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A MODIFIED METHOD OF SPECTRAL ANALYSIS OF RADIO SIGNALS USING THE OPERATOR APPROACH FOR THE FOURIER TRANSFORM

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Abstract. *The article proposes the improved method of spectral analysis of radio signals. The improvement is achieved due to the use of special operators in the signal conversion process. This allows you to distinguish the signal accurately and to determine its characteristics at the background of many airspace obstacles. The obtained graphical results fully confirm the advantages of the proposed method. The simulation results proved the advantage of the improved method of spectral analysis of radio signals; the advantage is achieved through the usage summing matrix functions in the process of signals conversion. The proposed improved method increases the accuracy of signals detection of secretly obtaining information means by 12%.*

Keywords: spectrum, radio monitoring, matrix functions, secretly obtaining information means, harmonic functions, Fourier transformation, Poisson operator

ZMODYFIKOWANA METODA ANALIZY WIDMOWEJ SYGNAŁÓW RADIOWYCH Z WYKORZYSTANIEM PODEJŚCIA OPERATORSKIEGO DLA TRANSFORMATY FOURIERA

Streszczenie. *W artykule zaproponowano udoskonaloną metodę analizy widmowej sygnałów radiowych. Poprawę osiąga się dzięki zastosowaniu specjalnych operatorów w procesie konwersji sygnału. Pozwala to na dokładne rozróżnienie sygnału i określenie jego charakterystyki na tle wielu przeszkód w przestrzeni powietrznej. Uzyskane wyniki graficzne w pełni potwierdzają zalety proponowanej metody. Wyniki symulacji wykazały przewagę udoskonalonej metody analizy widmowej sygnałów radiowych; zaletę uzyskuje się poprzez wykorzystanie funkcji macierzy sumującej w procesie konwersji sygnałów. Zaproponowana udoskonalona metoda zwiększa o 12% dokładność wykrywania sygnałów tajnego pozyskiwania informacji.*

Słowa kluczowe: widmo, monitoring radiowy, funkcje macierzowe, środki tajnego pozyskiwania informacji, funkcje harmoniczne, transformacja Fouriera, operator Poissona

Introduction

The rapid development of communication tools and technologies has led not only to positive but also to negative results. Nowadays, information is becoming more and more important than material or energy resources. Therefore, the primary task of competition is to gain access to information. 80-90% of the necessary information is obtained with the help of technical means. Technical means of secretly obtaining information have a great advantage over other methods of obtaining information. First, they do not require the operator's permanent presence in the city of the information leak. Secondly, the scout cannot be tracked, because he works in the passive mode of receiving information. And in the third, information is received in real-time and in a complete manner. It is possible to detect technical means of covertly obtaining information that works on a radio channel only by detecting radio signals of technical means of covertly obtaining information. The process of detecting a radio signal is the first step in detecting a leak. Most signals are analog, so it is impossible to process such signals in a digital system. To represent them accurately, we need numbers of infinite length. Therefore, it is necessary to convert an analog signal on a time interval into a sequence of numbers of a given length, taken at discrete moments of the time interval. Based on the above, an important task is to improve the methods of converting analog radio signals into digital ones using the Fourier transform. But the transformation always has errors and inaccuracies, therefore the scientific task of improving the methods of converting analog radio signals into digital ones using the Fourier transform with the analysis of mathematical conversion errors and determining ways to eliminate them is relevant.

1. Literature review

A significant number of publications are devoted to the issue of finding radio signals of the means of clandestine information acquisition (MCIA).

Thus, in works [1, 2], the issue of finding and locating means of secretly obtaining information with the help of automated software search complexes and additional devices is considered. However, in the very process of radio signal conversion, conversion errors are not considered. In works [3, 4], the classification of MCIA and features of their detection are given. The paper proves that receiver scanners capable of working under computer control have a significant advantage over other means of detecting radio signals. Using an external computer with software allows you to automate the process of searching and identifying technical devices for obtaining information. Unfortunately, no attention was paid to the methodology of signal processing and the process of converting radio signals.

