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EVALUATING MODIFIED PAIRING INSERTION HEURISTICS FOR EFFICIENT DIAL-A-RIDE PROBLEM SOLUTIONS IN HEALTHCARE LOGISTICS

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Abstract. *A subset of the Vehicle Routing Problem (VRP), the Dial-a-Ride Problem (DARP) is concerned with the effective route planning of cars employed to pick up and deliver passengers to designated destinations. For the transportation of elderly or incapacitated patients in Apizaco, Tlaxcala, Mexico, this research suggests using DARP. We propose to tackle this problem, a mathematical programming model and an insertion heuristic as a solution method. The objective is to optimize the trip time while adhering to the problem constraints. We ran a number of trials in different settings, accounting for different model parameter values. The outcomes demonstrate notable progress, with each of the generated routes having the least trip times. For instance, in a scenario with 20 patients, increasing vehicle speed from 30 km/h to 60 km/h reduced the total travel time from 30.66 minutes to 15.11 minutes. This significant improvement underscores the computational efficiency and practical applicability of the proposed heuristic approach in patient transportation systems.*

Keywords: dial-a-ride problem, healthcare logistics, insertion heuristic, patient transportation

OCENA ZMODYFIKOWANYCH HEURYSTYK WSTAWIANIA PAR W CELU WYSTARCZAJĄCEGO ROZWIĄZANIA PROBLEMU TRANSPORTU NA ŻĄDANIE W LOGISTYCE OPIEKI ZDROWOTNEJ

Streszczenie*. Problem transportu na żądanie (Dial-a-Ride Problem – DARP), będący podzbiorem problemu wyznaczania trasy pojazdu (Vehicle Routing Problem – VRP), dotyczy efektywnego planowania tras samochodów wykorzystywanych do odbierania i dostarczania pasażerów do wyznaczonych miejsc docelowych. W przypadku transportu osób starszych lub niepełnosprawnych w Apizaco w stanie Tlaxcala w Meksyku, badania sugerują zastosowanie problemu DARP. Proponujemy rozwiązanie tego problemu za pomocą modelu programowania matematycznego i heurystyki wstawiania jako metody rozwiązania. Celem jest optymalizacja czasu podróży przy zachowaniu ograniczeń związanych z problemem. Przeprowadziliśmy szereg prób w różnych warunkach, uwzględniając różne wartości parametrów modelu. Wyniki pokazują znaczący postęp, a każda z wygenerowanych tras charakteryzuje się najkrótszym czasem podróży. Na przykład w scenariuszu z 20 pacjentami zwiększenie prędkości pojazdu z 30 km/h do 60 km/h skróciło całkowity czas podróży z 30,66 minut do 15,11 minut. Ta znacząca poprawa podkreśla wydajność obliczeniową i praktyczną przydatność proponowanego podejścia heurystycznego w systemach transportu pacjentów.*

Słowa kluczowe: problem z transportem na żądanie, logistyka opieki zdrowotnej, heurystyka wstawiania, transport pacjentów

Introduction

Patient transportation for medical appointments is a significant logistical challenge, particularly in densely populated urban areas such as Apizaco, Tlaxcala. The primary objective is to minimize patients' waiting and travel times while adhering to vehicle capacity and time constraints. The Dial-a-Ride Problem (DARP) is a valuable tool for addressing this issue, as it enables the efficient planning of vehicle routes for picking up and dropping off passengers at specific locations within scheduled time frames.

The Dial-a-Ride Problem (DARP) was developed in the early 1980s to design bus routes based on passenger requests [17]. It is an NP-hard optimization problem and a variant of the pickup and delivery problem with time constraints.

Extending the vehicle routing problem with time windows to include pickups and deliveries increases the complexity of the problem, as the challenge shifts from transporting goods to moving them between multiple points. This extension eliminates the single origin or destination point typically seen in business contexts, making the problem more complex in real-world applications.

An important distinguishing feature of the Dial-a-Ride Problem is its application to passenger transportation, where "service quality" becomes a critical factor. This refers to minimizing the time passengers spend in the vehicle. Unlike freight transportation, where travel time is less critical, unnecessarily extending travel times in passenger transport leads to significant dissatisfaction.

In the DARP, vehicles are required to depart from a central depot, perform multiple pickups and drop-offs, and return to the depot. Effective management of these operations is essential to meet patient needs while optimizing resource utilization and adhering to operational constraints Fig. 1.

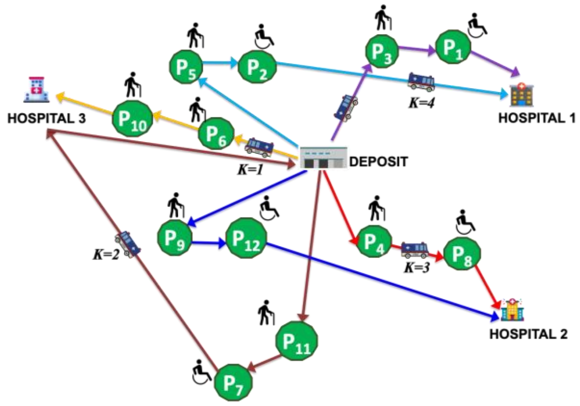

Fig. 1. Transportation of patients to hospitals

1. Literature review

The Dial-a-Ride Problem (DARP) is an extension of the Vehicle Routing Problem (VRP) that incorporates unique features, such as the need to pick up and drop off passengers at specific locations within predefined time windows. This problem has been extensively studied in the literature, with the aim of developing models and algorithms that balance operational efficiency and service quality [4, 11].

