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MODELING DYNAMIC AND STATIC OPERATING MODES OF A LOW-POWER ASYNCHRONOUS ELECTRIC DRIVE

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Abstract. *The article presents a mathematical model of the asynchronous motor in oblique coordinates, based on differential equations expressed in the standard Cauchy form. The differential equations of traditional models are implicitly formulated; therefore, during numerical implementation for prolonged processes, matrix coefficient rotation leads to significant time expenditure and the accumulation of errors during integration. This complex task is proposed to be addressed by ensuring that the differential equations of the electromechanical state are non-stiff and, importantly, written in standard Cauchy form. The standard Cauchy form is essential for analyzing asynchronous motors, as changes in the number of unknowns significantly restructure the coefficient matrix. This formulation of the equations is convenient for numerical integration, as explicit methods, which are considerably simpler than implicit methods, can be implemented. To create a mathematical model, coordinate transformations were performed based on the classical theory of electric machines. The advantage of the proposed method of using different coordinate axes is the possibility of analyzing new variables and obtaining constant coefficients in the equations of state of the electric motor. The model accounts for the electromagnetic interactions of the motor's electrical circuits and their nonlinearity, enabling the simulation of electromagnetic and electromechanical processes. Transitional operating modes of the asynchronous motor have been modeled and analyzed. The proposed model can be utilized for analyzing the operation of motors both as standalone elements and as components of an electromechanical system. It is demonstrated that this model aligns with classical electrical machine theory. Simulation results are provided, along with their analysis.*

Keywords: mathematical model, differential equations, numerical methods, asynchronous motor, electric drive

MODELOWANIE DYNAMICZNYCH I STATYCZNYCH TRYBÓW PRACY ASYNCHRONICZNEGO NAPĘDU ELEKTRYCZNEGO MAŁEJ MOCY

Streszczenie. *W artykule przedstawiono model matematyczny silnika asynchronicznego we współrzędnych ukośnych, oparty na równaniach różniczkowych wyrażonych w standardowej postaci Cauchy'ego. Równania różniczkowe tradycyjnych modeli są sformułowane w sposób niejawny, dlatego podczas implementacji numerycznej dla długotrwałych procesów rotacja współczynników macierzy prowadzi do znacznych nakładów czasu i akumulacji błędów podczas całkowania. To złożone zadanie proponuje się rozwiązać poprzez zapewnienie, że równania różniczkowe stanu elektromechanicznego są niesztywne i, co ważne, zapisane w standardowej postaci Cauchy'ego. Standardowa postać Cauchy'ego jest niezbędna do analizy silników asynchronicznych, ponieważ zmiany liczby niewiadomych znacząco zmieniają macierz współczynników. Takie sformułowanie równań jest wygodne dla całkowania numerycznego, ponieważ można zaimplementować metody jawne, które są znacznie prostsze niż metody niejawne. W celu stworzenia modelu matematycznego przeprowadzono transformacje współrzędnych w oparciu o klasyczną teorię maszyn elektrycznych. Zaletą zaproponowanej metody wykorzystania różnych osi współrzędnych jest możliwość analizy nowych zmiennych i uzyskania stałych współczynników w równaniach stanu silnika elektrycznego. Model uwzględnia oddziaływania elektromagnetyczne obwodów elektrycznych silnika i ich nieliniowość, umożliwiając symulację procesów elektromagnetycznych i elektromechanicznych. Zamodelowano i przeanalizowano przejściowe tryby pracy silnika asynchronicznego. Zaproponowany model może być wykorzystany do analizy działania silników zarówno jako samodzielnych elementów, jak i komponentów systemu elektromechanicznego. Wykazano, że model ten jest zgodny z klasyczną teorią maszyn elektrycznych. Przedstawiono wyniki symulacji wraz z ich analizą.*

Słowa kluczowe: model matematyczny, równania różniczkowe, metody numeryczne, silnik asynchroniczny, napęd elektryczny

Introduction

The progress of modern technologies inevitably leads to a reassessment of existing methodologies for the research and analysis of electrotechnical devices.