The work [5] deals with the general principle of radio monitoring, which allows you to analyze the radio-electronic situation in control areas, maintain a database of radio-electronic means, and use it for effective detection of radio signals, including short-term sessions of their operation. For example, when using remote control devices and intermediate storage of information. However, the peculiarity of detecting analog means of secretly obtaining information is not considered.

In works [6 – 8] attention is paid:

Search efficiency is determined not only by the parameters of the scanning receivers but also by the software installed on the external computer. This software not only controls the receiver but also performs preliminary radio monitoring. This allows the detection of dangerous radio signals, but no attention is paid to the accuracy and errors of radio signal conversion.

In the conditions of rapid development of computer technologies, which is shown in works [9, 11, 26, 33], attention should be focused on search software capable of qualitatively analyzing radio signals only based on qualitative input parameters. That is, based on the high-quality conversion of an analog signal into a digital one. Therefore, improving the search process based on the Fourier transform is relevant.

The issue of analysis of radio control (radio monitoring) systems with different technical parameters, which have one thing in common - they can only display and store panoramas of signal spectra on the radio, is discussed in works [10, 23–25]. The task of analyzing digital signals in legal communication channels is either not solved at all, or is performed formally. That is, autonomous devices cannot find analog means of secretly obtaining information. This becomes effective only with software search engines where the primary input signal is a digital signal obtained by converting an analog signal to digital using a Fourier transform. The capabilities of the main software complexes for finding technical means of secretly obtaining information are described in scientific works [12–15, 20–23]. Works [25, 33–36] provide an overview of new versions of devices for automated search systems.

Having analyzed the modern literature, we can conclude that there are many methods and programs for finding means of secretly obtaining information. The software used to analyze digital packets for radio control search tasks is constantly being improved [26–32]. However, universal software does not exist, because there is no mathematical apparatus for converting analog signals into digital without errors. One of the ways to improve the software is to improve the input parameters – the parameters for converting analog signals into digital ones, i.e. improved Fourier transformation. Therefore, the task of analysing the mathematical errors of converting analog radio signals into digital ones based on Fourier transformations and eliminating conversion errors is an urgent scientific task.

2. Application of summing matrix functions in the Fourier transformation

The main task of the programs of SMOI complexes is to obtain useful information from the signal taking account the noise. The signal can be not only a certain material carrier of information, but also any physical process, the parameters being changed in accordance with the transmitted message. To solve the problem, it is necessary to receive this signal, to recognize it, to establish its adequacy to a certain class and the degree of danger for the object and its information system.

Most signals have an analog nature, so it is impossible to process such signals in the digital system. For their accurate representation we need numbers of infinite length. We have to convert the analog signal on the time interval into a sequence of numbers of the given length, taken at discrete moments of the time interval.

The task of detecting SMOI is the problem of signals spectral analysis. A signal is a material carrier of information. The signal is characterized by the change in the physical quantity representing it later. Therefore, the natural mathematical model of the signal is the function of time $S(t)$. The dimension $S(t)$ is determined by the dimension of the corresponding physical quantity. In radio electronics, the mathematical model is widely used where $F(\omega)$ is the function of frequency, known as the signal spectrum.

The spectrum and the time function describe the same signal, so they are interrelated. Assuming that the function of time $S(t)$ is known, we investigate what the concept of spectrum means.

The spectrum of the signal $S(t)$ is the set of amplitudes and initial phases of harmonic oscillations of multiple frequencies, the sum of which is equal to the signal $S(t)$.

