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# ESTIMATION OF RENEWABLE ENERGY SOURCES UNDER UNCERTAINTY USING FUZZY AHP METHOD

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***Abstract.*** *Renewable energy sources are natural energy forms that are replenished on a human timescale and are seen as more sustainable and eco-friendlier compared to fossil fuels. These resources are crucial for lowering carbon emissions and addressing climate change, ensuring a cleaner and more sustainable energy future. Choosing renewable energy sources in an uncertain environment presents a complex decision-making challenge, as it involves assessing multiple factors under conditions of uncertainty, such as fluctuating energy prices, shifting government policies, varying resource availability (e.g., solar, wind, hydro), and technological advancements. Fuzzy multi-criteria decision-making approaches offer a structured way to evaluate different options based on several criteria while addressing these uncertainties. The region in question experiences a combination of sunny, windy, and rainy days throughout the year, but the availability of solar, wind, and hydro resources is subject to significant uncertainty. Solar energy varies by season and location, and weather patterns are hard to predict. While government incentives exist, they may change overtime. Wind resources can be inconsistent, with the region's average annual wind speed supporting wind power, but occasional periods of low wind intensity. Hydropower, though valuable, is affected by uncertain factors such as water availability, climate change, and regulatory or environmental considerations. In this article, the fuzzy AHP method with fuzzy Z-numbers is employed to assess renewable energy sources such as solar, wind, hydro based on criteria like economic factors, environmental impact, and technical feasibility.*

**Keywords**: renewable energy sources, fuzzy numbers, fuzzy AHP method, Z-numbers, selection under uncertainty

## OCENA ODNAWIALNYCH ŹRÓDEŁ ENERGII W WARUNKACH NIEPEWNOŚCI PRZY UŻYCIU ROZMYTEJ METODY AHP

***Streszczenie.*** *Odnawialne źródła energii to naturalne formy energii, które są uzupełniane w ludzkiej skali czasowej i są postrzegane jako bardziej zrównoważone i przyjazne dla środowiska w porównaniu z paliwami kopalnymi. Zasoby te mają kluczowe znaczenie dla obniżenia emisji dwutlenku węgla i przeciwdziałania zmianom klimatu, zapewniając czystszą i bardziej zrównoważoną przyszłość energetyczną. Wybór odnawialnych źródeł energii w niepewnym środowisku stanowi złożone wyzwanie decyzyjne, ponieważ wiąże się z oceną wielu czynników w warunkach niepewności, takich jak wahania cen energii, zmiany polityki rządowej, różna dostępność zasobów (np. energia słoneczna, wiatrowa, wodna) i postęp technologiczny. Rozmyte wielokryterialne metody podejmowania decyzji oferują ustrukturyzowany sposób oceny różnych opcji w oparciu o kilka kryteriów, przy jednoczesnym uwzględnieniu tych niepewności. Region, o którym mowa, doświadcza kombinacji słonecznych, wietrznych i deszczowych dni przez cały rok, ale dostępność zasobów energii słonecznej, wiatrowej i wodnej podlega znacznej niepewności. Energia słoneczna różni się w zależności od pory roku i lokalizacji, a wzorce pogodowe są trudne do przewidzenia. Chociaż istnieją zachęty rządowe, mogą one ulec zmianie w czasie. Zasoby energii wiatrowej mogą być niespójne, ze średnią roczną prędkością wiatru w regionie wspierającą energię wiatrową, ale sporadycznymi okresami niskiej intensywności wiatru. Energia wodna, choć cenna, jest uzależniona od niepewnych czynników, takich jak dostępność wody, zmiany klimatyczne oraz względy regulacyjne lub środowiskowe. W tym artykule zastosowano rozmytą metodę AHP z rozmytymi liczbami Z do przeprowadzenia oceny odnawialnych źródeł energii, takich jak energia słoneczna, wiatrowa i wodna, w oparciu o kryteria takie jak czynniki ekonomiczne, wpływ na środowisko i wykonalność techniczna*

**Słowa kluczowe:** odnawialne źródła energii, liczby rozmyte, rozmyta metoda AHP, liczby Z, wybór w warunkach niepewności

