http://doi.org/10.35784/iapgos.5533

received: 31.10.2023 | revised: 08.03.2024 | accepted: 18.03.2024 | available online: 31.03.2024

ELECTROMAGNETIC FIELD EQUATIONS IN NONLINEAR ENVIRONMENT

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Abstract. *The paper proposes electromagnetic field equations from the point of view of their adaptation to numerical methods. Maxwell's equations with partial derivatives are used, written concerning field vectors, which most fully reproduce the picture of physical processes in electrical engineering devices. The values of these vectors provide comprehensive information about the field at any spatio-temporal point. The concept of creating mathematical models of electrical devices adequate to physical processes has been developed. Mathematical transformations are carried out according to the rules of differential calculus. Dynamic processes in the elements of electrotechnical devices were analyzed using the apparatus of mathematical modeling. An algorithm for implementing differential equations with partial derivative numerical methods using computer simulation was implemented. The obtained results made it possible to understand the nature of electromagnetic phenomena in nonlinear media. The paper provides calculations of the field parameters in a flat ferromagnetic plate and the groove of the rotor of an electric machine.*

Keywords: electromagnetic field differential equations, mathematical model, numerical integration

RÓWNANIA POLA ELEKTROMAGNETYCZNEGO W ŚRODOWISKU NIELINIOWYM

Streszczenie. *W artykule zaproponowano równania pola elektromagnetycznego pod kątem ich adaptacji do metod numerycznych. Wykorzystano równania Maxwella z pochodnymi cząstkowymi, zapisywane względem wektorów pola, które najpełniej odtwarzają obraz procesów fizycznych w urządzeniach elektrycznych. Wartości tych wektorów dostarczają wyczerpujących informacji o polu w dowolnym punkcie czasoprzestrzennym. Opracowano koncepcję tworzenia modeli matematycznych urządzeń elektrycznych adekwatnych do procesów fizycznych. Przekształcenia matematyczne przeprowadzono zgodnie z zasadami rachunku różniczkowego. Do analizy procesów dynamicznych zachodzących w elementach urządzeń elektrotechnicznych wykorzystano modelowanie matematyczne. Zaimplementowano algorytm realizacji równań różniczkowych metodami numerycznymi pochodnych cząstkowych z wykorzystaniem symulacji komputerowej. Uzyskane wyniki pozwoliły zrozumieć naturę zjawisk elektromagnetycznych w ośrodkach nieliniowych. W artykule przeprowadzono obliczenia parametrów pola w płaskiej płycie ferromagnetycznej oraz w rowku wirnika maszyny elektrycznej.*

Słowa kluczowe: równania różniczkowe pola elektromagnetycznego, model matematyczny, całkowanie numeryczne

Introduction

Today, scientists are paying more and more attention to the methods of nonlinear issues in engineering calculations. The need to calculate spatial electromagnetic fields arises when solving a wide range of problems in electrical engineering, electronics, and telecommunications [1, 2]. These include magnetohydrodynamic energy generators, astrophysical objects, electromagnetic pulse propagation, plasma accelerators, mobile communications, etc.

Knowledge of research methods and description of electromagnetic fields and waves will make it possible to build field mathematical models of electrical and electronic devices. The simulation itself is a powerful means of researching the above devices. Thus, the construction of a mathematical model based on the equations of the electromagnetic field of a real physical object is realized by a skillful combination of the laws of electrical engineering with differential equations [5, 8].

Depending on the conditions of the problem and the use of the mathematical apparatus (ordinary differential equations or equations with partial derivatives), mathematical models can be considered as a Cauchy problem or a mixed problem.

1. Literature review

The analysis of recent studies shows that it is possible to unify equations and models by focusing them on the use of powerful numerical methods, in particular, explicit integration. This is realized by abandoning the traditional methods of the theory of electric circuits in favor of electromagnetic circuit methods and electromagnetic field methods in their close combination [1, 4].

The effectiveness of mathematical modeling in the study of transient processes in electrical devices directly depends on methodological (methods of computational mathematics) and technical factors in the presence of high-performance computers [5, 9].

