http://doi.org/10.35784/iapgos.3507
received: 18.02.2023 | revised: 15.03.2023 | accepted: 25.03.2023 | available online: 31.03.2023

MODIFICATIONS OF EVANS PRICE EQUILIBRIUM MODEL

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Abstract. *The paper regards the classical Evans price equilibrium model in the free product market in the aspect of regarding the opportunities for expanding (modifying) the model given that is aimed at perfecting the accuracy of its mathematical formulating. As an accuracy criterion, we have chosen a summary quadratic deviation of the calculated indices from the given ones. One of the approaches of modifying the basic Evans model is suggesting there is a linear dependence between price function and time as well as its first and second derivatives. In this case, the model will be described through differential equation of second order with constant coefficients, revealing some oscillatory process. Besides, it is worth regarding a non-linear (polynomial) dependence between demand, supply and price. The paper proposes mathematical formulating for the modified Evans models that have been approbated for real indices of exchange rates fluctuations. It also proves that increase of the differential and/or polynomial order of the given model allows its essential accuracy perfection. Besides, the influence of arbitrary restricting circumstances of the model on its accuracy is regarded. Each expanded Evans model is accompanied by mathematically formulated price and time dependence.*

Keywords: Evans model, market equilibrium, equilibrium price, differential equations, market modelling

MODYFIKACJE MODELU RÓWNOWAGI CENOWEJ EVANSA

Streszczenie. *W artykule rozpatrzono klasyczny model równowagi cenowej Evansa na wolnym rynku produktów w aspekcie możliwości rozbudowy (modyfikacji) danego modelu, zmierzającej do udoskonalenia dokładności jego matematycznego sformułowania. Jako kryterium dokładności wybraliśmy sumaryczne, kwadratowe odchylenie obliczonych indeksów od zadanych. Jednym z podejść do modyfikacji podstawowego modelu Evansa jest sugerowanie istnienia liniowej zależności między funkcją ceny a czasem oraz jej pierwszą i drugą pochodną. W takim przypadku model będzie opisany równaniem różnicowym drugiego rzędu o stałych współczynnikach, ujawniającym pewien proces oscylacyjny. Ponadto warto zwrócić uwagę na nieliniową (wielomianową) zależność pomiędzy popytem, podażą i ceną. W artykule zaproponowano matematyczne sformułowania dla zmodyfikowanych modeli Evansa, które zostały zaaprobowane dla rzeczywistych indeksów wahań kursów walutowych. Udowodniono, że zwiększenie rzędu różniczkowego i/lub wielomianowego danego modelu pozwala na jego zasadniczą poprawę dokładności. Ponadto rozważany jest wpływ dowolnych okoliczności ograniczających model na jego dokładność. Każdemu rozszerzonemu modelowi Evansa towarzyszy matematycznie sformułowana zależność cenowa i czasowa.*

Słowa kluczowe: model Evansa, równowaga rynkowa, cena równowagi, równania różnicowe, modelowanie rynku

Introduction

Investigating market balance is quite important both for economy and other modern science and technology studies, mathematics, modeling etc. The sense of market balance is in establishing certain market conjuncture, that is, certain steady demand and supply interrelation, or market “scissors”. Perceiving ways of establishing market balance enables realizing economy planning and forecasting tasks. These determine actuality of the investigation.

As there exists certain functional dependence of supply (descending function) and demand (ascending function) on the produce price, the cross point of curves for demand and supply corresponds to the balance point, the produce price in this point balancing supply and demand. The process of modeling market balance, or establishing equilibrium price, formulates the object of the investigation.

The Evans model [2] is one of the models for establishing equilibrium price. The model is described through differential equation of the first order with constant coefficients. The model admits that primary price for a product is known, and price function proper is related to time. Therefore, the solution of the problem in its present shape results in price change related to time.

Besides, the classical Evans model presumes linear relation of supply and demand to product price. Evidently, the presumption is quite simplified and the model itself requires additional investigation. For that reason, the Evans model for establishing equilibrium price is the subject of investigation. Eventually, the objective is to regard possibilities of expanding the Evans model for improving its accuracy.

The Evans model introduced nearly 100 years ago is now being scientifically criticized [5] for reason of staying efficient in a long perspective though it is not exact enough to describe the process of establishing equilibrium price for a short one. That is another ground to claim it should be additionally investigated and expanded (modified).

The market equilibrium problem stays quite a debate arising issue, numerous scientific approaches and methods being applied to its solution in as numerous works. For instance, the problem is regarded in [10] in which the equilibrium is investigated through labor market.