## Introduction

Renewable energy sources are forms of energy that are naturally replenished on a human timescale. These sources are regarded as more sustainable than fossil fuels since they generate minimal to no greenhouse gas emissions and are more sustainable than fossil fuels. The main types of renewable energy sources are solar energy, wind energy, hydropower (hydroelectric energy) etc. Solar energy is harnessed from the sun using technologies like photovoltaic panels or solar thermal systems. Advantages of this energy abundant and widely available, low environmental impact and zero greenhouse gas emissions during operation, can be used for electricity generation, heating, and even cooling. Challenges of solar energy are intermittent, as it depends on sunlight (i.e., not available at night or on cloudy days), requires a large amount of space for large-scale installations, efficiency can be affected by location and weather conditions. Wind energy is captured through turbines that convert wind movement into electricity. Wind farms can be located on land or offshore. Advantages of wind energy are abundant, particularly in coastal and open areas. Low environmental impact once turbines are installed, can produce large amounts of energy if wind conditions are favorable. The challenges of this energy are intermittent and variable, wind farms may have aesthetic and noise concerns, some locations may not have sufficient wind resources. Hydroelectric energy harnesses the energy from flowing or falling water, typically using dams or watermills, to generate electricity. Advantages of hydropower are highly reliable and can produce consistent energy, can serve as a base load energy source, provides additional benefits such as flood control and irrigation. The challenges of this energy have significant environmental impact, including disruption to ecosystems and fish migration, high initial capital costs for dam construction, suitable locations are limited, as it requires large bodies of water. Selecting the appropriate renewable energy source requires a holistic evaluation of multiple factors, including environmental impact, cost, technological feasibility, resource availability, and social acceptance. Each energy source has its own benefits and drawbacks, and the best choice depends on the special context, including geographical location, available infrastructure, and local policies. By carefully considering these factors and applying decision-making tools like multi-attribute analyzing and cost-benefit evaluation, stakeholders can make decisions that balance short-term requirements with long-term sustainability. The evaluation of renewable energy sources under conditions of uncertainty is a complex decision-making problem because it includes dealing with different criteria that can be difficult to quantify or predict with certainty. These factors include technological advancements, policy changes, economic conditions, environmental impacts, and social acceptance, among others. Uncertainty can arise from both external factors such as regulatory changes or fluctuating energy prices and internal factors like technological innovations or project-specific conditions. To handle these uncertainties, decision-makers can use different methods, such as fuzzy logic, scenario analysis, and multi-criteria decision-making (MCDM) methodologies. Determining suitable renewable energy sources under conditions of uncertainty using Z-numbers is a creative and efficient method for tackling the uncertainty and ambiguity inherent in these decision-making processes. Z-numbers combine fuzzy logic with probability theory to represent uncertain information in a more nuanced and flexible way compared to traditional methods.

Advantages of using Z-numbers for renewable energy selection are handling uncertainty better than traditional methods by combining fuzziness and probability, capturing both imprecision and risk associated with decision-making, making it more flexible, providing a more nuanced evaluation by considering not only the possible outcomes but also the degree of confidence in those outcomes. The structure of the article is formulated as follows. Section 1 presents literature review, and this literature review discusses various studies that have employed fuzzy MCDM methods for renewable energy source selection under conditions of uncertainty, highlighting their contributions, methodologies, and findings. In section 2 the basic definitions of fuzzy numbers and Z-numbers that applied in the selection process are given. In section 3 the technique under Z-numbers that is used for defining the best alternative for renewable energy sources is represented. The conclusion section of the research represents the main results.