The paper proposes a theoretically grounded mathematical apparatus, focused on the construction of optimal computing algorithms. The real designs of electrotechnical devices are quite complex in terms of geometry, which is related to the optimal use of conductor, structural, and insulating materials. When building a computational algorithm, the differential equations in the mathematical model must be approximated by difference schemes based on algebraic equations [13].

An essential feature of our class of problems from a computational point of view is the sharp heterogeneity of electrophysical properties, which gives rise to nonlinearity. Thus, the electrical conductivity of the construction materials of electrical engineering devices can change along spatial coordinates with a jump from zero to finite values, and in some cases to infinity. The equations of electrodynamics form the basis of mathematical models for this class of problems [12].

In this work, a mathematical apparatus is used for modeling [3], which is based on the theory of nonlinear differential equations, the solution of which is possible with the correct application of numerical methods oriented to computer technology. As is known, electrotechnical materials are characterized by isotropic and anisotropic properties. Let's first consider the principles of forming equations for an anisotropic medium, that is, one whose physical properties depend on the direction. In a nonlinear anisotropic medium, the values of electrical conductivities, dielectric, and magnetic permillivity are functions of the electromagnetic state and are described by diagonal matrices.

At the beginning of its development, the research and analysis methods of electrical devices were developed as methods in the timeless domain. Electrical devices were treated as ideal devices with linearized electromagnetic couplings, which resulted in incorrect results. That is, the mathematical apparatus inadequately described the physical processes in these devices.

The maximum use of steel in the magnetic conductors of electrical equipment led to the fact that the electromagnetic connections in the nominal states differed significantly from the linear ones. At the same time, calculation methods were based on approximate consideration of nonlinearities, which did not meet the requirements of practice. The application of nonlinear differential equations significantly complicated their integration. However, only such an approach for the analysis of processes in electrotechnical devices would ensure high accuracy of the calculation.

During the analysis of electrical devices by electromagnetic field methods, it is necessary to integrate differential equations with ordinary and partial derivatives in a single time-space. For this, equations with partial derivatives must be discretized using finite difference or finite element methods [14].

As you know, the consideration of the skin effect in the grooves of electric machines is based on the theory of electric circuits. Here, a multi-loop substitute scheme was used, determining whose parameters are complicated due to the increase in the number of links. This increased the order of the differential equations and their stiffness, which led to a loss of accuracy of the results. Summarizing the review of the literature available to us, we can state that the most promising direction of analysis and research of electromagnetic processes in electrotechnical devices is the application of electromagnetic field theory methods based on nonlinear differential equations. These are time-domain methods. Only in this way is it possible to carry out an adequate analysis with the help of mathematical models that describe physical processes as accurately as possible.

2. Researches methodology

Maxwell's equations for such a case take the form [3]. The function rot is the rotor of a three-dimensional vector field the coordinates of which are determined by the determinant of the third order. The first row is the coordinates of the coordinate axes *x*, *y*, *z*, the second corresponds to partial differentiation operators, and the third corresponds to the coordinate of the vector field [11, 12].

$$rot\vec{H} = \vec{\delta}$$
$$rot\vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{1}$$

$$\vec{\delta} = G\vec{E} + \varepsilon' \frac{\partial \vec{E}}{\partial t} \tag{2}$$

where $\vec{H}$ is the magnetic field intensity vector, A/m; $\vec{\delta}$ – vector of current density, G – matrix of electric conductivities the diagonal elements of which are determined from equation (3); $\vec{D}$ – electric induction vector; $\vec{E}$ – vector of electric field intensity, ε' – matrix of differential electric permillivity electric induction vector $D = f(E)$.

$$G = \begin{array}{|c|c|c|} \hline \gamma_x & & \\ \hline & \gamma_y & \\ \hline & & \gamma_z \\ \hline \end{array} \tag{3}$$

The elements of the matrix G are determined by the characteristics of the conductor in the direction of the anisotropy *x*, *y*, *z* main axes by formula (4):

$$\gamma_i(E) = \delta_i / E, \quad i = x, y, z \tag{4}$$

We find the electric field strength from the concepts of trigonometry.

The advantage of writing equation (1) in vector form is that it does not depend on the choice of spatial coordinate system. However, the expressions of the rotor components differ in different coordinate systems.