If the set of frequencies of harmonic oscillations is discrete, then the spectrum is discrete. If the set of frequencies is continuous, then the spectrum is continuous. Detailing the spectrum, we distinguish between amplitude and phase spectra. The amplitude spectrum is the set of amplitudes of harmonic oscillations of multiple frequencies. In the continuous spectrum, the characteristic of the amplitude spectrum is the amplitude spectral density $F(\omega)$, because it determines the dependence of an infinitesimal amplitude on the frequency at a fixed value of an infinitesimal frequency range $d\omega$. The phase spectrum is the set of initial phases of harmonic oscillations of multiple frequencies. In the continuous spectrum, the characteristic of the phase spectrum is the initial phase $\Phi(\omega)$. The signal spectrum exists if the signal $S(t)$ can be represented as the sum of harmonic oscillations. Representation $S(t)$ in the form of the sum of harmonic oscillations is called the Fourier spectral decomposition.

Due to this, the problem of converting signals into digital form in order to detect SMOI signals is provided by the usage of direct discrete Fourier transformation cosine:

$$\operatorname{Re} X[k] = \sum_{i=0}^{N-1} x[i]\cos\left(\frac{2\pi ki}{N}\right) \quad (1)$$

and direct discrete Fourier transformation sine

$$\operatorname{Im} X[k] = -\sum_{i=0}^{N-1} x[i]\sin\left(\frac{2\pi ki}{N}\right) \quad (2)$$

Formulas (1) and (2) are analogs of the corresponding continuous Fourier transformation cosine

$$\lambda_c(t) = \frac{1}{\pi}\int_0^\infty \lambda(u)\cos(ut)du \quad (3)$$

and the corresponding continuous Fourier transformation sine

$$\lambda_s(t) = \frac{1}{\pi}\int_0^\infty \lambda(u)\sin(ut)du \quad (4)$$

As the discrete Fourier transformation decomposes the studied signal (sample) into sinusoidal and cosine components, in this paper we propose to consider the Fourier transformation of the form

$$\hat{\lambda}(t) = \frac{1}{\pi}\int_0^\infty \lambda(u)\cos\left(ut + \frac{r\pi}{2}\right)du,\ r\in N \quad (5)$$

It is obvious that the introduced Fourier transformation of the form (5) turns into a sine of the Fourier transformation at odd values of r, and turns into a cosine of the Fourier transformation at even values of r.

Data obtained from digital media is usually written in the form of vectors, or matrices, which we will denote Λ. Therefore, the so-called summing function [34] constructed with the help of matrices Λ is of great importance in the Fourier transformation of the form (5). We will show how this summing function $\lambda(u)$ of Fourier transformation (5) is constructed.

So let f – some continuous 2π-periodic function corresponds to sound signals of a certain frequency in real conditions.
And

$$\frac{a_0}{2} + \sum_{k=1}^\infty \left(a_k(f)\cos(ku) + b_k(f)\sin(ku)\right) \quad (6)$$

is Fourier series of this continuous 2π-periodic function. Then by means of the matrix $\Lambda = \{\lambda_{n,k}\}$ (see, e.g. [31, p. 51]) without which data processing read from digital data carriers on the basis of the series (6) is not possible, we will construct the operator of the form

$$F(f,\lambda) = \frac{a_0}{2} + \sum_{k=1}^\infty \lambda_{n,k}\left(a_k(f)\cos(nx) + b_k(f)\sin(nx)\right) \quad (7)$$

Consider the specific examples of operators of the form (7). Namely:

1) if

$$\lambda_{n,k} = \left(1 + sk\left(1 + e^{-\frac{1}{n}}\right)\left(1 - e^{-\frac{1}{n}}\right)\right)e^{-\frac{k}{n}}$$

$$0 \le \rho \le \frac{1}{2},\ q \ge 1 \quad (8)$$

then from (7) we obtain the so-called generalized Poisson operator, properties having been considered in the works [6, 7, 11, 17, 19, 20, 30].

For visual representation of the obtained results, we construct the graph of the form (8) that confirms the conclusions that we obtained a generalized Poisson operator (Fig. 1).