The DARP has found significant applications in transporting elderly individuals, patients with reduced mobility, and users with specific needs. Studies by Toth and Vigo and by Masson et al. highlight how vehicle capacity constraints, travel time, and compatibility with specialized equipment affect route design. These works also explore the use of heterogeneous fleets and integrated transport systems to improve operational efficiency [14, 23].

In the field of medical services, Beaudry et al. and Parragh et al. have investigated route planning for patient transportation, emphasizing the additional challenges posed by prolonged waiting times, the need for personal assistance, and specific vehicle characteristics [1, 16]. Moreover, insertion heuristics, such as the Modified Pairing Insertion (MPI), have proven to be an effective methodology for solving moderate-sized problems. This technique, rooted in studies by Psaraftis and by Kubo and Kasugai, enables the generation of high-quality initial solutions that can be further refined using local optimization techniques [13, 17].

One of the earliest works on this subject is that of Psaraftis, who describes the application of exact algorithms and dynamic programming techniques to the dial-a-ride problem with multiple vehicles and time-varying windows [17, 18]. In a study of a patient transport issue in Austria, Tóth, Hajba, and Horváth used a mixed-integer linear programming (MILP) model and a large-neighborhood search method to solve the problem while accounting for vehicle capacity and drivers' rest times [22]. Kergosien et al. conduct research on a tabu search heuristic for dynamic patient movement between care units, with an emphasis on cost-effectiveness and efficiency [12]. Queiroz et al. investigate dynamic patient scheduling in emergency departments using advanced optimization techniques [19]. Vidal, Laporte, and Matl provide a concise guide on existing and emerging solutions to the vehicle routing problem, including the DARP, with an emphasis on service quality and efficiency [24]. In their analysis of issues relevant to human transportation, Doerner and Salazar-González highlight methodologies and applications within the DARP framework [5]. Conversely, Melachrinoudis, Ilhan, and Min offer a solution to the dial-a-ride problem for client transportation in a health-care organization, utilizing heuristic algorithms to increase efficiency [15].

Recent contributions have significantly expanded the modeling of DARP by integrating user-centric, operational, and economic perspectives. Gkiotsalitis and Nikolopoulou introduced an extended formulation with vehicle interchange points and perceived passenger travel times, capturing the disutility associated with vehicle crowding [10]. Dong, Rey, and Waller incorporated user utility-based accept/reject decisions, enhancing demand modeling by accounting for the relative attractiveness of travel modes [8]. Schenekemberg et al. examined hybrid logistics strategies that integrate private fleets with common carriers to address peak demand scenarios through outsourcing [20]. Sharif Azadeh et al. proposed a choice-driven DARP that incorporates profit maximization and assortment optimization, dynamically aligning service offerings with user preferences [21]. Furthermore, Dong et al. formulated a chance-constrained DARP with utility-maximizing demand and multiple pricing structures, offering a flexible framework for strategic planning in demand-responsive transport systems [6].

The following lists the current trends and advancements that have been studied.

Models of Integrated Services: Integration has potential to address both structural and attitudinal barriers to care delivery and to challenge stigma by communicating that health cannot be compartmentalized into physical and mental components. Specifically, stepped collaborative care interventions have been demonstrated to be feasible and effective in improving access to behavioural health services, outcomes, and patient and family satisfaction relative to existing care models [2].

Programing and tracking technology: The development of programming and tracking technologies has revolutionized how patient transport trips are organized and carried out. Mobile applications, geolocation information systems, and online platforms enable more effective coordination, real-time communication, and patient tracking throughout the entire transportation process [7].

Community-Based Transportation: Community-based transportation programs that mobilize local resources and volunteers to meet patients' transportation needs have proven to be successful in improving access to medical care in underserved rural and urban areas [3].

Transportation Tailored to Specific Populations: Transportation programs tailored to specific populations, such as elderly adults, patients with disabilities, and those with low incomes, have been developed. These programs aim to address the unique barriers these groups face in order to receive medical care and enhance their overall transportation experience [25].

Impact and Cost-Effectiveness Evaluation: The impact and cost-effectiveness evaluation studies have provided evidence about the advantages and viability of patient transportation programs. These studies highlight the significance of taking into account clinical outcomes, patient satisfaction, and associated costs when assessing the effectiveness of transportation interventions [26].

2. Problem description

In this context, the Dial-a-Ride Problem (DARP) involves route planning for a set of vehicles tasked with picking up patients from their homes and transporting them to medical appointments. Each patient has a specific origin and destination, as well as a designated time window during which they must be picked up and dropped off. Additionally, the vehicles have limited capacity and must start and end their routes at a central depot.

This proposal places particular emphasis on elderly patients and those with disabilities. These groups present additional logistical challenges due to their specific mobility and assistance needs. Elderly patients often require extra time to board and disembark from the vehicle, as well as additional support to ensure their comfort and safety during the journey. Moreover, they may have regular medical appointments, necessitating reliable and punctual transportation.