## 1. Literature review

The assessment and choice of renewable energy sources is very important in the context of achieving sustainable energy solutions. Given the multiple factors involved environmental, technical, economical, and social the decision-making process becomes highly complex, especially when these factors are uncertain or difficult to quantify. Multi-criteria decision-making techniques, integrated with fuzzy logic to account for uncertainty, have proven effective in navigating such complex decision environments. Choosing the most appropriate renewable energy source depends on multiple criteria such as environmental impact, technical feasibility, economic factors, and social acceptance. Classical methodologies of decision-making usually fail to capture the inherent uncertainties and imprecisions in these criteria. Uncertainty arises from factors like fluctuating market prices, unpredictable resource availability (e.g., wind or solar energy), technological advances, and changing policy environments. To address these uncertainties, decision-makers are turning to fuzzy logic and MCDM methodologies. Fuzzy logic, developed by Lotfi Zadeh in the 1960s, allows decision-makers to handle imprecise, vague, and incomplete information using linguistic terms like "low", "medium" and "high" instead of precise numerical values [20]. A foundational paper by Lotfi Zadeh that introduced fuzzy sets and fuzzy logic, laying the groundwork for its later applications in decision-making, including energy selection. Fuzzy logic helps model the uncertainty inherent in the decision-making problems, allowing for a more flexible and realistic representation of real-world situations. MCDM methodologies are utilized to estimate and rank multiple alternatives on base of several conflicting attributes. These methodologies largely apply in energy-related decision problems, when different attributes are required to be balanced to find the most suitable renewable energy option. The combination of fuzzy logic and MCDM techniques has been increasingly used to renewable energy source selection. By incorporating fuzzy numbers into MCDM frameworks, decision-makers can represent uncertainties more accurately, making the estimation of renewable energy options more realistic and practical. AHP is a widely used MCDM method that helps break down complex decision problems into smaller, more manageable parts. Fuzzy AHP integrates fuzzy logic with the AHP method to account for imprecise or subjective judgment in the pairwise comparison of criteria and alternatives. Several studies have applied Fuzzy AHP to renewable energy source selection under uncertainty. Tasri & Susilawati applied Fuzzy AHP to assess renewable energy sources in Indonesia [14]. They considered factors such as resource availability, environmental impact, and cost. The article determined that wind and solar powers were the most suitable options for region, depending on specific regional conditions. The employing of fuzzy AHP enabled a more accurate reflection of the experts' imprecise judgments. Hwang & Yoon proposed a fuzzy AHP-based approach for evaluating renewable energy sources in Korea, emphasizing cost, environmental benefits, and technology maturity [7]. The results highlighted wind and solar energy as the most attractive options for the region. TOPSIS is another widely used MCDM method, which ranks alternatives stated on their distance from the positive ideal and negative ideal solutions. Fuzzy TOPSIS allows for the incorporation of uncertainty by applying fuzzy numbers to construct the decision matrix. This method has been applied in many renewable energy selection studies. Shengul et al. employed fuzzy TOPSIS to assess renewable energy systems in Turkey [12]. By using attributes like energy efficiency, economical, and environmental impact, the study identified wind and solar power as the optimal sources. Fuzzy TOPSIS allowed the researchers to handle imprecision in experts' evaluations effectively. Fuzzy VIKOR is another MCDM method that aims to determine a compromise solution by considering the best alternatives that minimize distance from the ideal solution. This method has been utilized in many renewable energy selection problems where the criteria are conflicting. Kaya & Kahraman applied fuzzy VIKOR and AHP hybrid method to estimate renewable energy options for rural electrification in Turkey [8]. Wang et al utilized fuzzy VIKOR to select the most appropriate renewable energy source for a sustainable power policy in China [18]. They considered economic, environmental, and technological criteria. The fuzzy VIKOR approach allowed them to integrate experts' subjective judgments and provided a ranking of renewable energy sources, with solar and wind emerging as the top contenders. The ELECTRE method is offered for ranking options on the base of pairwise comparisons and elimination of less-preferred alternatives. Fuzzy approaches of the ELECTRE method can incorporate uncertainties in the decision criteria, and it has been applied to renewable energy decisions. Kaya and Kahraman used the fuzzy TOPSIS methodology in multi-attribute decision making for energy planning [9]. They concluded that wind and solar power were the more viable alternatives, but the decision varied by region due to differences in resource availability. Pohekar & Ramachandran apply MCDM to sustainable energy planning [11]. Tsoutsos et al explores various applications of fuzzy sets in decision-making related to energy policy and renewable energy source selection [17]. Kaygusuz research impact of environmental factor for energy usage and renewable energy politics in Turkey [10]. Terrados et al research regional energy planning utilizing SWOT analysis and long-term decision methods, affect factors on renewable energy development [15] and proposed integrated approach for renewable energy planning [16]. A comprehensive work by Zadeh, exploring the role of fuzzy logic in decision-making, applicable to renewable energy source selection [21]. Balezentiene et al apply fuzzy decision-making techniques to estimate renewable energy sources under conditions of uncertainty, considering various social, environmental, and technical factors [4]. Cristóbal use MCDM methodology in the estimation of a renewable energy planning in Spain [5]. Wang et al propose multi-attribute decision approach in planning sustainable energy [19]. Applying various types of fuzzy numbers such as type-1 fuzzy numbers, fuzzy Z-numbers [1], type-2 fuzzy numbers [3] for evaluation and selection process, organizations systematically estimate alternatives when considering vagueness and uncertainty in decision-making [6]. This approach allows decision-makers to make more aware and strong decisions that are arranged with organizational purposes and precedency [2].

### 1.1. Theoretical framework

**Definition 1.** Fuzzy numbers are a concept from fuzzy logic and fuzzy set theory, employed for representing uncertainty or vagueness in data. Unlike traditional or crisp numbers that have exact values, fuzzy numbers have ranges of possible values, typically represented by membership functions that describe the degree of truth or possibility of each value. A triangular fuzzy number is described by a triplet of values that represent the lower, modal, and upper limits of the number. These bounds form a triangle when plotted graphically. Fig. 1 illustrates the graphical representation of triangular fuzzy numbers.