The issue of differentiating the produce in accordance with equilibrium prices in the manufacture (the model for the simultaneous search with partial depth) is given in [9]. In [4] the stock exchange activity and equilibrium price in the shares market are introduced. Functioning and unique features of the equilibrium price described for transportation networks with perfect competition are regarded in [7], and, under oligopoly, in [1]. Thus, the problem of finding the solution being as well unique promotes another aspect for investigating market balance models.

In [6] market balance model is given as a quasi-variation inequality that is solved through iteration method. The model regards convergence of the model only in a certain point of the neighborhood for the current market state, that is, not solving the whole of the supposed set. Here authors once again deal with a really sophisticated issue of convergence for balance models, and eventually, of presenting the equilibrium price.

Investigating market balance issues, methods of game theory are applied, one of those approaches described in [8]. The latter is employed for complex games, and implies working out an enlarged market abstraction as well as search for the balance in this abstraction, all that resulting in return to the initial market. However, in this investigation framework, to regard the Evans equilibrium price model, the game methods are not to be employed.

The market equilibrium-based approach is also applied to investigate Internet of things: therefore, in [3] paradigm of peripheral calculations is further evolved. Besides, this paper, together with many above mentioned, deals with important restrictions made for the necessary solutions. Indeed, it is worth regarding the related model in connection with corresponding ready-made restrictions for those real processes and phenomena described by the model.

Market balance is also regarded through methods of mathematic programing and operation exploring. Therefore, the analogue for market balance could be considered the balance of optimization criteria that is mathematically described as superposition of objective functions. The example of that approach is given in [11] exploring economic activity for the subject of transportations market.

All above mentioned approaches seem quite promising for further exploration of market balance. However, they would hardly be employed to modify the Evans model for establishing equilibrium price and improving the model in its accuracy and relation to the real data of a certain market conjuncture. Therefore, this paper, to explore the model given, most frequently applies methods of differential equations and those of optimization. To make calculations and draw diagrams of the given functional correlations, appropriate program instruments will be applied.

1. Problem statement

The classical Evans model admits that demand $d = d(p) = d[p(t)]$ and supply $s = s(p) = s[p(t)]$ are linear functions for price $p = p(t)$. The main supposition of the given model implies that rate of price change that is its first derivative in time, is equal to the product of the difference between supply and demand multiplied by a coefficient $\gamma \in R$:

$$\frac{dp}{dt} = \gamma \cdot [d(p) - s(p)], p(0) = p_0 \quad (1)$$

where p_0 – is the initial price value for the moment of time $t = 0$.

Obviously, from the application point of view, supply and demand are more dependent on price. Although, whatever functions could be, describing similar dependencies, they can be presented as polynomial and of arbitrary order through polynomial approximation, for instance, through the Taylor series expansion.

Therefore, the following suggestion could be made:

$$d = D_m(p), s = S_n(p) \quad (2)$$

where $D_m(p)$, $S_n(p)$ are polynomials of m and n order correspondingly, $m, n \epsilon N$.

Thus, the modified Evans model is obtained for establishing equilibrium price (1)-(2), which will be called polynomial.

Besides, it is easily demonstrated that classical Evans model is presented as a differential equation of the first order of the form:

$$b \cdot \frac{dp}{dt} + c \cdot p = d \quad (3)$$

where $b, c, d \in R$.

That is, according to (3), the left part of the equation is a linear combination of price function $p = p(t)$ and its first derivative.

Another approach to this model modification as (3) is a suggestion that there exists linear dependence between price function $p = p(t)$, and both its first and second derivatives in time:

$$a \cdot \frac{d^2p}{dt^2} + b \cdot \frac{dp}{dt} + c \cdot p = d \quad (4)$$

where $a \in R$.

The Evans model as (4) is further called the Evans differential model.

Basically, the right part of the equation (4) is a polynomial of *0* order. Supposing, the right part (4) contains a polynomial of derivative t with a degree higher than 0 obtain the Evans polynomial differential model, its mathematical formula given below.

Explore the above described models in more detail.

2. The Evans classical and differential models

The Evans classical model for establishing equilibrium price is quite much explored. Its solution as (3) is obtained as follows:

$$p(t) = U(t) \cdot V(T)$$

$$b \cdot U \cdot \frac{dV}{dt} + b \cdot V \cdot \frac{dU}{dt} + cUV = d$$

$$U \cdot \left(b \cdot \frac{dV}{dt} + c \cdot V\right) + b \cdot V \cdot \frac{dU}{dt} = d$$