The implementation of advanced technologies and methods is a critical task for specialists in mathematical modeling. While the theoretical foundation built on the laws of electrodynamics is well-established, solving nonlinear differential equations requires new approaches, influenced by advances in numerical methods and computational technology. Thus, solving mathematical modeling problems for electrotechnical devices and systems has become an objective necessity [1].

Today, mathematical modeling is a powerful tool for investigating physical processes in electrotechnical devices, particularly in asynchronous motors or asynchronous electric drives.In practice, conducting full-scale experiments is often challenging and sometimes impossible.

Therefore, the ability to construct mathematical models of electrotechnical devices enables researchers to calculate transient processes through computer simulations. On one hand, these simulations provide comprehensive data on electrical and mechanical quantities; on the other, they allow researchers to predict device or system behavior under non-nominal or emergency conditions. In this way, researchers acquire complete numerical data, facilitating the proper operation and maintenance of electrotechnical equipment.

1. Literature review

The analytical study and analysis of transient processes in asynchronous motors present complex challenges, requiring the application of mathematical modeling and experimental methods. The mathematical framework for modeling electrical machines is well-developed, as evidenced by a substantial body of literature from global experts. Accurately describing the physical processes in electrical machines necessitates the use of nonlinear differential equations theory, encompassing both ordinary equations in electromagnetic circuits theory and partial differential equations in field theory. Numerical methods must be employed to solve these differential equations [2, 12].

Until recently, mathematical modeling methods for electrotechnical devices, including asynchronous motors, were primarily considered in a time-invariant domain. These devices were treated as idealized, with linearized electromagnetic interactions, which ultimately led to certain inaccuracies. In other words, the mathematical framework provided only approximate descriptions of physical processes due to the simplified calculations based on the analytical solutions of electromechanical state equations. Modeling rotating electrical machines remains a challenging task, as it requires consideration of phenomena such as core saturation, the rotational motion of electrical circuits, and the skin effect in deep-slot motors. Numerous approaches and model variations have been developed, all relying on differential calculus. Engineering research methods based on equivalent circuit models for asynchronous machines have become outdated and no longer provide sufficiently accurate information on electromechanical quantities. To accurately replicate real processes in electric drives, a model has been developed based on differential equations and the fundamental laws of electrodynamics [3, 4].

As noted in [2, 4], differential equations of traditional asynchronous machine models are often expressed in an implicit form, complicating numerical implementation, especially in the study of long-term processes.

Additionally, operations such as matrix coefficient rotation and the subtraction of similar values (total and active flux linkages) lead to unnecessary computational overhead and the accumulation of errors during numerical integration. In our work, we propose an innovative approach to modeling a low-power asynchronous electric drive using non-stiff differential equations written in the standard Cauchy form. This form greatly simplifies an essential stage of the study: the computational process during computer simulations, thus enabling practical analysis of prolonged dynamic transient processes.

The solution of a differential equation under given initial conditions is the time dependence of the function in the form of an array, the size of which, depending on the step and final integration time, can be $10^3 \ldots 10^5$ points. It is impossible to solve such equations by traditional methods due to the significant amount of computational work, so numerical methods should be used.

Equations with ordinary derivatives are solved by explicit or implicit Euler or Runge-Kutta methods. This is implemented using a PC using appropriate mathematical methods and programming. This is the basis for analyzing the operational characteristics of electrical machines. The developed mathematical model of the asynchronous electric drive will facilitate automated design and the operation of new device models, while the use of computer simulation eliminates the need for physical experimentation.

2. Researches methodology

The methodology for solving electromagnetic state equations relies on the theory of electromagnetic circuits and is efficient in terms of software-based numerical implementation. Currently, an essential task is to construct adequate mathematical models that satisfy the conflicting requirements of mathematically simple yet accurate descriptions of complex physical processes in electrotechnical devices. New models must account not only for individual elements but also for complex electrotechnical systems [4].