According to figure 1, the sum of products of magnetic voltages *Hdl* on all sides of the contour *abcda* is: $+H_y dy$ along side *ab*, $\left(H_z + \frac{\partial H_z}{\partial y} dy\right) dz$ along *bc*, $-\left(H_y + \frac{\partial H_y}{\partial z} dz\right) dy$ along *cd*, $-H_z dz$ along *da*.

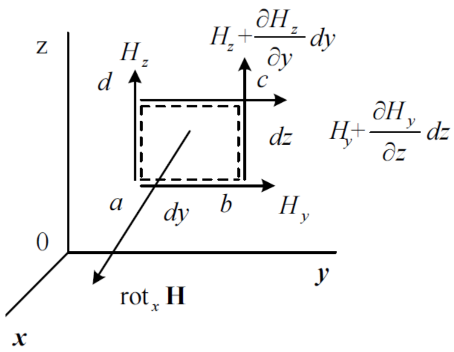

Fig. 1. To the explanation of the definition of rotH components in Cartesian coordinates

Regarding the *dydz* plane according to [3], we obtain the following equation (5):

$$\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\mathbf{x}_0 + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\mathbf{y}_0 + + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\mathbf{z}_0 = \delta_x \mathbf{x}_0 + \delta_y \mathbf{y}_0 + \delta_z \mathbf{z}_0 \tag{5}$$

where x_0, y_0, z_0 – orts.

The electric field strength is found from the concepts of trigonometry, which is shown by formula (6):

$$E = \sqrt{E_x^2 + E_y^2 + E_z^2} \tag{6}$$

The electric induction vector is determined from the characteristics of the dielectric according to equation (7):

$$\vec{D} = \varepsilon \vec{E} \tag{7}$$

where *ε* is the matrix of static electrical permillivity similar to expression (2) is determined from expression (8):

$$\varepsilon_i(E) = D_i / E, \quad i = x, y, z \tag{8}$$

The elements of the matrix ε' are determined by the characteristic of the dielectric $D = f(E)$.

Let's rewrite the magnetization characteristic $B=f(H)$ in another form [10] equation (9):

$$\vec{H} = N\vec{B} \tag{9}$$

where *N* is the diagonal matrix of inverse magnetic reluctance, defined similarly to (3).

The elements of the *N* matrix are determined from the magnetization curve in the directions of the main anisotropy axes.

Magnetic induction is found similarly to (5) and this is shown by formula (10):

$$B = \sqrt{B_x^2 + B_y^2 + B_z^2} \tag{10}$$

Then the magnetic reluctance are equal (11):

$$\nu_i(B) = H_i / B, \quad i = x, y, z \tag{11}$$

For software implementation of field calculation, it is necessary to approximate curves (4), (8), (11) and apply equations (1), (2). For this, it is necessary to know the initial conditions (the value of the field vectors at t=0) and the boundary conditions on the surface of the closed space throughout time.

Another case of nonlinearity is the electromagnetic field in a lattice ferromagnet. Such material is widely used for the construction of electrical devices as a magnetic conductor. Here we have a case of launching layers of ferromagnets and dielectrics. The calculation of the electromagnetic field in such an environment, taking into account the boundary conditions within homogeneous environments, is practically difficult to implement due to the large volume of the calculations. Therefore, in practical calculations, the flat-band environment is replaced by some anisotropic homogeneous environment, which is equivalent in electromagnetic terms to the first one.

In the transverse direction, we carry out the replacement, taking into account the serial connection of the ferromagnetic and non-magnetic gap magnetic resistances, and in the longitudinal direction – a parallel connection.

We apply the Cartesian coordinate system so that the *x*-axis passes across, and the *y*, *z* axis passes along the layers of the charged magnetic conductor.

Let's write the boundary conditions for the vectors $\vec{B}, \vec{H}$ at the interface of two homogeneous environments with the normal vector $\vec{n}_0$, respectively [12] equation (12), (13):

$$\left(\vec{B}_2 - \vec{B}_1\right)\cdot \vec{n}_0 = 0 \tag{12}$$

$$\vec{n}_0 \times \left(\vec{E}_2 - \vec{E}_1\right) = 0 \tag{13}$$

then

$$B_x = B_{fx} = B_{fd};\;\; H_y = H_{fy} = H_{dy};\;\; H_z = H_{fz} = H_{dz} \tag{14}$$

Here, the indices *f* and *d* indicate belonging to ferro- and diamagnets, and its absence – to an equivalent environment.