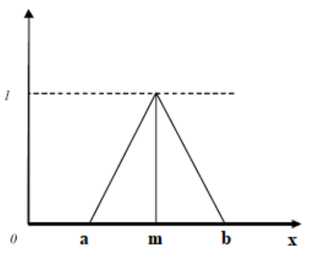


*Fig. 1. A triangular fuzzy number*

In this case $\tilde{A}$ is typically described as $\tilde{A}=(a,m,b)$ where $a$ is the lower limit (the smallest possible value), $m$ is the mode or peak point (the most likely or representative value), and $b$ is the upper bound (the largest possible value). Membership function $\mu_{\tilde{A}}(x)$ of triangular fuzzy number $\tilde{A}=(a,m,b)$ is determined as follows.

$$\mu_{\tilde{A}}(x)=\begin{cases}0, & x\le a\\ \dfrac{x-a}{m-a}, & a<x<m\\ \dfrac{b-x}{b-m}, & m<x<b\\ 0, & x\ge b\end{cases} \tag{1}$$

$a, m, b$ are real numbers and $a<m<b$.

**Definition 2.** The idea of Z-numbers refers to reliability of information [1]. $Z=(A,B)$ – number has two components – $\tilde{A}$ and $\tilde{B}$. The first component ($\tilde{A}$) represents a constraint on the values that can be assumed by a real-valued uncertain variable, and it is fuzzy number (usually triangular) that represents the degree of uncertainty or imprecision. The second component ($\tilde{B}$) indicates the reliability of the first component that determines how confident or certain we are about the fuzzy number. $\tilde{A}$ and $\tilde{B}$ are represented by linguistic terms. [13].

**Definition 3.** $Z$ valued comparison matrix is an extension of traditional pairwise comparison matrix used in decision-making methods, incorporating both fuzzy numbers (for uncertainty) and reliability data in the form of Z-numbers. In decision-making processes, especially in multi-criteria decision analysis, a pairwise comparison matrix is employed for comparing different alternatives or attributes. A traditional pairwise comparison matrix $A$ consists of elements $a_{ij}$ which represent the relative importance of criterion $i$ compared to criterion $j$. In the case of a Z valued pairwise comparison matrix, the elements $a_{ij}$ of the matrix are represented as Z-numbers instead of single crisp values. This allows each element in the matrix to carry both the imprecision (fuzziness) of the comparison as well as the reliability of the judgment. The general formula of a Z-number-valued matrix is represented below.

$$(Z_{ij}=(A_{ij},B_{ij}))=\begin{pmatrix}Z_{11}=(A_{11},B_{11}) & \dots & Z_{1n}=(A_{1n},B_{1n})\\ . & \dots & .\\ Z_{n1}=(A_{n1},B_{n1}) & \dots & Z_{nn}=(A_{nn},B_{nn})\end{pmatrix} \tag{2}$$

**Definition 4.** In the AHP, consistency refers to logical coherence of pairwise comparison judgments made by decision-makers when evaluating the relative significance of different attributes or alternatives. Consistency is important because AHP relies on different pairwise comparisons to derive the final decision. If the comparisons are inconsistent, the derived rankings or priorities may not be reliable. Consistency is estimated using the consistency ratio (CR), which is determined from the consistency index (CI). The CI represents how consistent the pairwise comparisons are relative to a random comparison matrix and for $Z$ numbers it calculated using the formula represented below [24].

$$CI(Z_{ij})=\max_{i<j<k}\min\left\{D\left(Z(1),\left(\frac{Z_{ik}}{Z_{ij}Z_{jk}}\right)\right)D\left(Z(1),\left(\frac{Z_{ij}Z_{ik}}{Z_{jk}}\right)\right)\right\} \tag{3}$$

The consistency ratio is defined by the ratio of consistency index and random consistency index (RI). The RI is determined by the size of the matrix and can be found from a table of values for various matrix sizes. The formula for CR is shown below.

$$CR=CI/RI \tag{4}$$

If CR < 0.10 then the consistency is considered acceptable. The pairwise comparisons are sufficiently consistent to produce reliable results. If CR ≥ 0.10 then the consistency is questionable, and the decision-maker may need to review and revise the comparisons. A high CR indicates that the judgments are inconsistent, leading to potentially unreliable priority values.

**Definition 5.** The Hellinger distance is a metric used to quantify the similarity between two probability distributions. It is commonly utilized in decision making with Z numbers to quantify how different two distributions are. The formula for the Hellinger distance is given below.

$$H(P,Q)=\frac{1}{\sqrt{2}}\sqrt{\sum_i\left(\sqrt{P(i)}-\sqrt{Q(i)}\right)^2} \tag{5}$$

$P(i)$ and $Q(i)$ are the probability mass functions (or density functions in the continuous case) of two distributions $P$ and $Q$.