Patients with disabilities, on the other hand, may require vehicles equipped with ramps or wheelchair lifts, specialized seating, and additional space for medical devices. Route planning must account for these needs to ensure that the assigned vehicles are appropriately equipped and that the additional time required for assistance does not adversely affect appointment schedules.

Time constraints are critical in this context, as delays can have severe health consequences for patients. Therefore, scheduling must be flexible enough to accommodate variations in service time required for each patient while simultaneously minimizing waiting and travel times.

Vehicle capacity is another significant limiting factor. Vehicles must have sufficient space to accommodate patients and their companions, if necessary, as well as any medical or mobility equipment required. This requires careful planning to prevent vehicle overloading and to ensure that all patients arrive at their appointments on time.

3. Method

Given a complete directed graph $G(V,E)$, consisting of a set V of n nodes and a set of arcs E:

- The set of nodes V can be divided into three subsets: the depot v_0, pickup points v^p_l, $l = 1,2,\ldots,m$, and delivery points v^d_l, $l = 1,2,\ldots,m$. The node v_0 represents the depot, which serves as the starting and ending point for the vehicle.
- There are m clients, each with a pair of pickup and delivery points (v^p_l, v^d_l), $l = 1,2,\ldots,m$, where v^p_l is the pickup point and v^d_l is the delivery point for client l.
- The total number of nodes is denoted as $n = |V| = 2m+1$. Associated with each arc $(i,j) \in E$, there is a weight D_{ij} which represents the distance between nodes i and j.

The objective is to find a permutation of nodes that minimizes the total travel distance:

$$\min \sum_{i=0}^{n} \left(D_{i,(i+1)}\right) + D_{n,1} \quad (1)$$

Subject to the following constraints:

$$v_0 = 1 \quad (2)$$

$$v_l^p < v_l^d, \forall l = 1,2,\dots,m \quad (3)$$

Constraint 3 specifies that the pickup point $v^p{}_l$ must be visited before the corresponding delivery point $v^d{}_l$

3.1. Parameters, variables, and model notation

The mathematical model formulation is described using the following notation:

Sets and Parameters:

n – Total number of locations (pickup and delivery points).

K – Total number of vehicles.

d_{ij}j – Total number of vehicles.between points i and j.

Q – Maximum capacity of each vehicle.

P – Set of pickup and delivery pairs. For example, if a patient is picked up at point p and dropped off at point d, then $(p,d) \in P$.

Decision Variables:

t_i – Time at which the vehicle arrives at point i.

q_i – Load of the vehicle upon arrival at point i (number of patients in the vehicle at that point).

x_{ijk} – Binary variable, equal to 1 if vehicle k travels directly from node i to node j, and 0 otherwise. This variable determines the sequence of nodes visited by each vehicle.

Objective Function: The objective function aims to minimize the total travel time of all vehicles' routes:

$$\min \sum_{k=1}^{K} \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} \cdot x_{ijk} \quad (4)$$

Flow Constraint for Each Pickup and Delivery Point. Each point must be visited exactly once by a vehicle.

$$\sum_{k=1}^{K} \sum_{j=1}^{n} x_{ijk} = 1, \ \forall i = 1,\dots,n \quad (5)$$

Flow Conservation Constraint: If a vehicle enters a point, it must also exit that point.

$$\sum_{j=1}^{n} x_{ijk} = \sum_{j=1}^{n} x_{jik}, \ \forall k = 1,\dots,K, i = 1,\dots,n \quad (6)$$

Capacity Constraint: The vehicle load at any given time must not exceed its maximum capacity.

$$0 \le q_i \le Q, \ \forall i = 1,\dots,n \quad (7)$$

Load Conservation Constraint: When a patient is picked up, the vehicle's load increases by 1, and when the patient is dropped off, the load decreases by 1. For the pickup point p and delivery point d of the same patient:

$$q_d = q_p - 1, \forall (p,d) \in P \quad (8)$$

Subtour Elimination Constraint: To avoid subtours, the arrival time at each point must respect the sequential order of the route:

$$t_j \ge t_i + d_{ij} - M \cdot \left(1 - x_{ijk}\right), \forall i,j = 1,\dots,n, k = 1,\dots,K \quad (9)$$

Where M is a sufficiently large constant.

Domain of Variables:

$$x_{ijk} \in \{0,1\}, \forall i,j = 1,\dots,n, k = 1,\dots,K$$
$$t_i \ge 0, \forall i = 1,\dots,n, \ q_i \ge 0, \forall i = 1,\dots,n \quad (10)$$

3.2. Solution method

Although the model's objective function optimizes total travel time, the arc costs (d_{ij}) used in the algorithm are directly based on Euclidean or network distances between geographic coordinates. As travel time is calculated from distance and average vehicle speed (i.e., time = distance/speed), distance remains a fundamental component embedded in the solution structure. Therefore, minimizing travel time inherently incorporates distance, and the proposed Modified Pairing Insertion (MPI) heuristic consistently preserves this relationship when evaluating routing decisions. This approach aligns with foundational DARP formulations, where distance and ride time are tightly linked [17], and where route length is explicitly included as a key quality-of-service criterion [4]. Furthermore, recent models confirm that routing cost is normally proportional to geographical distance and/or travel time [8], reinforcing the relevance of distance as a primary element in travel time optimization.