This objective can be achieved when physical processes are described by a minimal, coherent system of nonlinear differential equations in Cauchy's normal form, i.e., solved explicitly for the first derivatives of the target functions over time [1, 6].

The foundations of this approach, laid out in [5, 7], remain relevant today. Mathematical models of asynchronous electric drives are classified into four types based on structure: L-, Ψ-, N-, and A-models. Each model is best suited for a specific analysis method. For instance, L-models are ideal for loop current methods, while A-models are suited for node voltage methods, making them practical for analyzing electrotechnical systems with power transformers and motors, i.e., systems with load nodes. After reviewing the current state of mathematical models for asynchronous motors, it is evident that suitable mathematical models meeting modern practical requirements have been developed for motors with squirrel-cage rotors. These models are rigorously based on the principles of electromagnetic circuit theory. In our work, we demonstrate the construction of a model for a low-power asynchronous electric drive.

The differential equations for the electromagnetic and mechanical states are written in Cauchy's normal form, which is convenient for integration using explicit numerical methods, such as Euler's or Runge-Kutta methods. Experience in computer simulation has shown that analyzing the dynamic modes of an asynchronous motor in autonomous modes is best performed based on time-discretized differential equations, following the explicit method principle [9, 10].

To develop the mathematical model, we utilize coordinate transformations from electrical machine theory [8, 11].

The main advantage of applying various coordinate axes is the ability to analyze new variables with constant coefficients. For instance, in N- and Ψ-models, these are total flux linkages, which are less practically significant in electromechanical systems compared to currents, as represented in A-models. In the theory of asynchronous machines, concepts of collinear spatial vectors of the main magnetic field ψ_m and the magnetizing current i_m, are widely used, where their magnitudes are interconnected by the magnetization curve [12, 14]

$$\psi_m = \psi_m\left(I_m\right). \tag{1}$$

The projections of these vectors onto the coordinate axes are considered as their respective components. The choice of these axes simplifies the equations describing the electromechanical state of the asynchronous motor. In this work, we use oblique coordinate axes A and B, along with the corresponding transformation matrices for these axes [8].

$$\Pi = \frac{2}{\sqrt{3}} \begin{vmatrix} -sin\left(\gamma - 120^0\right) & sin\gamma \\ -sin\gamma & sin\left(\gamma + 120^0\right) \end{vmatrix} \tag{2}$$

$$\Pi^{-1} = \frac{2}{\sqrt{3}} \begin{vmatrix} sin\left(\gamma + 120^0\right) & -sin\gamma \\ sin\gamma & -sin\left(\gamma - 120^0\right) \end{vmatrix} \tag{3}$$

where γ is the rotor angle of the machine.

The relationship between phase and transformed variables is expressed as follows:

$$h_{п} = \Pi\, h_{ф}; \qquad h_{ф} = \Pi^{-1} h_{п} \tag{4}$$

The differential equations of the windings in an asynchronous motor, as known, are structurally similar to those for a transformer and are given as:

$$\frac{d\Psi_j}{dt} = U_j - R_j I_j \tag{5}$$

where Ψ_j, U_j, I_j ($j = S, R$) – denote the columns of total flux linkages, voltages, and currents of the stator ($j = S$) and rotor ($j = R$) windings, respectively. By multiplying equation (5) by transformation matrix (2) and taking into account expression (4), we obtain:

$$\frac{d\Psi_j}{dt} = U_j + \Omega_j \Psi_j - R_j I_j \tag{6}$$

where

$$h_j\left(h = \Psi, U, I;\quad j = S, R\right) = \left(h_{jx}, h_{jy}\right)_t \tag{7}$$