## 2. Research methodology

The research methodology for selecting renewable energy resources under conditions of uncertainty involves several key steps to ensure that the decision-making process is robust, accurate, and adaptable to varying conditions. A methodology fuzzy AHP for determining the most appropriate renewable energy resources considering uncertainty factors using fuzzy MCDM method. This article focuses on renewable energy resources like solar, wind, and hydro under conditions of uncertainty that may affect the decision-making process (e.g., environmental, technological, economic, social factors). This methodology provides a structured and transparent way to account for both uncertainty and imprecision in the selection of renewable energy resources. The methodology generally involves research design, criterion identification, data collection, data analysis, energy assessment, and decision-making about supplier selection based on the research and analysis results. Methodology typically includes research design, identification of criteria, collection of data, analysis of data, power estimation, decision-making regarding renewable energy selection based on the findings of the investigation and contemplations.

### 2.1. Participants

The selection problem of energy sources involves several stages to evaluate and choose the best energy source based on various criteria under uncertainty. Fuzzy AHP with Z-numbers is particularly useful in this context because it allows for dealing with vagueness or subjectivity in the decision-making process.

Step 1. Problem definition and goal setting. Identifying the decision problem involves clearly defining the goal. For example, the objective could be to select the most suitable energy source such as solar, wind, hydro, etc. based on various factors like cost, sustainability, efficiency, environmental impact, etc.

Step 2. Establishing criteria. Defining the evaluation criteria includes the different factors that will be used to assess the energy sources. For example, criteria might include economic (cost, investment, return on investment), environmental (impact on the environment, sustainability), technological (technological readiness, efficiency), social (acceptance, public opinion) factors.

Step 3. Pairwise comparison of criteria and alternatives. The decision-maker compares the significance of each attribute relative to the others. These comparisons are done using fuzzy numbers instead of crisp values, as it accounts for vagueness or imprecision in judgments. The pairwise comparisons for alternatives are made similarly based on each criterion.

Step 4. Calculating the fuzzy weights. The fuzzy pairwise comparison matrix is normalized to get the fuzzy weights for each criterion and sub-criterion.

Step 5. Synthesis of results. The fuzzy scores of the options are calculated by aggregating the fuzzy weights of the criteria and comparing the alternatives for each attribute. The ranking of options is provided considering aggregated fuzzy scores. This ranking helps in determining the most suitable energy source.

Step 6. Final decision. Based on the analysis, a final decision is made by selecting the energy source that best satisfies the goal and criteria. The selected energy source will have the highest weighted fuzzy score after considering all factors.

## 2.2. Participants

The selection of energy resources is a critical decision-making problem that includes evaluation and estimation of the most suitable energy options based on a range of factors. Decision makers use fuzzy multi-attribute decision making approaches using Z-numbers for estimation alternatives and criteria. Let's consider a simplified example with three energy sources – solar (S), wind (W), and hydro (H) and three criteria economic factors ($C_1$), environmental impact ($C_2$), and technical feasibility ($C_3$). Economic factors in energy resources include different types of costs such as initial cost, operating and maintenance costs, energy production cost, return on investment, upfront investment required for setting up the energy source e.g., building a solar power plant or a wind farm, strategical expenses related to the maintenance and operation of the energy system, cost per unit of energy completed, for example, cost per kilowatt-hour, expected financial return from the energy source over its operational lifetime. Environmental impact includes characteristics such as greenhouse gas emissions, land and resource use, biodiversity impact, sustainability. Greenhouse gas emissions is the amount of $CO_2$ or other harmful gases produced by the energy source (e.g., coal or oil vs. solar or wind). Land and resource use is the amount of land, water, and other resources required for energy production (e.g., land area for wind turbines, water for hydropower). Biodiversity impact includes the potential effects on local ecosystems and wildlife. Sustainability is whether the energy source is renewable (e.g., solar, wind, geothermal) or non-renewable (e.g., fossil fuels). Technical feasibility includes energy efficiency (the conversion efficiency of the energy source in producing usable energy), technology readiness (the level of technological development and maturity of the energy source) and grid compatibility (how easily the energy produced can be integrated into the existing power grid). Using Fuzzy AHP with Z-numbers allows us to handle the vagueness and ambiguity intrinsic in such decisions, leading to a more informed and reliable choice of energy source.

### 2.2.1. Solution of problem

The solution to the evaluation and selection problem for energy sources is a complicated, multi-attribute decision-making process. By using multicriteria fuzzy AHP with Z-numbers, we can systematically and transparently evaluate the various energy sources based on multiple criteria, including economic factors, environmental impact, and technical feasibility. The importance of weights of various attributes and the scores of qualitative defined by linguistic terms. In linguistic terms, when discussing the importance of weight in a fuzzy or vague sense, the realm of fuzzy semantics and gradience is entered. This typically involves how elements of meaning or structure can have variable, context-dependent importance or weight rather than fixed, binary values. Linguistic terms for the first part of Z-number that are determined by triangular fuzzy numbers are presented below.

