tau_1,\tau_1,\ldots,\tau_k)=g(\tau_1)\cdot g(\tau_2)\cdots g(\tau_k) \tag{13}$$

For this case the equation (10) takes the form of

$$\begin{aligned} z(t)=&\,\nu_1\cdot\sum_{n=-1}^{1} c_n\cdot e^{jn\omega_1 t}\int_{-\infty}^{\infty} g(\tau_1)\cdot e^{-jn\omega_1\tau_1}\,d\tau_1+\\ &+\nu_2\cdot\left(\sum_{n=-1}^{1} c_n\cdot e^{jn\omega_1 t}\int_{-\infty}^{\infty} g(\tau_1)\cdot e^{-jn\omega_1\tau_1}\,d\tau_1\right)\times\\ &\times\left(\sum_{n=-1}^{1} c_n\cdot e^{jn\omega_1 t}\int_{-\infty}^{\infty} g(\tau_2)\cdot e^{-jn\omega_1\tau_2}\,d\tau_2\right)+\\ &+\nu_3\cdot\left(\sum_{n=-1}^{1} c_n\cdot e^{jn\omega_1 t}\int_{-\infty}^{\infty} g(\tau_1)\cdot e^{-jn\omega_1\tau_1}\,d\tau_1\right)\times\\ &\times\left(\sum_{n=-1}^{1} c_n\cdot e^{jn\omega_1 t}\int_{-\infty}^{\infty} g(\tau_2)\cdot e^{-jn\omega_1\tau_2}\,d\tau_2\right)\times\\ &\times\left(\sum_{n=-1}^{1} c_n\cdot e^{jn\omega_1 t}\int_{-\infty}^{\infty} g(\tau_3)\cdot e^{-jn\omega_1\tau_3}\,d\tau_3\right) \end{aligned} \tag{14}$$

Please note that the integrals in equation (14) are the values of Bode magnitude and Bode phase plots (Bode plot) $W(j\omega)$ at the points $\omega = n\omega_1$, since, as is well known, the Bode plot $W(j\omega)$ is a Fourier transformation of impulse response $g(t)$, that is

$$W(j\omega)=\int_{-\infty}^{\infty} g(t)\cdot e^{-j\omega t}\,dt=\int_{0}^{\infty} g(t)e^{-j\omega t}dt \tag{15}$$

In view of the above, the equation (14) may be rewritten as

$$\begin{aligned}
&z(t)=\varphi_{-3}(\nu_3,c_{-1},W(-j\omega_1))\cdot e^{-j3\omega_1 t}+\\
&+\varphi_{-2}(\nu_3,\nu_2,c_{-1},c_0,W(-j\omega_1),W(0))\times e^{-j2\omega_1 t}+\\
&+\varphi_{-1}(\nu_3,\nu_2,\nu_1,c_{-1},c_0,c_1,W(-j\omega_1),W(0),W(j\omega_1))\times\\
&\times e^{-j\omega_1 t}+\\
&+\varphi_0(\nu_3,\nu_2,\nu_1,c_{-1},c_0,c_1,W(-j\omega_1),W(0),W(j\omega_1))+\\
&\varphi_1(\nu_1,\nu_2,\nu_3,c_{-1},c_0,c_1,W(-j\omega_1),W(0),W(j\omega_1))\times e^{j\omega_1 t}+\\
&+\varphi_2(\nu_2,\nu_3,c_0,c_1,W(0),W(j\omega_1))\cdot e^{j2\omega_1 t}+\\
&+\varphi_3(\nu_3,c_1,W(j\omega_1))\cdot e^{j3\omega_1 t}
\end{aligned} \tag{16}$$

The expression (16) shows that the signal $z(t)$, except for the first harmonica and constant component, stipulated for by the set in the form of a truncated series (8) signal $x(t)$, also contains the second and the third harmonics stipulated for by nonlinearity (9).