The equality of magnetic voltages in the transverse direction of the real and equivalent environment taking into account (14) can be expressed as dependence (15):

$$\left(\nu_f h_f + \nu_d h_d\right) B_x = \left(h_f + h_d\right) H_x \tag{15}$$

where ν are inverse magnetic reluctance, *h* are layer thicknesses.

From equation (15), we obtain the inverse static permillivity along the *x* axis. This is shown by equation (16):

$$\nu'_x = \xi \nu_f \tag{16}$$

where ξ is the anisotropy coefficient in the transverse direction.

$$\xi = \frac{h_f + \dfrac{\nu_d h_d}{\nu_f}}{h_f + h_d} \tag{17}$$

In the longitudinal direction, we obtain equation (18), which shows the equality of the magnetic fluxes:

$$\left(\frac{h_f}{\nu_f} + \frac{h_d}{\nu_d}\right) H_k = \left(h_f + h_d\right) B_k,\;\; k = y,z \tag{18}$$

From (19) here we find the coefficients ν'_x, ν'_y.

$$\nu'_k = \chi \nu_f,\;\; k = y,z \tag{19}$$

Where χ is the anisotropy coefficient of the later direction

$$\chi = \frac{h_f + h_d}{h_f + \dfrac{h_d \nu_f}{\nu_d}} \tag{20}$$

The value ν'_f is determined by the real induction field B_f in the ferromagnet according to equations (10), (11). The ratio of inductions in the transverse and tension in the longitudinal directions in ferromagnets and equivalent environment is determined in accordance with (14). The ratio (21) between the remaining projections is obtained by equations (9), (16), (19):

$$H_x = \xi H_{fx};\;\; B_y = B_{fy}/\chi;\;\; B_z = B_{fz}/\chi \tag{21}$$

According to the first expression (13) and the last two expressions (20), we get (22):

$$\nu_f = \sqrt{B_x^2 + \chi\left(B_y^2 + B_z^2\right)} \tag{22}$$

The values of B_x, B_y, B_z at each integration step are obtained from the equations of the electromagnetic field. Solving (21), for example, by Newton's method, we find according to (15)–(19) the magnetic reluctance of the matrix *N* in equation (8).

If we add electrical conductivities in the longitudinal direction, and supports in the transverse direction, then we get the expressions of the matrix elements (3). The electrical conductivity of the dielectric is zero

$$\gamma_x = 0;\;\; \gamma_k = \frac{h_f \gamma_f}{h_f + h_d},\;\; k = x,z \tag{23}$$

If the external electromagnetic field changes so slowly that the polarization process is proportional to its changes, then the ratio between the vectors does not depend on the time derivatives of these vectors, and such a field is called quasi-stationary. The magnetic field due to bias currents can be neglected, but the change of the magnetic field over time should be taken into account. As a rule, the conditions of quasi-stationarity are always fulfilled. In a quasi-stationary field in a conductive environment in the absence of a extraneous currents field, the current density vector is determined only by conduction currents [10, 12].

Maxwell's second equation is another form of Faraday's law and has the form of equation (24):

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{24}$$

According to (24), the magnetic induction changes over time, i.e. it generates an electric field.

The equation of the continuity of the magnetic field according to [3, 4] has the form (25):

$$\nabla \cdot \vec{B} = 0 \tag{25}$$

Equations (1), (24), (25) are the basic equations of the quasi-stationary electromagnetic field.

The relationship between the magnetic induction vector $\vec{B}$ and the magnetic field strength $\vec{H}$ is expressed through the medium parameters according to (11).

According to equations (5), the system of electromagnetic field equations in Cartesian coordinates consists of scalar equations (26) written in the projections of the electric and magnetic field intensity vectors:

$$\left.\begin{aligned}
&\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \gamma E_x; \quad \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{1}{\nu}\frac{\partial H_x}{\partial t};\\
&\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = \gamma E_y; \quad \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{1}{\nu}\frac{\partial H_y}{\partial t};\\
&\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = \gamma E_z; \quad \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{1}{\nu}\frac{\partial H_z}{\partial t}.
\end{aligned}\right\} \tag{26}$$

According to (1), (5), the differential equation (26) takes the form (2):

$$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0 \tag{27}$$

The complete system of electromagnetic field equations consists of the system of equations (23), (24), and (27) only imposes restrictions on the distribution of field vector projections in space.