Most combinatorial optimization problems are associated with complex scenarios where finding a solution is challenging. The Dial-a-Ride Problem (DARP) is NP-hard, meaning it does not have an exact solution strategy that can be executed in acceptable computational time.

For this case, an insertion heuristic is used to generate solutions without relying on any prior state while satisfying all the constraints defined in the model formulation. This heuristic is chosen for its simplicity and efficiency, as it constructs an initial solution by iteratively inserting nodes into a vehicle's route until all demands are met.

The heuristic insertion employed is the algorithm known as Modified Pairing Insertion (MPI), which simultaneously inserts a pair of pickup and delivery points [13] (see Fig. 2).

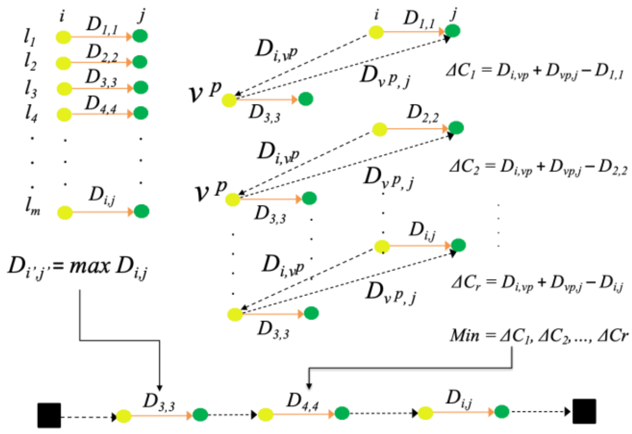

Fig. 2. Proposed solution methodology – modified pairing insertion (MPI)

3.3. Steps of the Modified Pairing Insertion (MPI) Algorithm

Step 1: Initialization

Begin with a subtour consisting solely of the depot node V_o.

Step 2: Pair Selection

Given the current subtour, select a pair of pickup and delivery points (v^p,v^d) that are not yet included in the subtour.

$$D_{i',j'} = \max D_{i,j}, \text{ where } i \in N_p, j \in N_d \quad (11)$$

Step 3: Insertion

Find the K pairs that minimize $D_{i,vp} + D_{vp,j} - D_{i,j}$ and denote them as:

$$(i_1,j_1), (i_1,j_1), \dots, (i_k,j_k), \dots, (i_K,j_K)$$

Step 4: Repeat

Return to Step 3 until all demands are satisfied and a complete route is constructed.

Fig. 3 presents the algorithm used to solve the vehicle routing problem (DARP) using an insertion heuristic. The goal is to generate optimal routes starting from a depot to serve multiple clients, where they are picked up and transported to specific destinations. This approach accounts for the limited capacity of the vehicle while aiming to minimize travel time.

Modified Pairing Insertion Algorithm for DARP

START

Input: Depot coordinates, coordinates of patients to be picked up, delivery point coordinates, total number of vehicles, and vehicle capacity.

Output: Set of generated routes and total time for each route.

```
for i=1i = 1i=1 to p patients do
    Calculate the time between the depot and the patients.
    Calculate the time between patients.
end for
while there are unpickued patients ≠ 0:
    Initialize an empty route R.
    Select the patient farthest from the depot.
    Insert the patient into route R.
    Update the vehicle's load.
    Insert the corresponding delivery point into the route.
    Update the vehicle's load.
    For each patient yet to be picked up:
        Calculate the time to insert a new patient into the current route.
        Insert the patient at the position that minimizes the accumulated
time.
```

```
        Update the vehicle's load.
      end for
      Finalize the route and begin a new one.
      Calculate the total time for the route.
    end while
    Display the generated routes and their total times.
```

Fig. 3. Modified pairing insertion (MPI) algorithm

This pseudocode defines an algorithm for solving the Dial-a-Ride Problem (DARP), aimed at finding the best way to assign clients to vehicles while optimizing total travel time. The insertion heuristic assists in identifying optimal routes during each iteration, and clients that cannot be initially assigned are reassessed later. The algorithm also considers vehicle capacity and time constraints between clients and the depot.

Benefits of the Modified Pairing Insertion (MPI)

Simplicity and Speed: The insertion heuristic is easy to implement and executes quickly, making it suitable for moderate-sized problems where a solution is needed within a reasonable time frame.

Flexibility: It can easily adapt to changes in problem parameters, such as the addition of new patients or adjustments to time windows.

Efficiency: While it does not guarantee an optimal solution, it produces high-quality solutions that can later be improved through local or global optimization techniques.

Limitations

Initial Solution Quality: The quality of the initial solution heavily depends on the insertion strategy. If the initial insertion is suboptimal, subsequent optimization may not significantly improve the solution.

Scalability: For very large-scale problems, the insertion heuristic may not be sufficient and might need to be combined with more advanced optimization techniques.

In summary, the insertion heuristic is a powerful and efficient tool for addressing the Dial-a-Ride Problem in the context of patient transportation in Apizaco, Tlaxcala.