Therefore, with the realization of the output signal $z(t)$ on the interval T, we set it by the truncated Fourier series, which includes a constant component and the first three harmonics, i.e.

$$\begin{aligned}
&z(t)=q_{-3}\cdot e^{-j3\omega_1 t}+q_{-2}\cdot e^{-j2\omega_1 t}+\\
&+q_{-1}\cdot e^{-j\omega_1 t}+q_0+q_1\cdot e^{j\omega_1 t}+\\
&+q_2\cdot e^{j2\omega_1 t}+q_3\cdot e^{j3\omega_1 t}
\end{aligned} \tag{17}$$

where $q_n,\ n=(-3,-2,-1,\ 0,\ 1,\ 2,\ 3)$ are the Fourier coefficients that shall be calculated by the formula (7).

Substituting the truncated series (17) in equation (16) instead of $z(t)$, will have an identity that is executed if and only if the coefficients of the same harmonics in the left and right parts of equations (16) considering (17) are the same, that is, when

$$\begin{cases}
\varphi_{-3}(\nu_3,c_{-1},W(-j\omega_1))=q_{-3}\\
\varphi_{-2}(\nu_3,\nu_2,c_{-1},c_0,W(-j\omega_1),W(0))=q_{-2}\\
\varphi_{-1}(\nu_3,\nu_2,\nu_1,c_{-1},c_0,c_1,W(-j\omega_1),W(0),W(j\omega_1))=q_{-1}\\
\varphi_0(\nu_3,\nu_2,\nu_1,c_{-1},c_0,c_1,W(-j\omega_1),W(0),W(j\omega_1))=q_0\\
\varphi_1(\nu_1,\nu_2,\nu_3,c_{-1},c_0,c_1,W(-j\omega_1),W(0),W(j\omega_1))=q_1\\
\varphi_2(\nu_2,\nu_3,c_0,c_1,W(0),W(j\omega_1))=q_2\\
\varphi_3(\nu_3,c_1,W(j\omega_1))=q_3
\end{cases} \tag{18}$$

Specifying the function $\varphi_{-3}(\cdot),\ \varphi_{-2}(\cdot),\ \varphi_{-1}(\cdot)$ $\varphi_0(\cdot),\ \varphi_1(\cdot),\ \varphi_2(\cdot),\ \varphi_3(\cdot)$ by performing all algebraic operations on the right part of the expression (16), instead of the indefinite system of equations (18) we obtain the system of equations

$$\begin{cases}
\nu_3\cdot c_{-1}^3\cdot W^3(-j\omega_1)=q_{-3}\\
\nu_2\cdot c_{-1}^2\cdot W^2(-j\omega_1)+3\cdot\nu_3\cdot c_{-1}^2\cdot c_0\times\\
\times W^2(-j\omega_1)\cdot W(0)=q_{-2}\\
\nu_1\cdot c_{-1}\cdot W(-j\omega_1)+2\cdot\nu_2\cdot c_{-1}\cdot c_0\times\\
\times W(-j\omega_1)\cdot W(0)+3\cdot\nu_3\cdot c_{-1}^2\cdot c_1\times\\
\times W^2(-j\omega_1)\cdot W(j\omega_1)+\\
+3\cdot\nu_3\cdot c_{-1}\cdot c_0^2\cdot W(-j\omega_1)\times W^2(0)=q_{-1}\\
\nu_1\cdot c_0\cdot W(0)+2\cdot\nu_2\cdot c_{-1}\cdot c_1\times\\
\times W(-j\omega_1)\cdot W(j\omega_1)+\nu_2\cdot c_0^2\times\\
\times W^2(0)+6\cdot\nu_3\cdot c_{-1}\cdot c_0\times\\
\times c_1\cdot W(-j\omega_1)\cdot W(0)\cdot W(j\omega_1)+\\
+\nu_3\cdot c_0^3\cdot W^3(0)=q_0\\
\nu_1\cdot c_1\cdot W(j\omega_1)+2\cdot\nu_2\cdot c_0\cdot c_1\times\\
\times W(0)\cdot W(j\omega_1)+3\cdot\nu_3\cdot c_{-1}\times\\
\times c_1^2\cdot W(-j\omega_1)\cdot W^2(j\omega_1)+\\
+3\cdot\nu_3\cdot c_0^2\cdot c_1\cdot W^2(0)\cdot W(j\omega_1)=q_1\\
\nu_2\cdot c_1^2\cdot W^2(j\omega_1)+3\cdot\nu_3\cdot c_0\times\\
\times c_1^2\cdot W(0)\cdot W^2(j\omega_1)=q_2\\
\nu_3\cdot c_1^3\cdot W^3(j\omega_1)=q_3
\end{cases} \tag{19}$$