2) if

$$\lambda_{n,k} = e^{-\frac{k}{n}} \quad (9)$$

then from (7) we get the Abel-Poisson operator [4, 7];

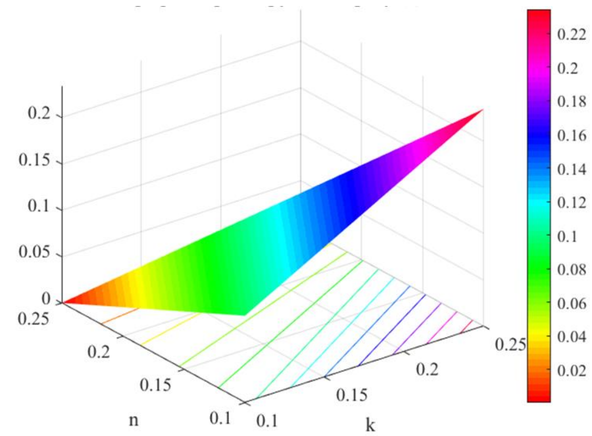

Fig. 1. Graphical representation of the form (8)

3) if

$$\lambda_{n,k} = 1 + \frac{k}{2}\left(1 - e^{-\frac{2}{n}}\right) e^{-\frac{k}{n}} \tag{10}$$

then from (7) we have the biharmonic Poisson operator [14, 16–18];

4) if

$$\lambda_{n,k} = \left(1 + \frac{k}{4}\left(3 - e^{-\frac{2}{n}}\right) \times \left(1 - e^{-\frac{2}{n}}\right) + \right. \\ \left. + \frac{k^2}{8}\left(1 - e^{-\frac{2}{n}}\right)\right) e^{-\frac{k}{n}} \tag{11}$$

then from (7) we have the threeharmonic Poisson operator [10] (Fig. 2).

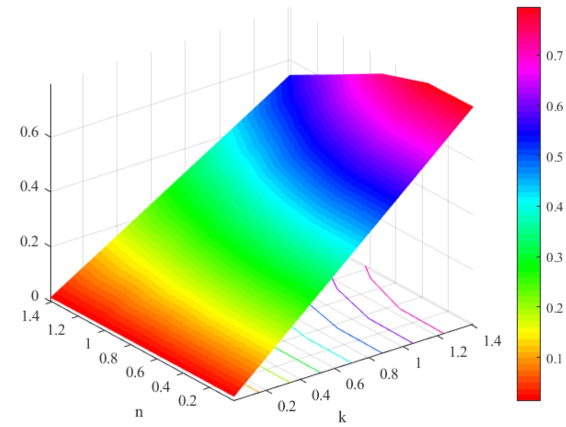

Fig. 2. Graphical representation of the form (11)

For the clearer representation of the obtained results, we construct the graph of the form (8), which confirms the conclusions that we obtained the threeharmonic generalized Poisson operator.

All the above considered examples of matrixes $\Lambda = \{\lambda_{n,k}\}$ as a result of the combination with Fourier series (6) give harmonic functions of the form (7), without which the processing of data read from digital media is not possible. At the same time, these harmonic functions of the form (7) are solutions of the corresponding integral-differential equations of elliptic type [1, 5, 9, 12, 21, 30]. The main difference between the integral and discrete Fourier transformation is that the integral Fourier transformation is defined on the whole domain of the function $\lambda(u)$, and the discrete Fourier transformation is defined only on the finite discrete set of points. Therefore, it is logical to move from the operators of the form (7) to the operators of the form

$$F_n(f;\lambda) = \frac{a_0}{2} + \sum_{k=1}^{n-1} \lambda_{n,k} \times \\ \times \left(a_k(f)\cos(ku) + b_k(f)\sin(ku)\right) \tag{12}$$