The resistance R_j and angular frequency Ω_j matrices have the forms (8) and (9):

$$R_j = \Pi_j R_{фj} \Pi_j^{-1} \tag{8}$$

$$\Omega_j = \Pi_j \frac{d\Pi_j^{-1}}{dt} \tag{9}$$

The current equations are written according to [5, 8]:

$$I_S = \alpha_S\left(\Psi_S - w_S \Phi_S\right) \tag{10}$$

$$I_R = \alpha_R\left(\Psi_R - w_R \Phi_R\right) \tag{11}$$

where α_S, α_R – the inverse leakage inductances of the stator and rotor phase windings; Φ_S, Φ_R – the columns of working fluxes of the windings.

$$\Phi_j\left(j = S, R\right) = \left(\Phi_{jA}, \Phi_{jB}\right)_t \tag{12}$$

Multiplying equations (10) and (11) by transformation matrix (2), considering the expression for $\psi_S = w_S\Phi_S$; $\psi'_R = w_R\Phi_R$, we obtain the equations in transformed coordinates, with the columns of Ψ_j, I_j ($j = S$, R) taking the form (7), and the columns of ψ_j – taking the form:

$$\psi_j\left(j = S, R\right) = \left(\psi_{jx},\ \psi_{jy}\right)_t \tag{13}$$

The use of oblique coordinates is beneficial when they are rig-idly linked to the stator winding.

$$\Omega_S = 0 \tag{14}$$

The rotor angular frequency matrix, according to (2), (3), and (9), is expressed as:

$$\Omega_R = \frac{\omega}{\sqrt{3}}\begin{array}{|c|c|}\hline -1 & -2 \\ \hline 2 & 1 \\ \hline\end{array} \tag{15}$$

Now let's consider the magnetic circuit state equations. The magnetizing current components are defined as the projections of vector i_m onto the A and B axes:

$$I_A = I_m\cos\alpha;\quad I_B = I_m\cos\left(\alpha - 120^0\right) \tag{16}$$

or as the sum of the phase currents along these axes:

$$I_A = \frac{2}{3}\sum_k I_k\cos\alpha_{kx} = I_{SA} + I_{Rx};\quad I_B = \frac{2}{3}\sum_k I_k\cos\alpha_{kz} = I_{SB} + I_{Rz} \tag{17}$$

Forming the magnetizing current column $I_m = \left(I_A, I_B\right)_t$, expressions (17) can be written in matrix form:

$$I_m = I_S + I_R \tag{18}$$

The components of columns (13) are found as the projections of vector ψ_m onto coordinate axes A and B:

$$\begin{gathered}\psi_A = \psi_{SA} = \psi_{Rx} = \psi_m\cos\alpha \\ \psi_B = \psi_{SB} = \psi_{Rz} = \psi_m\cos\left(\alpha - 120^0\right)\end{gathered} \tag{19}$$

By eliminating the trigonometric functions from equations (16) and (19), we obtain:

$$I_A = \tau\psi_A;\qquad I_B = \tau\psi_B \tag{20}$$

where τ is the static specific magnetic resistance, determined by the magnetization curve (1).

$$\tau = \frac{I_m}{\psi_m} = \tau\left(I_m\right) \tag{21}$$

The modulus of the magnetizing current is then:

$$I_m = 2\sqrt{\frac{I_A{}^2 + I_AI_B + I_B{}^2}{3}} \tag{22}$$

where

$$I_A = I_{SA} + I_{RA};\quad I_B = I_{SB} + I_{RB} \tag{23}$$

Considering equation (23), expressions from (20) can be rewritten in matrix form:

$$I_m = \tau\psi = I_S + I_R \tag{24}$$

To the equations describing the electromechanical state, the equation for the mechanical rotational motion of the rotor should be added:

$$\frac{d\omega}{dt} = \frac{p_0}{J}\left(M_E - M\right) \tag{25}$$

$$\frac{d\gamma}{dt} = \omega;\quad n = 9.55\omega \tag{26}$$

where p_0 is the number of magnetic pole pairs; J is the rotor's moment of inertia; M_E is the electromagnetic torque; M is the mechanical torque; γ is the rotor's angular position in electrical radians, n is the rotor's rotational speed. The expression for the electromagnetic torque M_E is presented in [1]:

$$M_E = \sqrt{3}p_0\left(\psi_A I_{SB} - \psi_B I_{SA}\right) \tag{27}$$

or

$$M_E = \sqrt{3}p_0\left(I_{Rx}I_{SB} - I_{Rz}I_{SA}\right)/\tau \tag{28}$$

Equations (6), (10), (11), (24), (25), (26), and (28) constitute the N-model of an asynchronous motor in oblique coordinates. The differential equations are defined in terms of flux linkages in the circuits and are represented in Cauchy's normal form. To perform modeling, parameters such as stator and rotor winding resistances R_S, R_R, inverse leakage inductances α_S, α_R of the windings, the magnetization curve (1) (or the main inverse inductance of the machine α_m), the moment of inertia J, the number of pole pairs p_0, the stator winding voltage U_S, and the static load torque M on the rotor shaft are required.

We now proceed to another model type, the A-model, where differential equations are also presented in Cauchy's normal form, with currents as the differentiating functions.

Differentiating equation (24) with respect to time:

$$\frac{d\tau}{dt}\psi + \tau\frac{d\psi}{dt} = \frac{dI_S}{dt} + \frac{dI_R}{dt} \tag{29}$$

We expand *dτ/dt as* a total derivative.

$$\frac{d\tau}{dt} = \frac{d\tau}{d\psi_m}\left(\frac{d\psi_m}{d\psi_x}\frac{d\psi_x}{dt} + \frac{d\psi_m}{d\psi_y}\frac{d\psi_y}{dt}\right) \tag{30}$$

Differentiating (21) with respect to ψ_m, we obtain an expression:

$$\frac{d\tau}{d\psi_m} = \frac{\rho - \tau}{\psi_m} \tag{31}$$

where ρ is the differential magnetic resistance, determined from the magnetization curve (1):

$$\rho = \frac{dI_m}{d\psi_m} = \rho\left(I_m\right) \tag{32}$$

The total working flux linkage ψ_m is determined similarly to the magnetizing current (22):

$$\psi_m = 2\sqrt{\frac{\psi_A^2 + \psi_A\psi_B + \psi_B^2}{3}} \tag{33}$$

Differentiating (33) with respect to ψ_x, ψ_y according to (21), (22), we obtain the expressions:

$$\begin{gathered}\frac{\partial\psi_m}{\partial\psi_A} = \frac{2}{3}\frac{2I_A + I_B}{I_m} \\ \frac{\partial\psi_m}{\partial\psi_B} = \frac{2}{3}\frac{I_A + 2I_B}{I_m}\end{gathered} \tag{34}$$

Differentiating equations (10) and (11) with respect to time and substituting the results into (6), we obtain:

$$\begin{cases}\dfrac{dI_S}{dt} = \alpha_S\left(U_S + \Omega_S\Psi_S - R_SI_S - \dfrac{d\psi}{dt}\right) \\ \dfrac{dI_R}{dt} = \alpha_R\left(\Omega_R\Psi_R - R_RI_R - \dfrac{d\psi}{dt}\right)\end{cases} \tag{35}$$

After substituting expressions (31), (34), and (35) into (29) and considering (20) and (21), the following differential equation is derived:

$$M\frac{d\psi}{dt} = \alpha_S\left(U_S + \Omega_S\Psi_S - R_SI_S\right) + \alpha_R\left(U_R + \Omega_R\Psi_R - R_RI_R\right) \tag{36}$$

where M is the linkage matrix for the increments of total and working flux linkages:

$$M = \begin{array}{|c|c|}\hline \alpha + \dfrac{2}{3}\dfrac{\rho - \tau}{I_m^2}\left(2I_A^2 + I_AI_B\right) & \dfrac{2}{3}\left(\rho - \tau\right)\left(2I_AI_B + I_A^2\right)/I_m^2 \\ \hline \dfrac{2}{3}\left(\rho - \tau\right)\left(2I_AI_B + I_B^2\right)/I_m^2 & \alpha + \dfrac{2}{3}\dfrac{\rho - \tau}{I_m^2}\left(2I_B^2 + I_AI_B\right) \\ \hline\end{array} \tag{37}$$

where

$$\alpha = \tau + \alpha_S + \alpha_R \tag{38}$$

Finally, bringing (36) to Cauchy's normal form, we obtain:

$$\frac{d\psi}{dt} = G_S\left(U_S + \Omega_S\Psi_S - R_SI_S\right) + G_R\left(\Omega_R\Psi_R - R_RI_R\right) \tag{39}$$

Here, G_S, G_R are the linkage matrices of the working and total flux linkages:

$$G_S = \alpha_S G; \quad G_R = \alpha_R G \tag{40}$$

where

$$G = \begin{array}{|c|c|} \hline T + b_A I_A & b_B I_A \\ \hline b_A I_B & T + b_B I_B \\ \hline \end{array} \tag{41}$$

where

$$b_A = b\left(2I_A + I_B\right); \; b_B = b\left(I_A + 2I_B\right); \; b = \frac{2}{3}\left(R - T\right) / I_m^2 \tag{42}$$

The values of R and T are determined from equations (43).

$$R = \frac{1}{\alpha_S + \alpha_R + \rho}, \quad T = \frac{1}{\alpha_S + \alpha_R + \tau} \tag{43}$$

By substituting (39) into (35) and considering (14), we obtain the differential equations for the asynchronous machine in oblique coordinates:

$$\left.\begin{aligned} \frac{dI_S}{dt} &= A_S\left(U_S - R_S I_S\right) + A_{SR}\left(\Omega_R \Psi_R - R_R I_R\right) \\ \frac{dI_R}{dt} &= A_{RS}\left(U_S - R_S I_S\right) + A_R\left(\Omega_R \Psi_R - R_R I_R\right) \end{aligned}\right\} \tag{44}$$

where

$$A_S = \alpha_S\left(1 - G_S\right); \; A_{SR} = -\alpha_S G_R; \; A_{RS} = -\alpha_R G_S; \; A_R = \alpha_R\left(1 - G_R\right) \tag{45}$$

In equation (44), the column of total flux linkages $\Psi_2 = \left(\Psi_{2A}, \Psi_{2B}\right)_t$ for the rotor winding is represented as shown in (46).

$$\Psi_{Rj} = \frac{I_{Sj} + I_{Rj}}{\alpha_m} + \frac{I_{Rj}}{\alpha_R}; \quad j = A, B \tag{46}$$

Magnetic system saturation is typically accounted for in dynamic and capacitive braking and overvoltage scenarios. Since we do not analyze these modes, saturation is not considered here. In this case, the coefficient matrix (41) simplifies to the scalar form (38). The three-phase asynchronous motor is a symmetrical machine, and the model is formulated for two phases. The network voltages are defined as sinusoidal functions: $u_A = 310 sin \omega t$, $u_B = 310 sin(\omega t\text{-}120°)$.

The algebraic-differential equations (25), (28), (44), and (46) constitute the A-model of the asynchronous motor.

3. Results

The methodology for calculating the inverse inductances and resistances is based on the catalog data of the electric motor and the parameters of the T-shaped equivalent circuit, as shown in works [9, 13]. The calculated parameters in real physical quantities will be used in the simulation of transient processes in the asynchronous motor of the 4A71A2 series.

Specifications: P_{nom} = 0.75 kW, n_{nom} = 2860 rpm, p_0 = 1, η = 0.8, $cos\varphi$ = 0.87, J = 0.008 kg·m², X_μ = 2.6 p.u., R'_1 = 0.12 p.u.