which may be used for making specific calculations as for the problem with identification of nonlinear characteristics $f_k(y)$ of the dynamic system, and for the problem of parameters identification of Bode plot $W(j\omega)$ its inertial component, the solution of which require the algorithms, which are to be built below.

We built an algorithm for solving the problem of identification of nonlinear dynamic system, the inertial impulse response $g(t)$ of which satisfies the condition of separability (13), the order polynomial nonlinearity $f_k(y)$ equals three, i.e., $k = 3$, and a model of the input signal $x(t)$ looks like a sinusoids with a frequency ω_1, "lifted" over the axis of time by a permanent component, c_0, i.e., has the form of (20). We generalize the results for the case when the nonlinearity has a random order, i.e., is approximated by a polynomial function (1), and the input signal contains a random number of harmonic components, i.e. is approximated by a cut-off Fourier series (3).

From the expressions (1)-(3), (10), (13), (14) for the above formulated conditions we will have

$$z(t)=\sum_{i=1}^{k}\nu_i\left(\sum_{n=-N}^{N}c_nW(jn\omega_1)e^{n\omega_1 t}\right)^i \tag{20}$$

It is easy to see that from the expression (20), setting $k = 3$, $N = 1$, we get the expression (16), which in turn during decomposition in cut Fourier series (17), we obtain the system of equations (19). In the case of random order of polynomial inequality, which equals k, this system will have $2k+1$ equations (as for the number of coefficients of Fourier truncated series, in which the output signal $z(t)$ will be decomposed to receive the harmonic identity with the right-hand side of equation (20)).

But if the expressions (20) are fair then there appears a question: "Is it possible to transform them in a way that they remain equitable on condition that the impulse response of dynamic system contains multiple exponential components which number, as we know from the theory of linear differential equations equals the order of differential equation, which describes the process in this dynamic system?"

Further, based on our work [10], we show that the answer to this question is positive.

Let the impulse response of the inertial part of nonlinear dynamical system can be represented as

$$g(t) = g_1(t) + g_2(t) \tag{21}$$

where both components are exhibitors that differ only by the index of power and coefficients, by which these exponentials are multiplied.

Substituting expression (21) in the expression (2), we get

$$\begin{aligned}
z(t) &= \sum_{i=1}^{k} \nu_i \int_{-\infty}^{\infty} \overset{i}{\cdots} \int_{-\infty}^{\infty} x(t-\tau_1)\ldots x(t-\tau_i)\times \\
&\times [g_1(\tau_1,\ldots,\tau_i) + g_2(\tau_1,\ldots,\tau_i)]d\tau_1\ldots d\tau_i = \\
&= \sum_{i=1}^{k} \nu_i [\int_{-\infty}^{\infty} \overset{i}{\cdots} \int_{-\infty}^{\infty} x(t-\tau_1)\ldots x(t-\tau_i)\times \\
&\times g_1(\tau_1,\ldots,\tau_i)d\tau_1\ldots d\tau_i + \\
&+ \int_{-\infty}^{\infty} \overset{i}{\cdots} \int_{-\infty}^{\infty} x(t-\tau_1)\ldots x(t-\tau_i)\times \\
&\times g_2(\tau_1,\ldots,\tau_i)d\tau_1\ldots d\tau_i]
\end{aligned} \tag{22}$$