Linear filtering methods are well-structured methods for which efficient computational schemes based on the fast convolution algorithms have been developed. Therefore, the next step of our study is to represent the operator of the form (7) in the form of the convolution. For this, the values of the Fourier coefficients

$$a_k(f) = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos(kt)dt \quad k = 0,1,2,...$$

and

$$b_k(f) = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin(kt)dt \quad k = 1,2,...$$

substitute in the right part (7). As a result, we get that

$$\begin{aligned} F(f;\lambda) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)dt + \\ &+ \sum_{k=1}^{\infty} \lambda_{n,k} \Big(\frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos(kt)\cos(ku)dt + \\ &+ \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin(kt)\sin(ku)dt\Big) = \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)dt + \frac{1}{\pi}\sum_{k=1}^{\infty}\lambda_{n,k}\int_{-\pi}^{\pi} f(t)\cos k(t-u)dt = \\ &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\left(\frac{1}{2} + \sum_{k=1}^{\infty}\lambda_{n,k}\cos k(t-u)\right)dt \end{aligned} \tag{13}$$

Having replaced the variable $t - u$ in the right part (13) we will have that

$$\begin{aligned} F(f;\lambda) &= \frac{1}{\pi}\int_{-\pi-u}^{\pi-u} f(u+\tau)\left(\frac{1}{2} + \sum_{k=1}^{\infty}\lambda_{n,k}\cos(k\tau)\right)d\tau = \\ &= \frac{1}{\pi}\int_{-\pi-u}^{-\pi} f(u+\tau)\left(\frac{1}{2} + \sum_{k=1}^{\infty}\lambda_{n,k}\cos(k\tau)\right)d\tau + \\ &+ \frac{1}{\pi}\int_{-\pi}^{\pi} f(u+\tau)\left(\frac{1}{2} + \sum_{k=1}^{\infty}\lambda_{n,k}\cos(k\tau)\right)d\tau - \\ &- \frac{1}{\pi}\int_{\pi-u}^{\pi} f(u+\tau)\left(\frac{1}{2} + \sum_{k=1}^{\infty}\lambda_{n,k}\cos(k\tau)\right)d\tau = I_1 + I_2 - I_3. \end{aligned} \tag{14}$$

To conclude, we have to show that $I_1 = I_3$. Having replaced the variable in the integral I_1: $\tau = t - 2\pi$ and thus considering 2π periodicity of the function $f(\cdot)$ we will receive that

$$\begin{aligned} I_1 &= \frac{1}{\pi}\int_{-\pi-u}^{-\pi} f(u+\tau)\left(\frac{1}{2} + \sum_{k=1}^{\infty}\lambda_{n,k}\cos(k\tau)\right)d\tau = \\ &= \frac{1}{\pi}\int_{\pi-u}^{\pi} f(u+t-2\pi) \times \\ &\times \left(\frac{1}{2} + \sum_{k=1}^{\infty}\lambda_{n,k}\cos k(t-2\pi)\right)dt = I_3 \end{aligned} \tag{15}$$

Using equation (15) to the right-hand side of the relation (14) we can write that

$$F(f;\lambda) = \frac{1}{\pi}\int_{-\pi}^{\pi} f(u+\tau)\left(\frac{1}{2} + \sum_{k=1}^{\infty}\lambda_{n,k}\cos(k\tau)\right)d\tau \tag{16}$$

Formula (16) gives the compact record of the summation matrix function $\lambda(u)=\lambda\left(\frac{k}{n}\right)$, constructed using the matrix $\lambda_{n,k}$.

If we denote by $\mathcal{E}(F_n)$ the degree of deviation from the expected signal when it is approximated by the operator $F_n(f;\lambda)$, then from the formula (4.2) of the work [8] we have that

$$\mathcal{E}(F_n)=\frac{4}{\pi^2}\frac{1}{n}\int_0^1\frac{|\lambda(u)|}{1-u}du+O\left(\frac{1}{n}\int_{1-\frac{2}{n\pi}}^{1}\frac{|\lambda(u)|}{1-u}du\right)+ \tag{17}$$
$$+O\left(\frac{1}{n}\int_0^1 u(1-u)\left|d\nu'(u)\right|\right)+O\left(\frac{1}{n}\right),\ n\to\infty$$