4. Results and discussion

The following objectives were established to evaluate the computational effectiveness, efficiency, and performance of the proposed algorithm. The purpose is to develop vehicle routing under the Dial-a-Ride Problem (DARP) framework to pick up and transport patients from various origins to different destinations, such as clinics or health centers, while aiming to reduce travel time.

The results for this model were obtained using an insertion heuristic like the previously described strategy. The procedure was implemented in Python 3.11.7 and executed on an MSI Cyborg 15 A12V PC equipped with a 9th-generation Intel Core i5 processor (4.00 GHz) and 8.0 GB of RAM. All scenarios were solved considering various locations, vehicle capacities, travel speeds, and numbers of vehicles.

The parameters for the scenarios were generated using a uniform distribution, with minimum and maximum values as shown in Table 1.

Fig. 4 illustrates the locations of the various patients, the facilities to which they need to be transported, and the depot location. The parameters used in the experiment were generated and verified to ensure accuracy and suitability for achieving meaningful results. If any parameter values were found to be unsuitable, new values were generated to guarantee the viability of the experiment.

To evaluate the algorithm's performance, several scenarios were randomly generated based on different instance sizes. Following the previously described code, the scenarios and their respective results are presented in Table 2. These results illustrate the routes generated and the required travel times.

Table 1. Parameters for scenario generation in the DARP model

Parameter	Range
Number of patients	[5,20]
Vehicle capacity	[1]
Number of vehicles	[3]
Average vehicle speed (km/h)	[30,60]

Other Parameters	Value
Deposit location	0 Latitude: (19.4161), Longitude: (-98.1439)
Patient coordinates	1 Pickup: (19.4226, -98.1570) Delivery: (19.4256, -98.1617) 2 Pickup: (19.4180, -98.1551) Delivery: (19.4171, -98.1615) 3 Pickup: (19.4224, -98.1496) Delivery: (19.4234, -98.1506) 4 Pickup: (19.4265, -98.1524) Delivery: (19.4256, -98.1617) 5 Pickup: (19.4231, -98.1598) Delivery: (19.4171, -98.1615) 6 Pickup: (19.4250, -98.1525) Delivery: (19.4234, -98.1506) 7 Pickup: (19.4190, -98.1520) Delivery: (19.4256, -98.1617) 8 Pickup: (19.4181, -98.1479) Delivery: (19.4171, -98.1615) 9 Pickup: (19.4210, -98.1485) Delivery: (19.4234, -98.1506) 10 Pickup: (19.4263, -98.1498) Delivery: (19.4256, -98.1617) 11 Pickup: (19.4202, -98.1484) Delivery: (19.4171, -98.1615) 12 Pickup: (19.4221, -98.1614) Delivery: (19.4234, -98.1506) 13 Pickup: (19.4203, -98.1538) Delivery: (19.4256, -98.1617) 14 Pickup: (19.4197, -98.1564) Delivery: (19.4171, -98.1615) 15 Pickup: (19.4180, -98.1525) Delivery: (19.4234, -98.1506) 16 Pickup: (19.4232, -98.1531) Delivery: (19.4256, -98.1617) 17 Pickup: (19.4238, -98.1612) Delivery: (19.4171, -98.1615) 18 Pickup: (19.4184, -98.1477) Delivery: (19.4234, -98.1506) 19 Pickup: (19.4257, -98.1576) Delivery: (19.4256, -98.1617) 20 Pickup: (19.4207, -98.1610) Delivery: (19.4171, -98.1615)

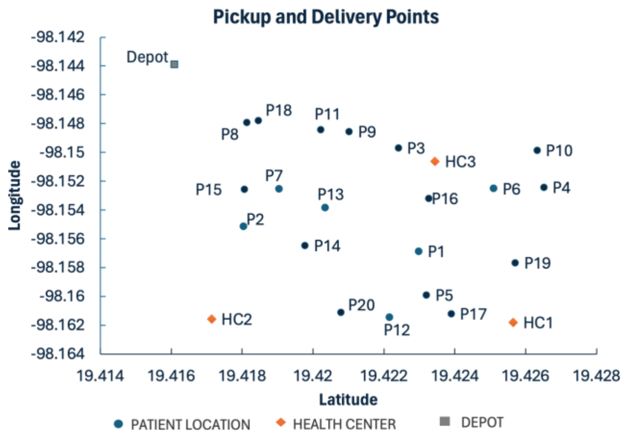

Fig. 4. Patient and delivery point locations

Table 2 (part 1). Variation in vehicle capacity (4 to 8 patients) and speed (30 to 60 km/h) with 3 vehicles

Vehicle Capacity	Vehicle Speed (km/h)	Number of Patients	Total Time (min)
4	30	5	8.6167
4	30	10	19.2401
4	30	12	23.4056
5	30	5	8.6167
5	30	10	16.7065
5	30	15	25.7080
6	30	5	8.6167
6	30	10	16.7065
6	30	15	24.4107
6	30	18	28.7074
7	30	5	8.6167
7	30	10	16.7065
7	30	15	24.4107
7	30	20	30.6584
8	30	5	8.6167
8	30	10	16.7065
8	30	15	24.4107
8	30	20	30.2227
4	40	5	6.46257
4	40	10	14.4301
4	40	12	17.5542
5	40	5	6.4625
5	40	10	12.5299
5	40	15	19.2810
6	40	5	6.4625
6	40	10	12.5299
6	40	15	18.3080
6	40	18	21.5305