Assuming that each component of the expression (28) holds the identity (13) – and if each component is exponential, then this identity, as we have mentioned above is always performed – and according to the execution of the property (11) for each component, we can present the expression (29) in the form of

$$\begin{aligned}
z(t) &= \sum_{i=1}^{k} \nu_i [\left(\int_0^{\infty} x(t-\tau) g_1(\tau) d\tau \right)^i + \\
&+ \left(\int_0^{\infty} x(t-\tau) g_2(\tau) d\tau \right)^i]
\end{aligned} \tag{23}$$

In turn, the expression (30) using a ratio (3), (13), (15) is easily reduced to the form

$$\begin{aligned}
z(t) &= \sum_{i=1}^{k} \nu_i [\left(\sum_{n=-N}^{N} c_n W_1(jn\omega_1) e^{n\omega_1 t} \right)^i + \\
&+ \left(\sum_{n=-N}^{N} c_n W_2(jn\omega_1) e^{n\omega_1 t} \right)^i]
\end{aligned} \tag{24}$$

where

$$\begin{cases} W_1(jn\omega_1) = W_1(s)\big|_{s=jn\omega_1} \\ W_2(jn\omega_2) = W_2(s)\big|_{s=jn\omega_2} \end{cases} \tag{25}$$

$$\begin{cases} W_1(s) = \int_0^{\infty} g_1(t) e^{-st} dt \\ W_2(s) = \int_0^{\infty} g_2(t) e^{-st} dt \end{cases} \tag{26}$$

Summarizing the expression (21) up to M components, i.e., presenting the impulse response of the inertial part of nonlinear dynamical system of the M-th order as

$$g(t) = \sum_{m=1}^{M} g_m(t) \tag{27}$$

on condition that each component of the expression (34) the property (11) is fair, we can easily come to the generalized expression

$$z(t) = \sum_{i=1}^{k} \nu_i [\sum_{m=1}^{M} \left(\sum_{n=-N}^{N} c_n W_m(jn\omega_1) e^{n\omega_1 t} \right)^i] \tag{28}$$

where

$$W_m(jn\omega_1) = W_m(s)\big|_{s=jn\omega_1}, \quad m = 1,2,\ldots,M$$

$$W_m(s) = \int_0^{\infty} g_m(t) e^{-st} dt \tag{29}$$

We start this extension with the appeal to expression (21), in which on the right side there is the sum of two exponents.

As we know from the theory of linear differential equations, in this case the differential equation systems will have the 2nd order and depending on whether its characteristic polynomial roots be of negative numbers or complex-conjugate pair of numbers with negative real parts, the processes in this system will be aperiodic of the 2nd order (curves 2 in Fig. 2) or oscillatory (curve 3 in Fig. 2). Our work [7] shows that the differential equations of 2nd order describe a wide class of real dynamic systems; it also presents the received conditions which describe the way to determine that.

In order to be able to match the level of complexity of the system of equations, we obtain as a result of solving the problem, formulated in the title, with the equations that we have received for nonlinear dynamical system with inertial part of the 1st order, i.e., the system of equations (19), also ask, as in the problem input $x(t)$ in the form (8), and the output signal Z after nonlinearities in the form (9).

3. Experimental research

To build an algorithm for the identification of mathematical model of nonlinear dynamic system, we set its input signal $x(t)$ as sinusoid with a frequency ω_1, "lifted" over the time axis by a constant component c_0, i.e., in the form

$$\begin{aligned}
x(t) &= c_0 + A\sin\omega_1 t = c_0 + A\frac{e^{j\omega_1 t} - e^{-j\omega_1 t}}{2j} = \\
&= \frac{A}{2j} e^{j\omega_1 t} + c_0 + \frac{A}{-2j} e^{-j\omega_1 t} = c_1 e^{j\omega_1} + c_0 + c_{-1} e^{-j\omega_1 t}
\end{aligned} \tag{30}$$

and nonlinearity we set by a polynomial function (9).