Where according to the work [8]

$$\lambda(u)=\begin{cases}\frac{1}{e^u}, & 0\le u\le 1-\frac{1}{n}\\ \frac{n(1-u)}{e^{1-\frac{1}{n}}}, & 1-\frac{1}{n}\le u\le 1\\ 0, & u\ge 1\end{cases} \tag{18}$$

$$\nu(u)=\begin{cases}\left(1-\frac{1}{e^u}\right)u^{-1}, & 0\le u\le 1-\frac{1}{n}\\ \left(1-\frac{n(1-u)}{e^{1-\frac{1}{n}}}\right)u^{-1}, & 1-\frac{1}{n}\le u\le 1\\ u^{-1}, & u\ge 1\end{cases} \tag{19}$$

To estimate the first term from the right-hand side (17), according to the representation (18) for the function $\lambda(u)$, we write it in the form

$$\frac{4}{\pi^2}\cdot\frac{1}{n}\int_0^1\frac{|\lambda(u)|}{1-u}du=\frac{4}{\pi^2}\times$$
$$\times\frac{1}{n}\int_0^{1-\frac{1}{n}}\frac{du}{e^u(1-u)}+\frac{4}{\pi^2}\cdot\frac{1}{n}\int_{1-\frac{1}{n}}^{1}\frac{n}{e^{1-\frac{1}{n}}}du \tag{20}$$

We note that for the second term from the right-hand side of (20) we have the obvious estimate

$$\frac{4}{\pi^2}\cdot\frac{1}{n}\int_{1-\frac{1}{n}}^{1}\frac{n}{e^{1-\frac{1}{n}}}du=O\left(\frac{1}{n}\right) \tag{21}$$

To find the first integral from the right-hand side of the form (20), we use the formula (3) [23]

$$\int_\alpha^\beta\frac{e^{-\eta x}}{x+\gamma}dx=e^{\gamma\eta}\{\mathrm{Ei}[-(\gamma+\beta)\eta]-\mathrm{Ei}[-(\gamma+\alpha)\eta]\} \tag{22}$$

where $-\gamma<\alpha$ or $-\gamma>\beta$, $\mathrm{Ei}(x)$ is the integral exponential function [23]. Then, according to (20)

$$\int_0^{1-\frac{1}{n}}\frac{e^u}{1-u}du=e^{-1}\left(\mathrm{Ei}(1)-\mathrm{Ei}\left(\frac{1}{n}\right)\right) \tag{23}$$

From [23] we know that Ei(1) = 1.895 117 816…,

$$\mathrm{Ei}(x)=C+\ln x+\sum_{k=1}^{\infty}\frac{x^k}{k\cdot k!} \tag{24}$$

where $C = 0.577$ is Euler's constant. Hence

$$\mathrm{Ei}\left(\frac{1}{n}\right)=C+\ln\frac{1}{n}+\sum_{k=1}^{\infty}\frac{1}{k\cdot k!n^k} \tag{25}$$

Obviously, the series on the right-hand side of (25) converges and, in addition, we have the estimate

$$\sum_{k=1}^{\infty}\frac{1}{k\cdot k!n^k}<\sum_{k=1}^{\infty}\frac{1}{n^k}=\frac{1}{n-1}=O\left(\frac{1}{n}\right),\ n\to\infty \tag{26}$$

Therefore, comparing the relations (20)–(26), we obtain that

$$\frac{4}{\pi^2}\cdot\frac{1}{n}\int_0^1\frac{|\lambda(u)|}{1-u}du=\frac{4}{\pi^2}\frac{\ln n}{e\cdot n}+O\left(\frac{1}{n}\right) \tag{27}$$

Similarly, we can prove the validity of the estimate

$$\frac{1}{n}\int_{1-\frac{2}{n\pi}}^{1}\frac{\lambda(u)}{1-u}du=O\left(\frac{1}{n}\right),\quad n\to\infty \tag{28}$$

for the same function $\lambda(u)$ given by means of the relation (18).