Equally – (1, 1, 1; 1),
Very low – (1, 2, 3; 1),
Low – (2, 3, 4; 1),
Ordinary – (3, 4, 5; 1),
Moderate – (4, 5, 6; 1),
High – (5, 6, 7; 1),
Very high – (6, 7, 8; 1),
Absolutely – (7, 8, 9; 1),
Extremely – (8, 9, 10; 1).

Linguistic terms for the second (reliability) part of Z-fuzzy numbers are shown below.

Absolutely reliable – (1, 1, 1; 1),
Powerful reliable – (0.7, 0.8, 0.9; 1),
Very much reliable – (0.6, 0.7, 0.8; 1),
Much reliable – (0.5, 0.6, 0.7; 1),
Reliable – (0.4, 0.5, 0.6; 1),
Less reliable – (0.3, 0.4, 0.5; 1),
Very low reliable – (0.2, 0.3, 0.4; 1),
High unreliable – (0.1, 0.2, 0.3; 1),
Absolutely unreliable – (0, 0.1, 0.2; 1)

The relative necessity of these linguistic terms must align with the opinions and preferences of the decision-makers. Economic factors of solar panels moderately high, for wind turbines high, and for hydropower very high. Environmental impact factor affects solar is low, for wind energy moderate, for hydropower high. Technical feasibility factors for solar panels moderate, for wind energy moderate, and for hydropower is high. Basic steps of solving selection problems are shown below.

Step 1. Structuring decision matrix is presented in table 1.

*Table 1. Decision matrix of alternatives and criteria*

| | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|
| *S* | (5, 6, 7; 0.5,0.6,0.7) | (2, 3, 4; 0.4,0.5,0.6) | (4, 5, 6; 0.6,0.7,0.8) |
| *W* | (5, 6, 7; 0.4,0.5,0.6) | (4, 5, 6; 0.3,0.4,0.5) | (4, 5, 6; 0.5,0.6,0.7) |
| *H* | (6, 7, 8; 0.5,0.6,0.7) | (5, 6, 7; 0.4,0.5,0.6) | (5,6,7; 0.3,0.4,0.5) |

Step-2. Structuring pairwise comparison matrix of criteria as shown in table 2.

*Table 2. Pairwise comparison matrix*

| | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|
| $C_1$ | (1, 1, 1; 0.5,0.6,0.7) | (3, 4, 5; 0.5, 0.6, 0.7) | (0.2,0.25,0.29; 0.5, 0.6, 0.7) |
| $C_2$ | (0.2,0.25,0.33; 0.5, 0.6, 0.7) | (1, 1, 1; 0.5,0.6,0.7) | (0.2,0.25,0.33; 0.5, 0.6, 0.7) |
| $C_3$ | (3.5, 4, 4.5; 0.5, 0.6, 0.7) | (3, 4, 5; 0.5, 0.6, 0.7) | (1, 1, 1; 0.5,0.6,0.7) |

Step 3. Determining weights of criteria. Using Z-numbers in determining weights of criteria allows to incorporate both subjective assessments and certainty levels into decision-making process. This method is particularly useful when decision-makers are uncertain about the comparisons or have varying levels of confidence about the importance of different criteria. Unlike fuzzy numbers which represent uncertainty in terms of a single value and a membership function, Z-numbers are a generalized form that incorporates both the accuracy (confidence level) and possibility (degree of belief or uncertainty) about a particular assessment or value. Appropriate methodology, such as the eigenvector approach, is utilized to determine the weights for the criteria. Finding eigenvalues of Z-matrices involves solving the characteristic equation in the context of Z-numbers. This means extending the concept of determinants and characteristic polynomials to Z-numbers. The computational

approach often involves the use of algorithms designed for fuzzy systems, where operations on Z-numbers are defined and used to compute the eigenvalues and eigenvectors. Once the eigenvalues are found, the corresponding eigenvectors are determined by solving the equation $Zv = \lambda v$ for each eigenvalue $\lambda$. In this equation, $Z$ is $n \times n$ matrix, $\lambda$ is a scalar (eigenvalue), and $v$ is eigenvector. Using Z-lab program we define eigenvectors that represent weights of criteria. Weight of economic factors criteria ($C_1$) is

$$w_1 = (0.2019, 0.2027, 0.2028; 0.6091, 0.7701, 0.7873)$$

Weight of environmental impact criteria ($C_2$) is

$$w_2 = (0.0941, 0.0944\ 0.0945; 0.6223, 0.7328, 0.7371)$$

Weight of technical feasibility criteria ($C_3$) is

$$w_3 = (0.7007, 0.7029, 0.704; 0.5356, 0.6781, 0.6822)$$

Step 4. Normalization of decision matrix. This process ensures that the performance values are comparable across criteria and alternatives. The goal is to handle the fuzziness of the input data and derive a consistent ranking of alternatives for decision-making.

