http://doi.org/10.35784/iapgos.7384

received: 17.02.2025 | revised: 05.03.2025 | accepted: 15.03.2025 | available online: 31.03.2025

MODIFIED COSINE-QUADRATIC REFLECTANCE MODEL

Oleksandr Romanyuk[1], Volodymyr Lytvynenko[2], Yevhen Zavalniuk[1]
[1]Vinnytsia National Technical University, Vinnytsia, Ukraine, [2]Kherson National Technical University, Kherson, Ukraine

Abstract. *In the article, a new light reflectance model is proposed. The model is based on the modified cosine-quadratic bidirectional reflectance distribution function. The concept of bidirectional reflectance distribution function is analyzed. The disadvantages of existing physically accurate and empirical bidirectional reflectance distribution functions, including classical cosine-quadratic function, are discussed. The necessity of new empirical distributive functions development is justified. The comparison of double integrals of hemispherical reflectivity of reference Blinn function and classical cosine-quadratic function is provided. Based on the comparison, the new expression of cosine-quadratic distributive function calculation is obtained. The proposed expression makes it possible to significantly increase the accuracy of glare epicentre reproduction and is intended for use in highly productive three-dimensional rendering systems.*

Keywords: bidirectional reflectance distribution function, rendering, shading, visualization

ZMODYFIKOWANY COSINUS-KWADRATOWY MODEL ODBICIA ŚWIATŁA

Streszczenie. *W artykule zaproponowano nowy model odbicia światła. Model ten opiera się na zmodyfikowanej dwukierunkowej funkcji rozkładu odbicia. Przeanalizowano koncepcję dwukierunkowej funkcji rozkładu odbicia. Rozważono główne niedociągnięcia istniejących, fizycznie dokładnych i empirycznych dwukierunkowych funkcji rozkładu odbicia, w szczególności klasycznej funkcji cosinus-kwadratowej. Uzasadniono potrzebę opracowania nowych empirycznych funkcji rozkładu. Dokonano porównania całek podwójnych współczynnika odbicia półkulistego funkcji odbicia Blinna i standardowej funkcji cosinus-kwadratowej. Na podstawie porównania uzyskano nowe wyrażenie umożliwiające obliczenie funkcji cosinus-kwadratowej. Proponowane wyrażenie pozwala znacząco zwiększyć dokładność odwzorowania epicentrum olśnienia oraz strefy tłumienia olśnienia i jest przeznaczone do stosowania w wysokowydajnych systemach renderowania trójwymiarowego.*

Słowa kluczowe: dwukierunkowa funkcja rozkładu odbicia, renderowanie, cieniowanie, wizualizacja

Introduction

Modern 3D rendering systems require accurate and efficient light reflection models. Existing methods have limited precision in reproducing glare, reducing image realism.

The main requirement for interactive three-dimensional rendering systems [23], which usually are designed to operate in a real-time mode, is the formation of graphics scenes of acceptable quality for a limited interval of time.

The process of three-dimensional image formation includes the execution of the sequence of complex steps, which together form the graphics pipeline. The possibility of highly productive image formation depends on the key aspects of the implementation of particular graphics pipeline steps. In particular, the rendering stage includes up to 80% [21] of calculations for the formation of three-dimensional scenes. The rendering process includes the determination of colour intensity at every point of the object surface, for this, the surface reflectance model, normalized normal vector, lighting vector, and reflection vector are calculated. The usage of relatively simple light reflectance models can provide an essential increase in three-dimensional image formation productivity.

At the same time, the usage of simplified light reflectance models leads to a decrease in the quality of the scene's object visual features reproduction. Therefore, the development of new models, which will provide a highly productive and sufficiently accurate representation of features of light reflection from materials, is actual.

The proposed modified BRDF function improves glare accuracy while minimizing computational costs, making it highly relevant for video games, VR, and visualization. This development meets the current demands of the computer graphics and real-time rendering industries.