$$b \cdot \frac{dV}{dt} + c \cdot V = 0$$

$$\frac{dV}{V} = -\frac{c \cdot dt}{b}$$

$$V = \exp\left(-\frac{c \cdot t}{b}\right)$$

$$\frac{dU}{dt} = \frac{d}{b} \cdot \exp\left(\frac{c \cdot t}{b}\right)$$

$$dU = \frac{d}{c} \cdot \exp\left(\frac{c \cdot t}{b}\right) d\left(\frac{c \cdot t}{b}\right)$$

$$U = \frac{d}{c} \cdot \exp\left(\frac{c \cdot t}{b}\right) + c_0, c_0 = const$$

$$p(t) = \left(\frac{d}{c} \cdot \exp\left(\frac{c \cdot t}{b}\right) + c_0\right) \cdot \exp\left(-\frac{c \cdot t}{b}\right) = $$

$$= \frac{d}{c} + c_0 \cdot \exp\left(-\frac{c \cdot t}{b}\right)$$

As $p(0) = p_0$, then:

$$\frac{d}{c} + c_0 = p_0$$

$$c_0 = p_0 - \frac{d}{c}$$

Eventually:

$$p(t) = \frac{d}{c} + \left(p_0 - \frac{d}{c}\right) \cdot \exp\left(-\frac{c \cdot t}{b}\right) \quad (5)$$

The Evans differential model as (4) is a non-homogeneous differential equation of second order, with constant factors. In such a statement and with the model's economic interpretation, the model given is analogous to the model of damped oscillations, therefore the discriminant of the characteristic equation (6) in the related homogenous differential equation should be negative:

$$a \cdot k^2 + b \cdot k + c \cdot p = 0 \quad (6)$$

$$D = b^2 - 4ac < 0$$

$$b \in (-2\sqrt{ac}; 2\sqrt{ac})$$

In this case, the roots of the characteristic equation:

$$k_{1,2} = \frac{-b \pm i\sqrt{-D}}{2a}, i = \sqrt{-1}$$

Correspondingly, real *Re*(*k*) and imaginary *Im*(*k*) parts of these roots:

$$Re(k) = \frac{-b}{2a}$$

$$Im(k) = \frac{\sqrt{-D}}{2a}$$

Then the solution of equation (4) will be:

$$p(t) = (c_1 \cdot cos(Im(k) \cdot t) + c_2 \cdot sin(Im(k) \cdot t)) \times$$
$$\times \exp(Re(k) \cdot t) - d/c \quad (7)$$

where $c_1, c_2 = const$.

Evidently, (7) proves, initial assumption $p(0) = p_0$ will not be sufficient for determining constants c_1, c_2. Therefore, additional initial statement is needed. The latter, for instance, could be the rate of price change (first derivative of $p(t)$ function for time t), or it could be price function proper in a certain moment of time t.

Further presume that factual values, both initial and final, of observations of price values are known:

$$p(0) = p_0; p(t_n) = p_n \quad (7a)$$

where n is quantity of observations, $t_n \neq 0$ is an n moment of time t.

Put (7a) in expression (7):

$$\begin{cases} p(0) = (c_1 \cdot cos0 + c_2 \cdot sin0) \cdot 1 - \frac{d}{c} = p_0, \\ p(t_n) = (c_1 \cdot cos(Im(k) \cdot t_n) + c_2 \cdot sin(Im(k) \cdot t_n)) \times \\ \times \exp(Re(k) \cdot t_n) - \frac{d}{c} = p_n. \end{cases}$$

$$\begin{cases} c_1 = p_0 + d/c, \\ (c_1 \cdot cos(Im(k) \cdot t_n) + c_2 \cdot sin(Im(k) \cdot t_n)) \cdot \exp(Re(k) \cdot t_n) = \\ = p_n + d/c. \end{cases}$$

$$\begin{cases} c_1 = p_0 + d/c, \\ c_2 \cdot sin(Im(k) \cdot t_n) = \frac{p_n + d/c}{\exp(Re(k) \cdot t_n)} - c_1 \cdot cos(Im(k) \cdot t_n). \end{cases}$$

$$\begin{cases} c_1 = p_0 + d/c, \\ c_2 = \frac{(p_n + d/c) \cdot \exp(-Re(k) \cdot t_n) - (p_0 + d/c) \cdot cos(Im(k) \cdot t_n)}{sin(Im(k) \cdot t_n)}. \end{cases}$$

Substituting c_1, c_2 into (7), obtain ultimate Evans differential model.

3. The Evans polynomial classical model

Similarly, to the main statement (1) of the Evans classical model, put it as follows:

$$\frac{dp}{dt} = \gamma \cdot [D_m(p) - S_n(p)], p(0) = p_0 \quad (8)$$

The right part of expression (8) is a polynomial of order m. For the expression, admit that $m = n$ and put it as:

$$T_n(p) = D_n(p) - S_n(p) \quad (9)$$

where $T_n(p)$ is a polynomial of order n.