Table 2 (part 2). Variation in vehicle capacity (4 to 8 patients) and speed (30 to 60 km/h) with 3 vehicles

Vehicle Capacity	Vehicle Speed (km/h)	Number of Patients	Total Time (min)
7	40	5	6.4625
7	40	10	12.5299
7	40	15	18.3080
7	40	20	22.9938
8	40	5	6.4625
8	40	10	12.5299
8	40	15	18.3080
8	40	20	22.6670
4	50	5	5.1700
4	50	10	11.5440
4	50	12	14.0434
5	50	5	5.1700
5	50	10	10.0239
5	50	15	15.4248
6	50	5	5.1700
6	50	10	10.0239
6	50	15	14.6464
6	50	18	17.2244
7	50	5	5.1700
7	50	10	10.0239
7	50	15	14.6464
7	50	20	18.3950
8	50	5	5.1700
8	50	10	10.0239
8	50	15	14.6464
8	50	20	18.1336
4	60	5	4.3083
4	60	10	9.6200
4	60	12	11.7028
5	60	5	4.3083
5	60	10	8.3532
5	60	15	12.8540
6	60	5	4.3083
6	60	10	8.3532
6	60	15	12.2053
6	60	18	14.3537
7	60	5	4.3083
7	60	10	8.3532
7	60	15	12.2053
7	60	20	15.3292
8	60	5	4.3083
8	60	10	8.3532
8	60	15	12.2053
8	60	20	15.1113

As vehicle capacity increases, the total time required for the same number of patients tends to decrease. This is because higher-capacity vehicles can transport more patients in a single trip, thereby optimizing operational time.

Similarly, as vehicle speed increases, the total time reduces significantly, particularly when managing a larger number of patients. This demonstrates that faster vehicles can reduce travel times even with a high number of patients.

Graphs of the routes were generated for some of the experimental results, showing how the routes vary depending on vehicle capacity, the number of patients, and speed. Fig. 5 displays the routes generated for a specific experiment with a vehicle capacity of 6 passengers, a speed of 40 km/h, and 18 patients.

The depot is marked with a black square in the upper left corner of the graph. It serves as the starting and ending point for all vehicles. Figure 5 – 7 displays three different vehicles, each represented by distinct shapes and colors:

Vehicle 1: Shown with blue lines, pickups are marked with blue circles, and deliveries are marked with blue rhombus.

Vehicle 2: Shown with green lines, pickups are marked with blue circles, and deliveries are marked with blue rhombus.

Vehicle 3: Shown with red lines, pickups are marked with blue circles, and deliveries are marked with blue rhombus.

Each vehicle follows a specific route, starting with pickups (indicated by a cross) and then proceeding with deliveries (indicated by a circle). The lines connecting these points represent the paths taken by the vehicle between locations. Numbers adjacent to the locations (e.g., R1, R2, etc.) likely serve as identifiers for the pickup and delivery points visited by the vehicle.

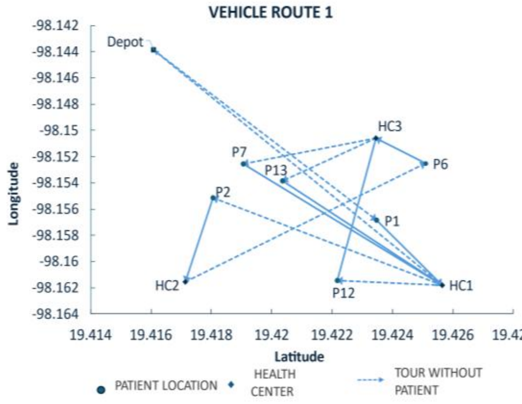

Fig. 5. Vehicle 1 routes obtained (Deposit → P1 → HC1 → P2 → HC2 → P6 → HC3 → P7 → HC1 → P12 → HC3 → P13 → HC1 → Deposit, Time spent traveling: 58.00 min.)

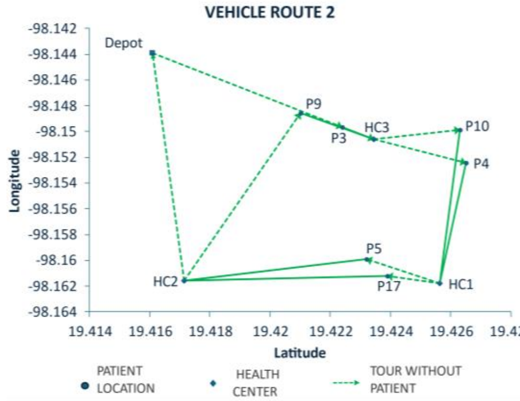

Fig. 6. Vehicle 2 routes obtained (b) Deposit → P3 → HC3 → P4 → HC1 → P5 → HC2 → P9 → HC3 → P10 → HC1 → P17 → HC2 → Deposit, Time spent traveling: 35.42 min.)