Let us find the estimate for the integral $\frac{1}{n}\int_0^1 u(1-u)\left|d\nu_n'(u)\right|$ where the function $\nu(u)$ is given by means of the relation (19).

From formula (19) it follows that

$$\nu''(u)=\frac{1}{e^u} \tag{29}$$

for all $u\in[0;1]$. If $u\in\left[\frac{1}{n};1-\frac{1}{n}\right]$, then $\nu'(u)=\frac{1}{ue^u}-\frac{1-e^{-u}}{u^2}$, hence

$$\nu''(u)=-\frac{1}{ue^u}-\frac{2}{u^2e^u}+\frac{2}{u^3}\left(1-\frac{1}{e^u}\right)\ge 0 \tag{30}$$

If $u\in\left[1-\frac{1}{n};1\right]$, then

$$\nu'(u)=-\left(1-\frac{n}{e^{1-\frac{1}{n}}}\right)\frac{1}{u^2},\ \ \nu'(u)=\frac{2n}{u^3e^{1-\frac{1}{n}}}<0 \tag{31}$$

Taking into account the estimate (29) and the obvious inequality $\frac{u-u^2}{e^u}\le\frac{1}{4},\ u\ge 0$, we get that

$$\int_0^{1/n}u(1-u)\left|d\nu'(u)\right|=\int_0^{1/n}\frac{u(1-u)}{e^u}du\le\frac{1}{4}\int_0^{1/n}du=\frac{1}{4n} \tag{32}$$

Since the function $\nu(u)$ is downward convex on the interval $\left[\frac{1}{n};1-\frac{1}{n}\right]$, it follows from (30) and the following obvious inequalities that

$$u-u^2\le\frac{1}{4},\ u\in R \tag{33}$$

$$e^{-u}\le 1,\ \ e^{-u}\le 1-u,\ u\ge 0 \tag{34}$$

we will have the estimate

$$\int_{\frac{1}{n}}^{1-\frac{1}{n}}u(1-u)\left|d\nu'(u)\right|\le\frac{1}{4}\left(\nu'\left(1-\frac{1}{n}\right)-\nu'\left(\frac{1}{n}\right)\right)=$$
$$=\frac{\left(1-\frac{1}{n}\right)}{4\left(1-\frac{1}{n}\right)^2e^{1-\frac{1}{n}}}-\frac{1}{4\left(1-\frac{1}{n}\right)^2}+\frac{1}{4\left(\frac{1}{n}\right)^2}-\frac{1+\frac{1}{n}}{4\left(\frac{1}{n}\right)^2e^{\frac{1}{n}}}\le \tag{35}$$
$$\le\frac{1}{4}\left(\frac{1}{1-\frac{1}{n}}+1\right)\le C_1,(C_1=\mathrm{const})$$

Since for all $u\in\left[1-\frac{1}{n};1\right]$ the function $\nu(u)$ is convex upwards, then, taking into account (31) and inequalities (33), (34), we will have:

$$\int_{1-\frac{1}{n}}^{1} u(1-u)\left|d\nu'(u)\right| \le \frac{1}{4}\left(\nu'\left(1-\frac{1}{n}\right)-\nu'(1)\right)=$$

$$=\frac{1}{4}\left(\frac{e^{-\left(1-\frac{1}{n}\right)}}{1-\frac{1}{n}}-\frac{1-e^{-\left(1-\frac{1}{n}\right)}}{\left(1-\frac{1}{n}\right)^{2}}+1-\frac{2}{e}\right)\le \qquad (36)$$

$$\le\frac{1}{4}\left(\frac{1}{1-\frac{1}{n}}+\left(1-\frac{2}{e}\right)\right)\le C_2,(C_2=\text{const})$$