*Table 3. Normalized decision matrix with Z-numbers*

| | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|
| $S$ | (0.5,0.7,0.9; 0.5,0.6,0.7) | (0.01,0.02,0.03; 0.4,0.5,0.6) | (0.01,0.1,0.3; 0.6,0.7,0.8) |
| $W$ | (0.5,0.7,0.9; 0.4,0.5,0.6) | (0.3, 0.5, 0.7; 0.3,0.4,0.5) | (0.01,0.2,0.4; 0.5,0.6,0.7) |
| $H$ | (0.3,0.4,0.5; 0.5,0.6,0.7) | (0.1,0.3,0.5; 0.4,0.5,0.6) | (0.3,0.4,0.5; 0.3,0.4,0.5) |

Step 5. Normalization of weighted decision matrix. This is a matrix that incorporates both the normalized performance of alternatives across criteria and the weights assigned to each criterion. This matrix is used for aggregating the scores for each alternative, and eventually, rank them. Normalizing weighted decision matrix represented below.

*Table 3. Weighted normalized decision matrix*

| | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|
| $S$ | (0.1,0.15,0.19; 0.34,0.46,0.55) | (0.00,0.002,0.003; 0.26,0.34,0.41) | (0.01,0.15,0.26; 0.35,0.51,0.55) |
| $W$ | (0.1,0.15,0.19; 0.27,0.39,0.47) | (0.03,0.05, 0.07; 0.21,0.32,0.4) | (0.01,0.12,0.26; 0.3,0.44,0.52) |
| $H$ | (0.05,0.06,0.14; 0.34,0.46,0.55) | (0.01,0.03,0.05; 0.28,0.4,0.48) | (0.17,0.31,0.47; 0.19,0.3,0.37) |

Step 6. Ranking alternatives. For ranking alternatives, we determine Hellinger distance for each alternative. The Hellinger distance is a metric that measures the similarity (or dissimilarity) between two probability distributions. In the context of ranking alternatives in decision-making processes, the Hellinger distance can be used to measure the distance between the normalized and weighted scores of each alternative and some idealized or reference distribution. This helps in ranking the alternatives by determining how close each one is to the ideal. The Hellinger distance is particularly useful when comparing the distributions of scores for different alternatives and can be helpful in decision-making models that use fuzzy logic or probabilistic distributions. Using formula (5) Hellinger distance for each option is determined.

Proximity between option $S$ and Z (1) is 2.43.
Proximity between option $W$ and Z (1) is 2.52.
Proximity between option $H$ and Z (1) is 2.47.

Result for solar energy is $S$ = 2.43.
Result for wind energy is $W$ = 2.52.
Result for hydropower is $H$ = 2.47.

The ranking indicate that solar energy is the most suitable, followed by hydropower and wind energy, depending on the weights assigned to each criterion.

This methodology allows for a systematic, transparent, and flexible decision-making process in renewable energy resource selection, which is particularly useful under conditions of uncertainty.