It means that the Evans model put as (8)-(9) is the Evans polynomial classical model of order n.

Considering (8) and (9), put:

$$\frac{dp}{dt} = \gamma \cdot T_n(p)$$

or

$$\frac{dp}{T_n(p)} = \gamma \cdot dt \quad (10)$$

Suppose, $p_i, i = \overline{1,n}$ are roots (they generally differ) of polynomial $T_n(p)$. Then, the equation is like:

$$T_n(p) = \alpha \cdot \prod_{i=1}^{n}(p - p_i) \quad (11)$$

where α is higher polynomial coefficient $T_n(p)$.

Substitute (11) into expression (10):

$$\frac{dp}{\prod_{i=1}^{n}(p-p_i)} = \alpha\gamma \cdot dt \quad (12)$$

Apply the method of undetermined coefficients $A_i \in R$ to the left part of expression (12)

$$\frac{dp}{\prod_{i=1}^{n}(p-p_i)} = \sum_{i=1}^{n} \frac{A_i \cdot dp}{p-p_i} = \alpha\gamma \cdot dt \quad (13)$$

To better demonstrate the given model as (13), put $n = 2$. Obtain:

$$\frac{1}{(p-p_1)(p-p_2)} dp = \left(\frac{A_1}{p-p_1} + \frac{A_2}{p-p_2}\right) dp = \alpha\gamma \cdot dt \quad (14)$$

In (14) apply necessary transformations:

$$\frac{A_1}{p-p_1} + \frac{A_2}{p-p_2} = \frac{A_1 p - A_1 p_2 + A_2 p - A_2 p_1}{(p-p_1)(p-p_2)} =$$

$$= \frac{(A_1+A_2)\cdot p - (A_1 p_2 + A_2 p_1)}{(p-p_1)(p-p_2)} = \frac{1}{(p-p_1)(p-p_2)}.$$

Whereas:

$$\begin{cases} A_1 + A_2 = 0 \\ A_1 p_2 + A_2 p_1 = -1 \end{cases}$$

By the Cramer method:

$$\Delta = \begin{vmatrix} 1 & 1 \\ p_2 & p_1 \end{vmatrix} = p_1 - p_2$$

$$\Delta_1 = \begin{vmatrix} 0 & 1 \\ -1 & p_1 \end{vmatrix} = 1$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 \\ p_2 & -1 \end{vmatrix} = -1$$

$$A_1 = \frac{\Delta_1}{\Delta} = \frac{1}{p_1 - p_2}$$

$$A_2 = \frac{\Delta_2}{\Delta} = \frac{-1}{p_1 - p_2}$$

Therefore, (14) is put as:

$$\left(\frac{1}{p-p_1} - \frac{1}{p-p_2}\right) dp = \alpha\gamma \cdot (p_1 - p_2) \cdot dt$$

$$\frac{d(p-p_1)}{p-p_1} - \frac{d(p-p_2)}{p-p_2} = \alpha\gamma \cdot (p_1 - p_2) \cdot dt$$

$$\ln|p - p_1| - \ln|p - p_2| = \alpha\gamma \cdot (p_1 - p_2) \cdot t + $$
$$+ \ln(\bar{c}), \bar{c} > 0$$

$$\left|\frac{p-p_1}{p-p_2}\right| = \bar{c} \cdot e^{\alpha\gamma\cdot(p_1-p_2)\cdot t}$$

As $p(0) = p_0$, then

$$\bar{c} = \left|\frac{p_0 - p_1}{p_0 - p_2}\right|$$

Let be $p_1 < p_2$.

Then with $p \in R \backslash [p_1; p_2]$, it comes to:

$$p - p_1 = \bar{c} \cdot e^{\alpha\gamma\cdot(p_1-p_2)\cdot t} \cdot (p - p_2)$$

$$p \cdot \left(1 - \bar{c} \cdot e^{\alpha\gamma\cdot(p_1-p_2)\cdot t}\right) = p_1 - \bar{c} \cdot e^{\alpha\gamma\cdot(p_1-p_2)\cdot t} \cdot p_2$$

$$p(t) = \frac{p_1 - \bar{c}\cdot e^{\alpha\gamma\cdot(p_1-p_2)\cdot t}\cdot p_2}{1 - \bar{c}\cdot e^{\alpha\gamma\cdot(p_1-p_2)\cdot t}} \quad (15)$$

And with $p \in (p_1; p_2)$:

$$p - p_1 = -\bar{c} \cdot e^{\alpha\gamma\cdot(p_1-p_2)\cdot t} \cdot (p - p_2)$$

$$p \cdot \left(1 + \bar{c} \cdot e^{\alpha\gamma\cdot(p_1-p_2)\cdot t}\right) = p_1 + \bar{c} \cdot e^{\alpha\gamma\cdot(p_1-p_2)\cdot t} \cdot p_2$$

$$p(t) = \frac{p_1 + \bar{c}\cdot e^{\alpha\gamma\cdot(p_1-p_2)\cdot t}\cdot p_2}{1 + \bar{c}\cdot e^{\alpha\gamma\cdot(p_1-p_2)\cdot t}} \quad (16)$$

Thus, the Evans polynomial classical model is obtained as (15)-(16).