3. Results

For a complete picture of the field, the basic equations (1), (24), (25) should be supplemented with initial and boundary conditions. The initial conditions are unknown at $t=+0$ and represent the spatial position of the field vectors $H=H(x,y,z)$ and $E=E(x,y,z)$ in time. The equations relating to the boundary conditions establish the dependence between the sought-after functions on both sides of the integration boundary.

We will exclude the electric components in the equations of the electromagnetic field, since the boundary conditions are easier to determine relative to the magnetic field vectors. Taking into account this factor, we will make the operation $\nabla \times$ from the left and right parts (1), (2). Then we get the equation:

$$\nabla \times \nabla \times \vec{H} = \gamma \nabla \times \vec{E} \tag{28}$$

We take the vector $\nabla \times \vec{E}$ in the right part of (28) from Maxwell's second equation (24), replacing *B* with *H* according to (11), then we get equation (29):

$$\nabla \times \nabla \times \vec{H} = -\frac{\gamma}{\nu}\frac{\partial \vec{H}}{\partial t} \tag{29}$$

According to [3], expression (29) takes the form

$$\frac{\gamma}{\nu}\frac{\partial \vec{H}}{\partial t} = \nabla^2 \vec{H} - \nabla\left(\nabla \cdot \vec{H}\right) \tag{30}$$

Taking into account (24), (28), equation (30) takes the form (31):

$$\frac{\partial \vec{B}}{\partial t} = \frac{1}{\gamma}\nabla^2 \vec{H} \tag{31}$$

According to (31), the electric and magnetic field strengths are mutually perpendicular [7].

Figure 2 shows the results of calculating the electromagnetic field in a ferromagnetic plate made of E4A steel with a thickness of β = 2.4 mm. Curve 1 corresponds to a saturated magnetic system, curve 2 to an unsaturated magnetic system. Calculations were carried out in the MAPLE program.

Boundary and initial conditions have the form of equations:

$$H(0, t) = H(\beta, t) = 750 sin 314t; \quad H(z, 0) = 0 \tag{32}$$

Dependence (9) is determined by the main magnetization curve for this grade of steel.

The scalar form of this equation consists of equations written for the projections of the vector $\vec{H}$ on the coordinate axis (33):

$$\frac{\partial \vec{B}}{\partial t} = \frac{1}{\gamma}\frac{\partial^2 H}{\partial z^2} \tag{33}$$

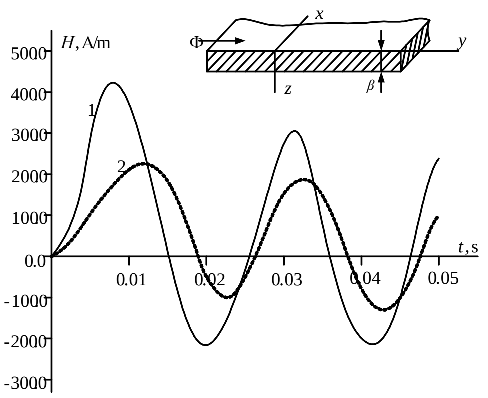

Fig. 2. Rated stress curves magnetic field inside the sheet

Consider another nonlinear problem in the field of electrodynamics. We are talking about the phenomenon of current displacement (skin effect) in rotating electric machines with deep rotor grooves. In such machines, eddy currents in the body of current conductors perform operational functions, and therefore must be taken into account with high accuracy.

Taking into account that the current in the conductor flows only along its length, and the vector of magnetic field tension, by the accepted assumption $\mu_{Fe} = \infty$, is perpendicular to the walls of the groove, we get a clear example of a plane electromagnetic wave.

By choosing a rectangular coordinate system, you can write down the conditions (34):

$$\vec{E} = \vec{x}_0 E; \qquad \vec{H} = \vec{y}_0 H \tag{34}$$

Then we come to equations (23), (26), which, taking into account the dimensions of the groove, take the form (35):

$$-\frac{\partial H}{\partial z} = \frac{b}{a}\gamma E; \qquad -\frac{\partial E}{\partial z} = \frac{1}{\nu}\frac{\partial H}{\partial t} \tag{35}$$

where *a*, *b* are respectively the width of the groove and the current conductor.