## 3. Discussion and conclusion

The estimation of renewable energy sources under uncertainty is a critical task for policymakers and energy planners. This study demonstrates the application of the fuzzy AHP method as a powerful tool to manage and evaluate the inherent uncertainties in decision-making processes related to renewable energy investments and implementation. This article focuses on the evaluation and ranking of renewable energy resources under vague conditions, using a fuzzy AHP methodology. Considering the growing significance of sustainable energy solutions and the necessity for informed decision-making amid uncertainty, this article presents an effective framework for evaluating and ranking renewable energy options based on various attributes. The fuzzy MCDM approach successfully addresses the inherent uncertainty in evaluating renewable energy resources. Traditional decision-making methods often struggle with subjective judgments and imprecise data, especially when dealing with environmental, economic, and social factors. By employing fuzzy sets, the approach captures and models this uncertainty, providing more robust and realistic evaluations. Comprehensive evaluation with multiple criteria considers various criteria such as economic factors, environmental affect, technical feasibility, social acceptance, which are crucial for the evaluation of renewable energy resources. Utilizing fuzzy pairwise comparisons, it allows stakeholders to express preferences in a more flexible manner, thus incorporating expert knowledge and experience into the decision-making process. The application of AHP, combined with fuzzy logic, enables the systematic comparison of alternatives. The study demonstrates how these methods can be used to derive a clear ranking of renewable energy options, even when dealing with fuzzy or vague data from decision-makers. The fuzzy MCDM framework provides decision-makers with a clearer understanding of the trade-offs involved in evaluating the best renewable energy resource. It highlights the importance of considering both technical and socio-economic factors, thus facilitating better-informed decisions in energy policy formulation. The findings have important implications for policymakers, as they offer a structured approach to selecting renewable energy resources that aligns with sustainable development goals. The results can be used to guide investment in renewable energy infrastructure, ensuring that the chosen alternatives provide the greatest benefits in terms of cost-effectiveness, environmental sustainability, and social acceptability. The paper also opens avenues for future research, such as incorporating more advanced fuzzy decision-making techniques, incorporating new criteria (e.g., public health), and applying the framework to other regions or countries with different energy landscapes. Additionally, it suggests integrating dynamic and evolving uncertainties over time, considering technological advancements in renewable energy. The fuzzy MCDM approach used in this study provides a powerful tool for the selection of renewable energy resources under vagueness. It bridges the gap between subjective human judgments and objective decision-making by offering a flexible, scalable, and comprehensive framework. As the world continues to shift towards cleaner energy solutions, the methods explored in this research can help guide policymakers and stakeholders toward more effective power choices. The selection of renewable energy sources in the context of uncertain environments is a complex decision-making task that involves multiple conflicting criteria such as cost, environmental impact, resource availability, and technological feasibility. Traditionally, multi-criteria decision-making methods have limitations when it comes to handling the inherent vagueness and imprecision involved in real-world energy planning, particularly when experts' judgments or data are vague or incomplete. To address this issue, Z-numbers (a combination of fuzzy numbers and probabilistic uncertainty) have emerged as a powerful tool for modeling uncertainty in decision-making. Z-numbers offer an improved framework by allowing for both fuzzy membership (imprecision) and probabilistic certainty (confidence) in decision parameters. This framework is particularly suitable for energy planning, where both imprecisions e.g., high cost, medium efficiency and uncertainty e.g., unknown future prices, technological advances, or resource availability play significant roles. In renewable energy selection, the traditional approach of using crisp values to evaluate attributes such as economic factors, environmental impact, and social acceptance is often insufficient to capture the vagueness and uncertainty associated with such values. Fuzzy numbers allow for representing linguistic expressions e.g., "medium cost" or "low environmental impact", but they do not account for how certain or uncertain those values are. Z-numbers overcome this limitation by adding a probabilistic element, which expresses the degree of confidence or reliability in the fuzzy number's accuracy. The use of Z-numbers in the selection of renewable energy sources offers a novel and robust approach for dealing with uncertainty in decision-making. By integrating fuzzy logic to represent imprecision and probabilistic values to quantify uncertainty, Z-numbers offer a more realistic and flexible representation of decision criteria in renewable energy planning. This approach enhances the decision-making process by allowing for better modeling of expert knowledge and real-world uncertainties, leading to more informed and reliable decisions. Key benefits of using Z-numbers include more accurate representation of uncertainty and imprecision, enhanced flexibility in decision models, improved support for policymakers in identifying optimal renewable energy sources. However, the approach also faces challenges such as the complexity of data collection, the subjectivity of expert input, and the computational demands of working with Z-numbers.

In this paper, evaluation and selection of appropriate energy sources using Z-numbers is characterized by various attributes such as, economic factors, environmental impact, and technical feasibility. For making decision on energy sources evaluation is utilized fuzzy aggregation methodology under Z-numbers considering high vagueness as suitable for the problem. Three alternatives - solar energy, wind energy, and hydropower (*S*, *W*, *H*) and three criteria economic factors, environmental impact, and technical feasibility ($C_1$, $C_2$, and $C_3$) is utilized for defining best energy source for region. By employing the methodology with Z-numbers, it was defined that solar energy is the most appropriate energy source for region. In conclusion, the fuzzy AHP method provides a robust framework for estimating and evaluating renewable energy sources under uncertainty. By incorporating fuzzy logic, this method accounts for uncertainties and imprecision in the decision-making process, making it particularly suitable for evaluating renewable energy projects that involve complex trade-offs and multiple uncertainties. Considering both uncertainties and multiple criteria in renewable energy decision-making, the fuzzy AHP method proves to be an effective tool in the sustainable energy transition, helping to pave the way for cleaner, more efficient, and socially accepted energy systems.

For comparing of this article results research application of type-2 fuzzy TOPSIS method for estimating renewable energy sources research is being worked on. The TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) method is a popular multi-criteria decision analysis approach, often used to rank and choose alternatives based on distance to an ideal solution. In the context of renewable energy, this could involve evaluating various renewable energy sources based on various criteria. Type-2 fuzzy sets are more advanced and handle uncertainty better. By using Type-2 Fuzzy TOPSIS, this method will provide a more reliable and precise evaluation of renewable energy sources, helping decision-makers choose the most appropriate technology based on multiple factors under uncertainty.

Future research could focus on standardizing Z-number applications in renewable energy selection, developing hybrid models that combine Z-numbers with other decision-support tools, and improving computational methods to handle large-scale energy planning problems more efficiently.

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