The Evans polynomial differential model is generally put as:

$$a \cdot \frac{d^2p}{dt^2} + b \cdot \frac{dp}{dt} + c \cdot p = Q_w(t), \quad (17)$$

where $Q_w(t)$ is polynomial of order $w > 0$.

With characteristic equation (6), equation (17) is solved:

$$p(t) = (c_1 \cdot cos(Im(k) \cdot t) + c_2 \cdot sin(Im(k) \cdot t)) \times$$
$$\times \exp(Re(k) \cdot t) + \bar{Q}_w(t), \quad (18)$$

where $c_1, c_2 = const$; $\bar{Q}_w(t)$ is polynomial of order $w > 0$.

To calculate values of coefficients for polynomial $\bar{Q}_w(p)$, substitute it into expression (17) and apply the method of undetermined coefficients. Thus, obtain the Evans polynomial differential model as (17)-(18).

Compare the accuracy of the obtained Evans models.

4. Comparing the accuracy of different Evans models with the condition of known parameters

Reveal the notion of the condition of known parameters in the examples (3) and (4). Suppose, parameters *b*, *c*, *d* of these both models are known, or obtained from the previous calculations, whereas parameter *a* of model (4) is unknown. The latter will be named hyperparameter, and changing its value manage the comparative accuracy of model (4) related to model (3).

For practical demonstration of comparing the accuracy of the Evans model for establishing equilibrium price, apply the indices for currency rate of Ukrainian hryvna (UAH) in relation to the US dollar, according to the National Bank of Ukraine (NBU) (table 1).

Table 1. Currency exchange rates for the pair UAH-USD (NBU data for the first half of 2021) [12]

Days	Months					
	January	February	March	April	May	June
1	28.2746	28.1324	27.9456	27.8226	27.733	27.4674
2	28.431	28.0603	28.0007	27.8324	27.7339	27.4381
3	28.4028	28.0589	27.933	27.9555	27.7205	27.3449
4	28.2847	27.995	27.8477	27.939	27.7641	27.34
5	28.2038	27.8885	27.7564	27.8384	27.6744	27.2914
6	28.046	27.7711	27.7091	27.8923	27.6318	27.1923
7	27.9705	27.6651	27.7431	27.9768	27.6273	27.1764
8	28.0609	27.6426	27.7016	27.9094	27.6142	27.0906
9	28.0524	27.7665	27.7486	27.9335	27.555	27.1068
10	28.0524	27.8384	27.7305	28.0156	27.4368	27.0404
11	28.1926	27.844	27.6434	27.9765	27.4166	26.9957
12	28.1544	27.9671	27.6525	27.9592	27.4665	26.9258
13	28.2035	27.8304	27.6978	27.9783	27.4572	27.0275
14	28.2561	27.9038	27.6828	28.0087	27.4281	27.1712
15	28.1648	27.8461	27.7184	28.0096	27.4553	27.1935
16	28.1665	27.8468	27.6871	28.0576	27.5461	27.305
17	28.1524	27.9304	27.7295	28.0642	27.526	27.2737
18	28.1652	27.8976	27.8706	27.9014	27.5004	27.4589
19	28.1929	27.9492	27.9698	27.8558		27.3964
20		27.9301	27.9698	27.7715		27.1763
21			27.9679	27.7867		
22			27.9694	27.75		
23			27.8852			

First, apply classical Evans model (3). Through least squares method (LSM) calculate values of parameters b, c, d for this model and draw diagrams of factual and calculated values (Fig. 1).

In the given case $b = 1.1296$, $c = 0.0596$, $d = 1.6508$, and model (3) itself looks as:

$$1.1296 \cdot \frac{dp}{dt} + 0.0596 \cdot p = 1.6508.$$

Its aggregate quadratic deviation is 8.4789.

Then model (4) for same data and according to condition of known parameters will be as:

$$a \cdot \frac{d^2p}{dt^2} + 1.1296 \cdot \frac{dp}{dt} + 0.0596 \cdot p = 1.6508,$$

where $a \in R$ is hyperparametre.