After recalculating according to [9, 13], the following output data for simulation were obtained: α_S = 88.2 H^{-1}, α_R = 31.9 H^{-1}, α_m = 0.93 H^{-1}, R_S = 11.3 Ω, R_R = 5.9 Ω.

For simulating transient processes, specifically for calculating electromechanical quantities, that is, integrating differential equations, the FORCE 2.0 software package was used, and for constructing graphical dependencies, the GRAPHER software package was employed.

The integration of the equations was performed using the explicit numerical Euler method [1, 8] with a time discretization step of 0.02/360. It is assumed that the electric drive is powered by an electrical network of unlimited capacity.

To confirm the theoretical assumptions, the start of the electric drive with a nominal load torque M_{nom} = 2.5 Nm to the regulated speed was simulated. The next simulation of the emergency is the disappearance of the mains voltage at a time of 2 s. Therefore, the operation of the automatic reserve activation after 0.5 s was simulated, which leads to repeated self-starting of the engine. At a time of 2.5 s, an increase in the load on the engine shaft by 50% to a value of 3.75 Nm is simulated. According to classical calculations, the nominal motor current is 1.6 A, and the nominal torque is 2.5 N·m.

Figures 1, 2, 3, and 4 show the calculated time dependences of the rotor rotation speed, the motor stator phase current, and the electromagnetic torque. Figure 5 shows the static mechanical characteristics.

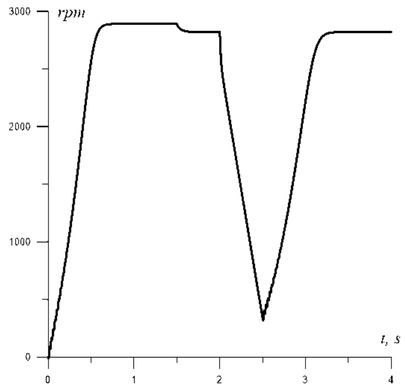

Fig. 1. Rotor speed graph

As can be seen from Fig. 1, the electric drive reaches a steady nominal speed of n = 2885 rpm in 0.8 seconds. When the load on the shaft increases, the speed decreases to 2920 rpm. At the moment t = 2 seconds, the motor shuts down due to an emergency disconnection from the network, and because of the increased torque on the shaft during coast-down, its speed decreases more sharply. After 0.5 seconds, the reserve is activated, and the motor accelerates back to the previous speed.

The amplitude of the starting current in Fig. 2 and Fig. 3 is approximately 15 A, while the effective value is 10.5 A. The starting current multiplicity relative to the nominal is 10.5/1.6 = 6.5, which falls within the range of 5 to 7, thus confirming the theory of direct motor starting. The nominal effective value of the phase current on the graph is 1.5 A, which is close to the calculated nominal value. With the increase in load, the current rises to an amplitude value of 3.2 A, decreases to zero when there is no voltage on the windings, and upon reapplication of voltage, it takes a shape similar to that during starting.