Excluding the electric field intensity from system (35), we obtain the equation (36):

$$\frac{\partial H}{\partial t} = c^2 \frac{\partial^2 H}{\partial z^2} \tag{36}$$

where

$$c^2 = \frac{a\nu}{b\gamma} \tag{37}$$

The derivative $\frac{\partial \Phi}{\partial t}$ can be represented as (38):

$$\frac{\partial \Phi}{\partial t} = \frac{\partial}{\partial t}\int_s \vec{B} dS \tag{38}$$

Where *S* is the surface through which the induction flux determines the flux Φ.

Taking into account formula (11) and (35), expression (38) takes the form (39):

$$\frac{\partial \Phi}{\partial t} = l\int_0^{\xi} \frac{1}{n}\frac{\partial H}{\partial t} dz = l\int_{\xi}^{0} \frac{\partial E}{\partial z} dz = \left[E(0) - E(\xi)\right] l \tag{39}$$

Hence, the tension of the groove part is equal to (40):

$$U = E(0) \cdot l \tag{40}$$

This voltage reflects surface phenomena throughout the grooved part of the conductor. Its calculation is related to the calculation of the electromagnetic field in the groove.

Consider the method of determining the boundary conditions for the electromagnetic field equation (29). According to Biot-Savar law, the strength of the magnetic field *H* depends on the current flowing in the conductor, as well as on the distance from it, so the boundary conditions must be sought on the basis of this law. For practical use, another form of its recording is more convenient – the law of full current. So, knowing the strength of the current in the groove, we determine the boundary conditions for the magnetic field tension vector using the full current law [6, 11]

$$\int \vec{H} d\vec{l} = I \tag{41}$$

Choosing the path of integration along the line that passes along the surface of the groove and in the body of the iron, we replace the integral (41) by the sum

$$\int \vec{H}\ d\vec{l} = \int_a \vec{H}_a d\vec{l} + \int_c \vec{H}_c d\vec{l} = 0 = I \tag{42}$$

where $\vec{H}_a, \vec{H}_c$ are the magnetic field intensity vectors on the groove surface and in the steel; *a* – the width of the groove opening zone; *c* is the length of the trajectory in the ferromagnet; *I* – conductor current.

We write the integral along the path that passes along the bottom of the groove (42):

$$\int_a \vec{H}_a d\vec{l} + \int_c \vec{H}_c d\vec{l} = 0 \tag{43}$$

The magnetic voltage drop in steel is close to zero. Therefore, it can be assumed with sufficient accuracy that the integral in steel is zero. In the groove opening zone, vector *H* has only a tangential component ($\vec{H} = \vec{y}_0 \cdot H_y$ – in Cartesian coordinates), and $H_y = const$.

Then we determine the boundary conditions from (42), (43). The initial conditions of the integrating functions are assumed to be zero.

In the upper and lower parts of the groove according to Fig. 2

$$H_y = H(0) = \frac{I}{a}; \quad H_c = H(10) = 0 \tag{44}$$

We find the electric field strength from equation (35). The equation has the form (45):

$$E = -\frac{a}{\gamma b}\frac{\partial H}{\partial z} \tag{45}$$

Having the value *E*(0), the voltage *U* of the conductor is found according to (40). Equations (36) together with the boundary conditions (44) constitute the boundary value problem in the time domain for the differential equations of the electromagnetic field. Therefore, the current line voltage is determined by the value of the electric field intensity vector on the surface of the groove (z=0).

The described method makes it possible to solve the problem of displacement of the current into the groove in the general case. This problem is solved on the basis of numerical methods of analysis using computer technology [14]. At the same time, the differential equations must be replaced by finite-difference equations. Let's draw up an explicit differential scheme for determining the field strength at the nodes of the spatial grid at different moments of time.

$$H(z,t+\Delta t) = H(z,t) + \\ +c\left[H(z-\Delta z,t) - 2H(z,t) + H(z+\Delta z,t)\right] \tag{46}$$

where c is found according to (37).