Thus, comparing the relations (32), (35) and (36), we obtain

$$O\left(\frac{1}{n}\int_{0}^{1} u(1-u)\left|d\nu'(u)\right|\right)=O\left(\frac{1}{n}\right) \qquad (37)$$

Finally, substituting (27), (28), and (37) into the right-hand side of the equality (19), we obtain that

$$\mathcal{E}(F_n)=\frac{4}{\pi^2 e}\cdot\frac{\ln n}{n}+O\left(\frac{1}{n}\right),\quad n\to\infty \qquad (38)$$

If we denote the degree of deviation from the expected signal when it is approximated by the operator $F(f,\lambda)$ through $\mathcal{E}(F)$, then, using the methods of the work [28, 35], we can show that:

$$\mathcal{E}(F)=\mathcal{E}(F,n)=O\left(\frac{1}{n}\right) \qquad (39)$$

Using the obtained asymptotic equations (38) and (39), we make the geometric interpretation of the comparative analysis of the deviation degree from the expected signal when it is approximated, respectively, by the operators $F(f,\lambda)$ and $F_n(f,\lambda)$.

The program for obtaining simulation results and the graphical representation of the results are given below.

```
1.import matplotlib.pyplot as plt - library (third-party) for plotting graphs
2.from math import log
3.def error_triangular(n : float):
4.   return log(n)/n
5.def error_rectangular(n : float):
6.   return 1/n
7.x = [i for i in range(2, 100)] # on the X-axis
8.t_y = [error_triangular(i) for i in range(2, 100)]
9.r_y = [error_rectangular(i) for i in range(2, 100)]
10.plt.plot(x, t_y, "^b", label="Error of triangular methods")
11.plt.plot(x, r_y, "vr", label="Error of rectangular methods")
12.plt.xlabel("n")
13.plt.ylabel("Main part of asymptotic mark of error")
14.plt.legend()
15.plt.show()
```

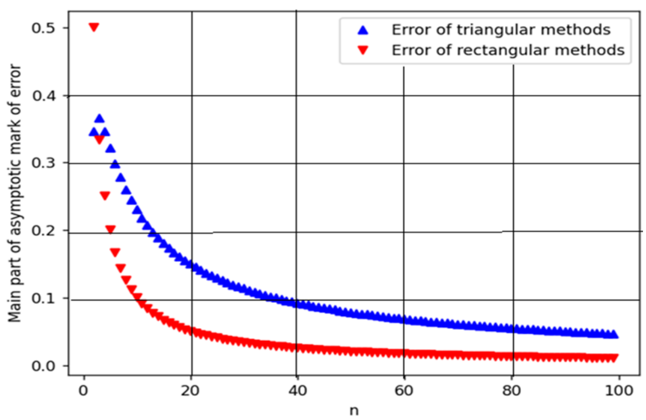

Fig. 3. Behavior of the main elements of the asymptotic estimate of the error using the approximation of triangular and rectangular methods

Graphical representation of the results of the program is shown in Fig. 3.

As we can see from the graph, when using operators $F(f,\lambda)$ of the form (7) we can obtain the reduction in the degree of deviation in the approximation by 12% compared with the operators $F_n(f,\lambda)$ of the form (12).

3. Conclusions

In the course of the conducted research, the process of detecting the radio signal as the means of covertly obtaining information, a contradiction between the existing methods of converting radio signals, and the need for error-free conversion of signals into digital form was revealed. An improved method is proposed that allows converting an analog signal on a time interval into a sequence of numbers of a given length, taken at discrete moments of the time interval. The improvement consists in the method of spectral analysis of radio signals due to the use of special Fourier transform operators. The advantage is achieved by the use of summing matrix functions in the process of signal conversion. This allows you to accurately distinguish the signal and determine its characteristics against the background of many obstacles in the air space. The proposed improved method increases the accuracy of detecting signals utilizing covert information acquisition by 12%. The graphical results obtained fully confirm the advantages of the proposed method and theoretical calculations. The simulation results proved the advantages of the improved method of spectral analysis of radio signals.

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