To calculate a, apply LSM to the appropriate model (4):

$$68119.62351 \cdot \frac{d^2p}{dt^2} + 1.1296 \cdot \frac{dp}{dt} + 0.0596 \cdot p = 1.6508$$

that is, $a = 68119.62351$.

In this case, aggregate quadratic deviation will be 4.0413. The diagrams of factual and calculated values of the given model are given in Fig. 2.

Therefore, the index of aggregate quadratic deviation, with known parameters, for differential Evans model (4) is smaller than the corresponding index for the classical model of establishing equilibrium value (3). That demonstrates better accuracy of model (4) related to model (3).

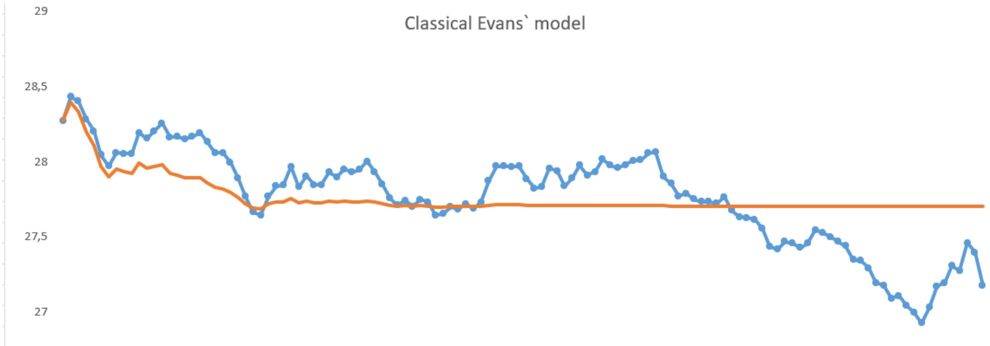

Fig. 1. Factual and calculated values for classical Evans model

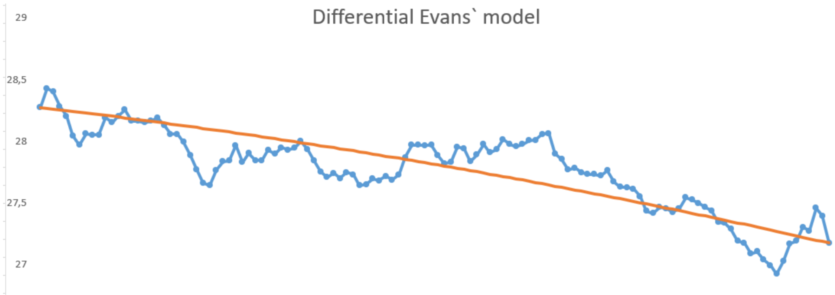

Fig. 2. Factual and calculated values for differential Evans model with known parameters

5. Comparing the accuracy of different Evans models with unknown parameters

Condition of unknown parameters means that parameters a, b, c, d of both models (3) and (4) are unknown and they basically all are hyperparameters. In this case, each of these models is built separately, then to compare their accuracy.

For model (3) in this case nothing matters and previously obtained results stay unchanged. On the contrary, model (4) now is built according to similar principle, that of model (3).

To calculate values a, b, c, d apply LSM to the related model. Obtain model (4) as:

$$79.35184 \cdot \frac{d^2p}{dt^2} - 0.7389 \cdot \frac{dp}{dt} + 0.5405 \cdot p = -15.0491$$

Diagrams for factual and calculated values for the given model are in Fig. 3.

However, in this case aggregate quadratic deviation will be 14.3321. That is, the accuracy of model (4) with unknown hyperparameters is lower than the accuracy of both classical and differential Evans models with known parameters. That implies the need for improving the model.

Turn to differential Evans model as (7). Now, assume that all its parameters are hyperparameters, without any numerical restrictions, either limiting, for the initial and final values. Apply LSM to the given model. Obtain:

$$p(t) = (27.2783 \cdot \cos(0.00087 \cdot t) + \\ +0.000001 \cdot \sin(0{,}00087 \cdot t)) \cdot \exp(-0{,}000242 \cdot t) + \\ +26.2753/28.275$$

Diagrams of factual and calculated values of the given model are in Fig. 4.

In the given case, aggregate quadratic deviation will be only 3.8334, that is the best value among all mentioned models. That is the ground to assume that differential Evans model is most accurate among all similar models of establishing equilibrium price, with unknown parameters and without limiting restrictions.

Regard polynomial Evans model as (15)-(16). As roots p_1, p_2 basically depend on the accuracy of creating supply and demand functions, they are not to be included into hyperparameters of the model. Instead, put p_1, p_2 as equal to minimal and maximum values for selected factual price indices. In this case, condition $p \in (p_1; p_2)$ will be realized, so polynomial Evans model will be as (16).