It is known that

$$W(j\omega) = W_1(s)\big|_{s=j\omega} = \frac{K_1}{1 + j\omega T_1} \tag{31}$$

Substituting an expression (25) in equation (19), we obtain the system of equations

$$\begin{cases}
\nu_3 c_{-1}^3 \left(\frac{K_1}{1 - j\omega_1 T_1} \right)^3 = q_{-3} \\
\nu_2 c_{-1}^2 \left(\frac{K_1}{1 - j\omega_1 T_1} \right)^2 + 3\nu_3 c_{-1}^2 c_0 \left(\frac{K_1}{1 - j\omega_1 T_1} \right)^2 K_1 = q_{-2} \\
\nu_1 c_{-1} \left(\frac{K_1}{1 - j\omega_1 T_1} \right) + 2\nu_2 c_{-1} c_0 \left(\frac{K_1}{1 - j\omega_1 T_1} \right) K_1 + \\
+3\nu_3 c_{-1}^2 c_1 \left(\frac{K_1}{1 - j\omega_1 T_1} \right)^2 \left(\frac{K_1}{1 + j\omega_1 T_1} \right) + \\
+3\nu_3 c_{-1} c_0^2 \left(\frac{K_1}{1 - j\omega_1 T_1} \right) K_1^2 = q_{-1} \\
\nu_1 c_0 K_1 + 2\nu_2 c_{-1} c_1 \left(\frac{K_1}{1 - j\omega_1 T_1} \right) \left(\frac{K_1}{1 + j\omega_1 T_1} \right) + \nu_2 c_0^2 K_1^2 + \\
+6\nu_3 c_{-1} c_0 c_1 \left(\frac{K_1}{1 - j\omega_1 T_1} \right) K_1 \left(\frac{K_1}{1 + j\omega_1 T_1} \right) + \nu_3 c_0^3 K_1^3 = q_0 \\
\nu_1 c_1 \left(\frac{K_1}{1 + j\omega_1 T_1} \right) + 2\nu_2 c_0 c_1 K_1 \left(\frac{K_1}{1 + j\omega_1 T_1} \right) + \\
+3\nu_3 c_{-1} c_1^2 \left(\frac{K_1}{1 - j\omega_1 T_1} \right) \left(\frac{K_1}{1 + j\omega_1 T_1} \right)^2 + \\
+3\nu_3 c_0^2 c_1 K_1^2 \left(\frac{K_1}{1 + j\omega_1 T_1} \right) = q_1 \\
\nu_2 c_1^2 \left(\frac{K_1}{1 + j\omega_1 T_1} \right)^2 + 3\nu_3 c_0 c_1^2 K_1 \left(\frac{K_1}{1 + j\omega_1 T_1} \right)^2 = q_2 \\
\nu_3 c_1^3 \left(\frac{K_1}{1 + j\omega_1 T_1} \right)^3 = q_3
\end{cases} \tag{32}$$

So, there are only 5 indeterminates and there are 7 equations in the system (32). To determine these 5 indeterminates, it is sufficient to take any 5 equations from the system (26) and create a system of five equations with five indeterminates, solving which by using a standard soft MATLAB allows to obtain the numerical values of all parameters of nonlinear dynamical system, and the two equations from the system (26) which will remain not used, may serve as criteria of the correctness for solving the problem of identification.

4. Conclusions

There had been developed a method for synthesis of mathematical models of nonlinear dynamic systems with non-linear characteristics in the kind of polynomials and models of the inertial part in the form of Bode plot, based on the algorithm of transferred of multiple Volterra integrals, set in the time domain, to one-fold integrals, for the solution of which there shall be used the Bode plot of the inertial part of these systems.

There had been done the generalization of the suggested class of the mathematical models into the nonlinear dynamical system with a random order of both, their nonlinear characteristics and the characteristics of their inertial properties.

On the example of nonlinear dynamic systems with the third-order of nonlinear characteristics and the 1st and 2nd orders of the inertial part of these systems, there had been specified an algorithm for their parametric identification.

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