In the case of rigid differential equations, we use an implicit difference scheme, which for equation (36) has the form

$$dH(z,t) = aH(z-\Delta z,t+\Delta t) + \\ +bH(z,t+\Delta t) + cH(z+\Delta z,t+\Delta t) \tag{47}$$

where

$$a = c = 1; \quad d = -\frac{\gamma}{\nu_0}\frac{(\Delta z)^2}{\Delta t}; \quad b = d - 2 \tag{48}$$

Here $\nu_0 = \frac{1}{\mu_0} = \frac{1}{4\pi}\cdot 10^7 \ \frac{m}{H}$ is the magnetic constant.

In equation (47), the known values of H and E at point z at time t are related to three unknown values of the same functions at points $z–\Delta z$, z, $z+\Delta z$ at time $t+\Delta t$.

Figure 3 shows the time dependence of the voltage per unit length of the groove section of the current conductor in the state of the given current, calculated according to formulas (45), (46).

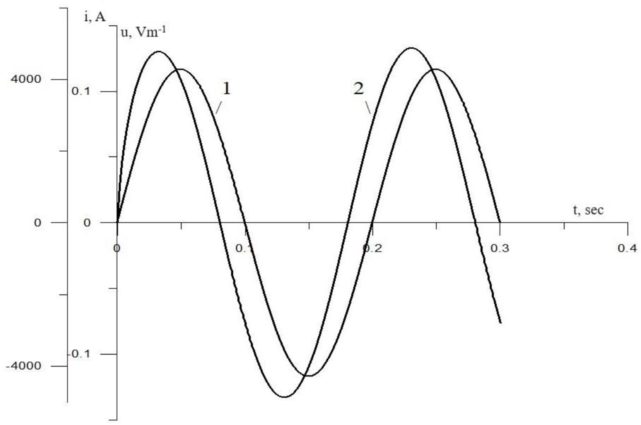

Fig. 3. Calculated curves of current (1) and voltage per unit length (2) of the slot current conductor in the state of sudden switching on the current source I = 4282sin(31.4t). Groove dimensions: h×l×a=0.038 m×0.23 m×0.005 m

Figure 4 shows the spatial discretization of the groove and the distribution of the field strength in it. The segment *ab* corresponds to the voltage, and *bc* is the internal electromotive force.

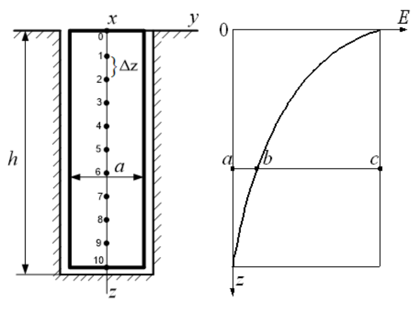

Fig. 4. Spatial discretization of the groove and distribution field strength in it

Figure 3 shows that the effect of current displacement is observed in the deep groove of the rotor of the electric machine. On the surface of the groove, the voltage is the greatest, which has a positive effect on the starting characteristics of electric asynchronous motors.

4. Conclusions

The proposed equations and methods of the theory of electromagnetic circuits and the electromagnetic field take into account such complex phenomena as the saturation of the steel of magnetic conductors, the surface effect in current conductors, the asymmetry of electrical circuits, the mutual rotational movement of electrical circuits, etc. The differential equations are written in the normal Cauchy form and are non-rigid and can be integrated by explicit methods that are simple in computer implementation. The equations appearing in the models make it possible to calculate with sufficient accuracy the quantitative characteristics of the object under study, to predict its behavior in various nominal and emergency modes, as well as to carry out optimization.

The calculation process is carried out based on the solution of the two-point boundary value problem of the differential equations of the electromechanical state. The differential equations of the electromechanical state are integrated based not on the given initial conditions, but on those that exclude the transient reaction and make it possible to enter directly into the steady process, bypassing the transient one.

The use of mathematical modeling and computer simulation methods makes it possible to abandon field experiments, which in many cases are difficult and expensive. Practical tasks of calculation and analysis of transient processes in electro-technical devices should be carried out only by mathematical modeling methods, which will allow them to be correctly designed and operated. The proposed method can be adapted to construct the equations of the electromagnetic field in moving media.

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