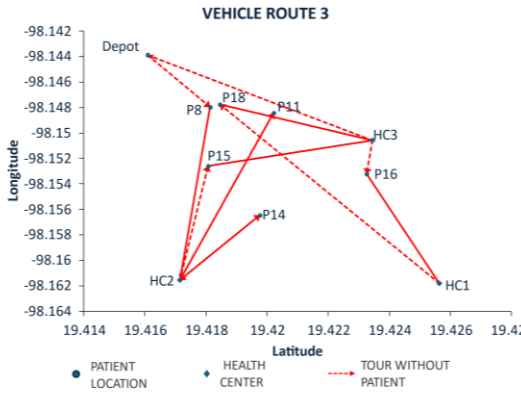

Fig. 7. Vehicle 3 routes obtained (c) Deposit → R8 → HC2 → R11 → HC2 → R14 → HC2 → R15 → HC3 → R16 → HC1 → R18 → HC3 → Deposit, Time spent traveling: 48.11 min.)

The graph axes display latitude and longitude coordinates, indicating that the pickup and delivery points are located on a real map. This visualization shows the relative geographic positions of each point. The routes appear to be designed to minimize travel time or distance. Vehicles collect from multiple points before performing deliveries, optimizing the routes based on client locations.

The graph indicates that the vehicles follow significantly different routes, although some overlap may occur in areas visited by more than one vehicle. This suggests that the routes are designed to distribute the workload among the vehicles evenly.

Fig. 8 – 12 presents the generated graphs depicting the routes corresponding to each vehicle, with pickups (circles) and deliveries (rhombus). The scenarios consider different vehicle capacities: 4, 7, and 8 passengers, with 12, 20, and 20 patients, respectively, and a speed of 40 km/h. Table 3 shows the routes obtained for every scenario.

Observing the last two graphs reveals a change in the total travel time, despite generating the same routes, due to the variation in vehicle speed. A lower vehicle capacity means fewer items can be transported (or fewer clients served) in a single trip. This limitation increases the number of trips or vehicles needed to serve all clients. Consequently, the routing process becomes more complex because routes must be optimized for multiple trips. Additionally, this results in longer delivery or service times overall, as more trips are required, leading to inefficiencies and potentially higher fuel consumption.

In contrast, higher vehicle capacity allows each vehicle to serve more clients in a single trip, reducing the total number of trips or vehicles required. This typically results in more direct routes and fewer total kilometers traveled, improving overall efficiency.