Besides, with p_0, p_1, p_2, it is possible to state $\bar{c}$. Therefore, hyperparameters of the model will only be α, γ. Calculate them applying LSM.

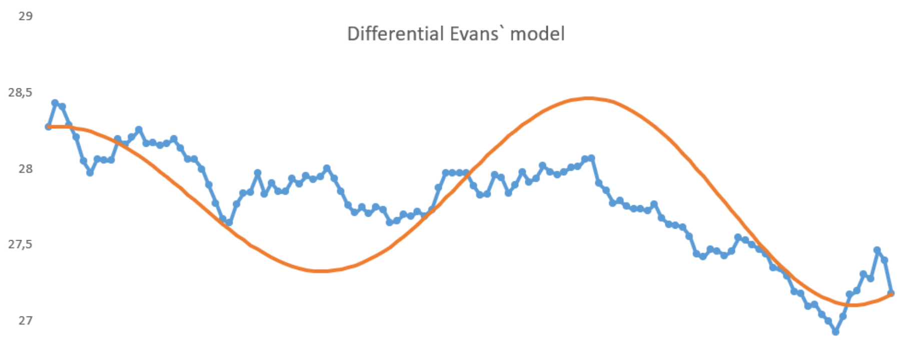

Fig. 3. Factual and calculated values of differential Evans model with unknown parameters

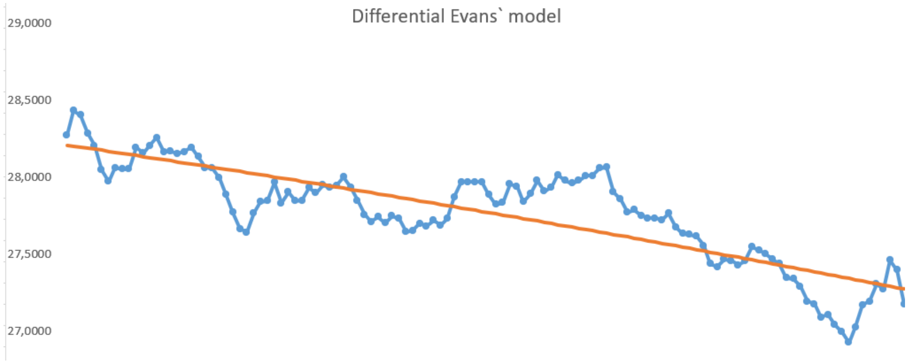

Fig. 4. Factual and calculated values for differential Evans model with unknown parameters and without limiting restrictions

Obtain $\alpha = 0.09675, \gamma = 0.19352$. The Evans model will be as follows:

$$p(t) = \frac{26.9258 + 8.624 \cdot e^{0.09675 \cdot 0.19352 \cdot 1.5052 \cdot t} \cdot 28.431}{1 + 8.624 \cdot e^{0.09675 \cdot 0.19352 \cdot 1.5052 \cdot t}}$$

Aggregate quadratic deviation in the given case will be 4.59736. Diagram of the model is given in Fig. 5.

During approbation of polynomial differential Evans model (17)-(18) its parameters will be presented with $a, b, c_1, c_2, Im(k)$, and also the coefficients of polynomial $\bar{Q}_w(t)$ admitting that $w = 1$.

Minimizing aggregated quadratic deviation, obtain hyperparameters:

$$a = 0.56;\ b = 1.62;\ c_1 = 0.334, c_2 = 1.583, Im(k) \approx 0$$

Besides:

$$\bar{Q}_w(t) = -0.0073 \cdot t + 28.2$$

Also, $Re(k) = -1.441$, aggregated quadratic deviation of the model is 3.99841 – graphically – Fig. 6.