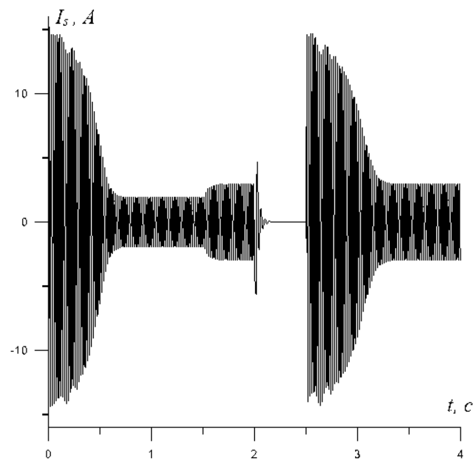

Fig. 2. Stator current dependence during the transient process

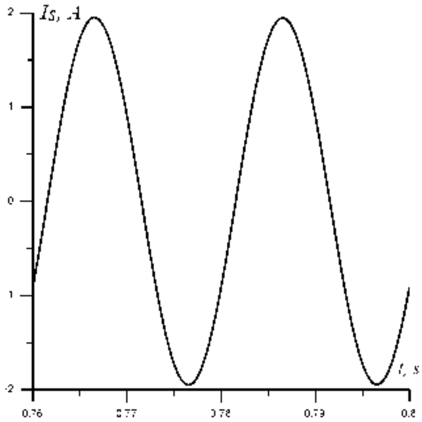

Fig. 3. Stator current in nominal mode for a time interval of 0.76...0.8 s

Figure 4 shows the time dependence of torque. The averaged starting torque during the start is approximately 6 Nm, which is 2.4 times greater than the nominal torque and also aligns with the theory of asynchronous machines. The maximum torque is 8 Nm. At steady speed, the electromagnetic and mechanical torques are balanced.

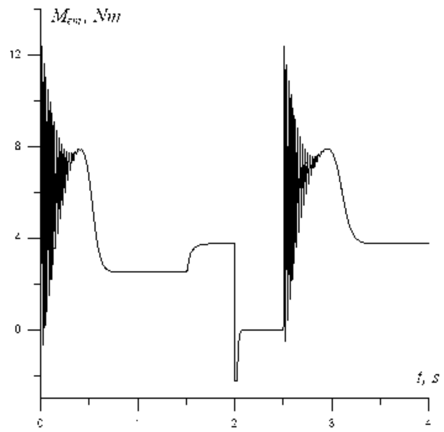

Fig. 4. Electromagnetic torque dependence during the transient process

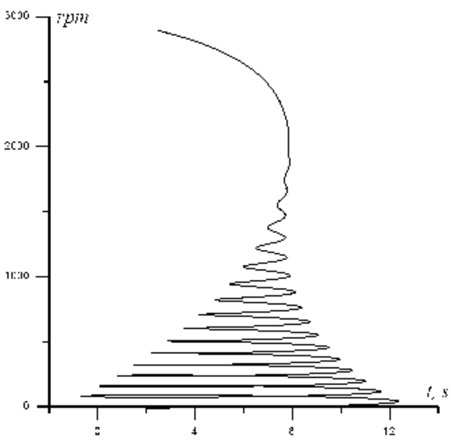

Fig. 5. Mechanical characteristic of the asynchronous motor

The static characteristic shown in Fig. 5 reflects the characteristics $n = f(t)$, $M_{em} = f(t)$ and $t = 0…1.5$ s and corresponds to the mechanical characteristic of the motor, for example, constructed using the Kloss formula.

4. Conclusions

The developed computer program, based on the mathematical model, is efficient in terms of memory usage and computational requirements, and it is capable of performing calculations with predefined accuracy. This mathematical model of the asynchronous motor can be utilized both in standalone mode and for the analysis of an electromechanical system that includes a supplying transformer and other electric motors. To achieve this, the model's equations must incorporate those of a three-phase transformer, as well as the equations governing other motors and the power supply node, where voltage calculations are also performed. The model's input equations facilitate the accurate calculation of quantitative characteristics of the analyzed system, enabling predictions of its behavior under various nominal and emergency conditions. Furthermore, the proposed mathematical model of the asynchronous motor allows for the assessment of how motor parameters, load conditions, and supply voltage impact its mechanical characteristics, starting current, torque, and other relevant quantities.

The proposed mathematical model of an asynchronous motor based on differential equations written in the normal Cauchy form, which takes into account the electromagnetic coupling of electrical circuits and their nonlinearity, allows us to determine the influence of motor parameters, load, and network voltage on its mechanical characteristics, starting current, and torque. The transient processes during start-up, reaching the regulated speed, and changing the load are analyzed, which allows us to determine the optimal operating modes of the motor for its correct operation. It is fair to state that the graphical dependencies coincide with the theoretical assumptions and methods of calculating electric machines in static modes.

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