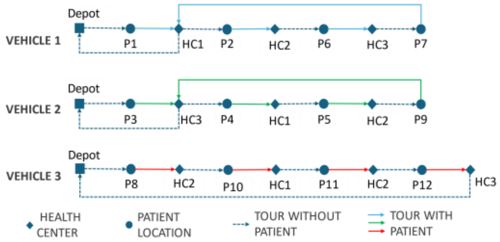

Fig. 8. Scenario 1: capacity of 4 passengers, speed of 40 km/h, and 12 patients

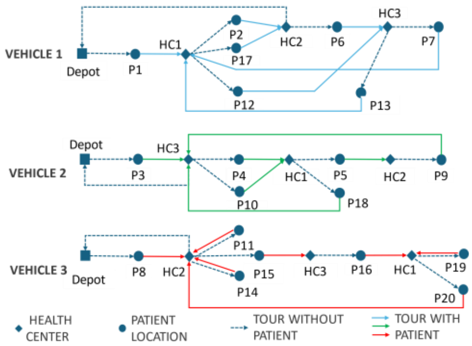

Fig. 9. Scenario 2: capacity of 7 passengers, speed of 40 km/h, and 20 patients

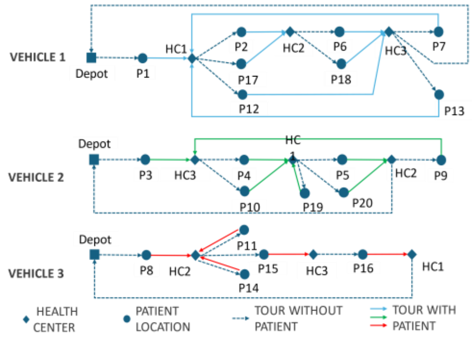

Fig. 10. Scenario 3: capacity of 8 passengers, speed of 40 km/h, and 20 patients

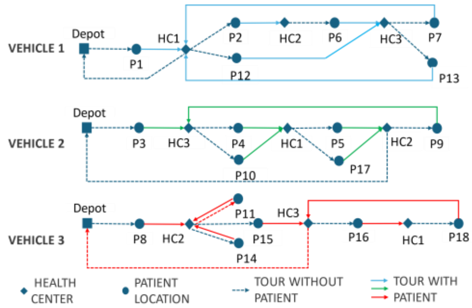

Fig. 11. Scenario 4: capacity of 6 passengers, speed of 30 km/h, and 18 patients

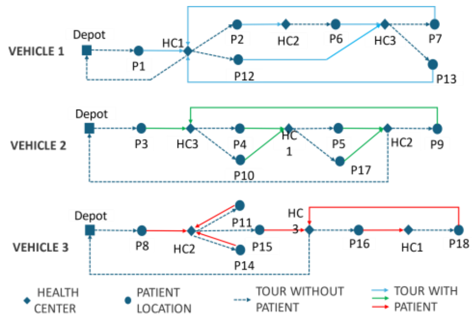

Fig. 12. Scenario 5: capacity of 6 passengers, speed of 60 km/h, and 18 patients

Table 3. Routes obtained for every scenario

Scenario	Vehicle	Route	Total Time (min)
Scenario 1: Capacity of 4 passengers, Speed of 40 km/h, and 12 patients	1	Depot → C1 → HC1 → C2 → HC2 → C6 → HC3 → C7 → HC1 → Depot	6.69
	2	Depot → C3 → HC3 → C4 → HC1 → C5 → HC2 → C9 → HC3 → Depot	4.63
	3	Depot → C8 → HC2 → C10 → HC1 → C11 → HC2 → C12 → HC3 → Depot	6.24
Scenario 2: Capacity of 7 passengers, Speed of 40 km/h, and 20 patients	1	Depot → C1 → HC1 → C2 → HC2 → C6 → HC3 → C7 → HC1 → C12 → HC3 → C13 → HC1 → C17 → HC2 → Depot	8.30
	2	Depot → C3 → HC3 → C4 → HC1 → C5 → HC2 → C9 → HC3 → C10 → HC1 → C18 → HC3 → Depot	7.63
	3	Depot → C8 → HC2 → C11 → HC2 → C14 → HC2 → C15 → HC3 → C16 → HC1 → C19 → HC1 → C20 → HC2 → Depot	7.06
Scenario 3: Capacity of 8 passengers, Speed of 40 km/h, and 20 patients	1	Depot → C1 → HC1 → C2 → HC2 → C6 → HC3 → C7 → HC1 → C12 → HC3 → C13 → HC1 → C17 → HC2 → C18 → HC3 → Depot	10.49
	2	Depot → C3 → HC3 → C4 → HC1 → C5 → HC2 → C9 → HC3 → C10 → HC1 → C19 → HC1 → C20 → HC2 → Depot	6.58
	3	Depot → C8 → HC2 → C11 → HC2 → C14 → HC2 → C15 → HC3 → C16 → HC1 → Depot	5.60
Scenario 4: Capacity of 6 passengers, Speed of 30 km/h, and 18 patients	1	Depot → C1 → HC1 → C2 → HC2 → C6 → HC3 → C7 → HC1 → C12 → HC3 → C13 → HC1 → Depot	10.66
	2	Depot → C3 → HC3 → C4 → HC1 → C5 → HC2 → C9 → HC3 → C10 → HC1 → C17 → HC2 → Depot	7.23
	3	Depot → C8 → HC2 → C11 → HC2 → C14 → HC2 → C15 → HC3 → C16 → HC1 → C18 → HC3 → Depot	10.81
Scenario 5: Capacity of 6 passengers, Speed of 60 km/h, and 18 patients	1	Depot → C1 → HC1 → C2 → HC2 → C6 → HC3 → C7 → HC1 → C12 → HC3 → C13 → HC1 → Depot	5.33
	2	Depot → C3 → HC3 → C4 → HC1 → C5 → HC2 → C9 → HC3 → C10 → HC1 → C17 → HC2 → Depot	3.61
	3	Depot → C8 → HC2 → C11 → HC2 → C14 → HC2 → C15 → HC3 → C16 → HC1 → C18 → HC3 → Depot	5.41

Route planning must be robust enough to handle unforeseen events such as traffic, adverse weather conditions, or mechanical issues with vehicles. This necessitates the implementation of a real-time monitoring and communication system to adjust routes and schedules as needed, ensuring reliable and efficient service for elderly patients and those with disabilities in Apizaco, Tlaxcala.

5. Conclusions and future work

To provide a high-quality service that considers minimum travel time, a practical case study of the Dial-a-Ride Problem (DARP) in the transportation of vulnerable patients from their place of residence to their respective health centers was presented. A mathematical programming model was developed, along with a heuristic solution method, to address real-world constraints such as vehicle capacity, service times, and routing order.

The results obtained demonstrate that the proposed approach is suitable for small to moderate-sized instances.

Several specific conclusions can be drawn from the experimental results:

- Increasing the vehicle speed from 30 km/h to 60 km/h resulted in a significant reduction in total travel time, for example, from 30.66 minutes to 15.11 minutes in a scenario with 20 patients.
- Greater vehicle capacities reduced the number of required trips, allowing more patients to be served per route and minimizing total operational time.
- The Modified Pairing Insertion heuristic produced feasible and efficient solutions across multiple test scenarios, confirming its applicability for healthcare logistics with limited computational resources.
- Graphical analyses validated the route distribution among vehicles and demonstrated the influence of input parameters – such as capacity and speed – on total service time.

These findings highlight the potential of the proposed model and heuristic to improve service quality in patient transportation systems, particularly in urban and semi-urban contexts such as Apizaco, Tlaxcala.

For future work, simulation techniques could be incorporated to account for traffic conditions, service hours, and variable demand, allowing for the modeling of real-world uncertainties and enhancing route programming analysis. Additionally, future research should consider real-world variables such as unexpected events (e.g., road closures, accidents) and variable service times, which were not included in the current deterministic model. These aspects are critical for improving the realism and robustness of patient transportation planning. Furthermore, performance metrics beyond travel time, such as operating costs, carbon emissions, and the integration of electric vehicles, should be considered to support more sustainable and cost-effective solutions.

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