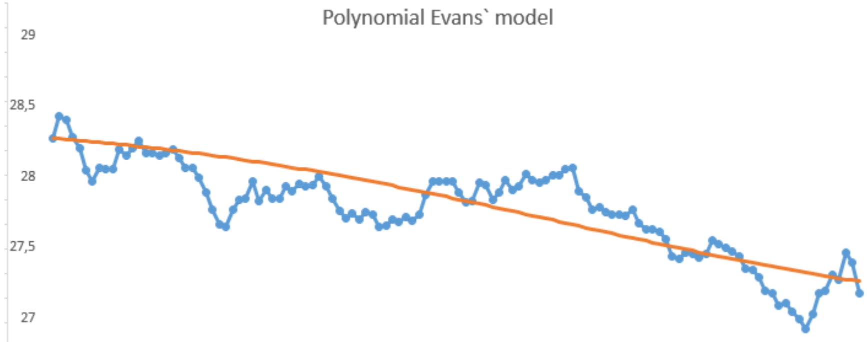

Fig. 5. Factual and calculated values of polynomial classical Evans model

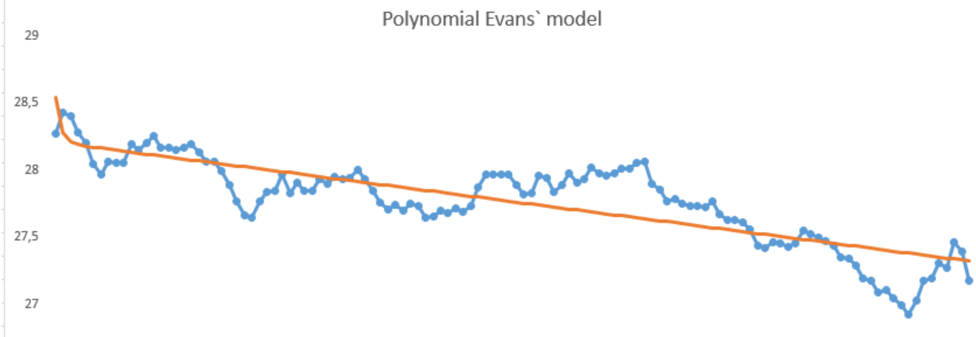

Fig. 6. Factual and calculated values of polynomial differential Evans model

6. Discussion

Make accuracy gradation, 1 standing for the most accurate model etc., for all models, in table 2.

Obviously, those models which are laid minimum of restrictions, are more accurate as to those that assume restrictions. However, it does not at all mean that, with creating this or that Evans model, the restrictions could be utterly rejected. Analyzing from economic or mathematical view, certain additional conditions related to the model can be either needed or desired. Therefore, while rejecting those conditions, the model can become one that vaguely describes real process or phenomenon it is expected to reproduce. In this case, the accuracy of the model is seen as an arbitrary criterion, as the model itself basically loses its primary purpose.

Table 2. Accuracy gradation of Evans models

Name of the model	Aggregate quadratic deviation	Gradation number
Classical	8.4789	5
Differential with known parameters	4.0413	3
Differential with unknown parameters and limiting restrictions	14.3321	6
Differential with unknown parameters and without limiting restrictions	3.8334	1
Polynomial classical	4.59736	4
Polynomial differential	3.99841	2

Therefore, the conclusion is that constructing Evans models of establishing equilibrium price it is worth looking for a balance between the accuracy of the model and those restrictions laid upon it.

The paper generally applies 2 main approaches, differential and polynomial, to modify classical Evans model. Evidently, other new or combined similar approaches can be proposed, together with criteria of the model's accuracy and perfection.

In addition, obtained solutions for each model are unique. It is evident that though the state of market equilibrium is theoretically unique numerical methods to calculate hyperparameters cannot be applied uniquely. It means each case produces more than one set of hyperparameters of the model. Basically, the fewer parameters the model possesses, the fewer sets of its values can be obtained.

7. Conclusions

The problem of market balance and modeling the establishing of equilibrium price at the market of products is a subject of scientific interest for scholar studies and applied specialties in many branches. The problem is regarded by both mathematics and economists, as well as by financial experts, investigators in modeling, computing and informational technology. There are quite many market balance models and also methods and ways of their description. In these studies, both scientists and practical workers besides applying popular approaches most regard essential restrictions of the models, and those are related to their applied character.

This study analyzes possible ways of modifying one of classical market balance models that is the Evans model. That one was essentially expanded and principal models were created to establish equilibrium price, those similar to the Evans model: differential, polynomial and differential polynomial. To this effect, the investigation applied method of increasing order for those differential equations relating to a model. In addition, the paper regards the problem analyzes the restrictions laid upon the model. Therefore, the created models were made mostly accurate according to the criterion of minimized aggregate quadratic deviation.

Obtained results allow admitting the highly promising approaches applied in the paper. New Evans models of increased orders were exposed to demonstrate higher accuracy in establishing market equilibrium price. Further studies are supposed to concentrate on creating more accurate functional relations stated between supply and demand, and product price. That will help in specifying the statement of the related Evans model. Another way to create specified models of equilibrium price is their approbation for real data, hitherto there exists the ability to calculate the related values of hyperparameters of the model.

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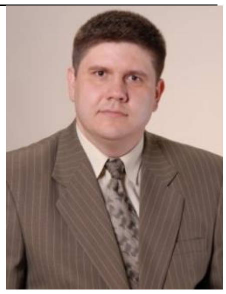

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