Main contribution to this paper:

- Modified Cosine-Quadratic Reflectance Model – A new BRDF improves glare reproduction on surfaces by refining the cosine-quadratic function.
- Optimized Reflectance Parameters – A novel approach using double integral calculations ensures precise glare epicentre rendering.
- Enhanced Glare Accuracy in 3D Graphics – The model improves glare reproduction accuracy up to 4.5 times over traditional methods.
- Efficient Real-Time Implementation – Designed for high-performance rendering, the model balances computational efficiency and visual fidelity.

The rest of the article consists of the following sections: Section Two – Reviews existing BRDF models, their limitations, and the need for improved accuracy; Section Three – presents the new BRDF model, its mathematical formulation, and parameter optimization; compares the proposed model with existing BRDFs, analyzing accuracy and efficiency through graphical results; and in the Conclusions – Summarizes key findings, emphasizing improved glare rendering and real-time application potential.

1. Literature analysis

The light reflectance models are based on the bidirectional reflectance distribution functions.

The bidirectional reflectance distribution function (BRDF) [8, 10, 19, 20, 24] is determined as the ratio of the radiance of light, outcoming from the surface point to incoming irradiance. BRDF is formally defined as [7–9, 17, 27, 29].

$$\frac{dL_r(\vec{V})}{L_i(\vec{L})\cos\theta_i d\omega_i} \tag{1}$$

where $L_r(\vec{V})$ – outcoming radiance, $L_i(\vec{L})$ – incoming irradiance, $\vec{L}$ – vector between the light source and surface point, $\vec{V}$ – vector between the eye of the observer (camera) and surface point, θ_i – the angle between normal vector to surface point $\vec{N}$ and $\vec{L}$, ω_i – the differential solid angle of illumination.

The most commonly used types of analytical bidirectional reflectance distribution functions, that are used in three-dimensional rendering, are empirical and physically accurate BRDFs [7, 11, 20].

Empirical BRDFs are simple and usually are obtained experimentally, they only approximately represent the key aspects of light reflection from the surfaces of objects.

As a rule, empirical BRDFs usually are based on the usage of the cosine of the angle between determined vectors. The advantage of this approach is the possibility of angle cosine calculation through vector dot product. Fig. 1 shows the main vectors that are used during BRDF calculation [20].

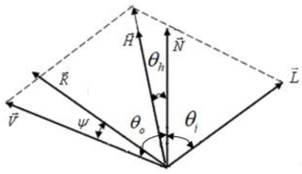

Fig. 1. Main vectors for BRDF calculation $\vec{L}$ *is an illumination vector,* $\vec{V}$ *– light reflection to observer (camera) vector,* $\vec{N}$ *– normal vector to surface point,* $\vec{R} = 2(\vec{L} \cdot \vec{N})\vec{N} - \vec{L}$ *is a vector of mirror reflection,* $\vec{H}$ *– halfway vector between* $\vec{L}$*,* $\vec{V}$*,* ψ *– the angle between* $\vec{R}$*,* $\vec{V}$*,* θ_h *– the angle between* $\vec{N}$*,* $\vec{H}$*,* θ_i *– the angle between* $\vec{N}$*,* $\vec{L}$*,* θ_o *– the angle between* $\vec{N}$*,* $\vec{V}$

The most famous empirical BRDF is Phong BRDF [1, 7, 8, 16], which lies in raising the cosine of the ψ angle between vectors $\vec{R}$, $\vec{V}$ to a power of n

$$\cos(\psi)^n \tag{2}$$

n is a surface shininess coefficient that determines the level of glare concentration at the surface material depending on its optical features.

Blinn BRDF [7, 8, 16] lies in the usage of the cosine of the angle between $\vec{N}$, $\vec{H}$ vectors instead of $\cos(\psi)$.

Phong BRDF and Blinn BRDF are characterized by their broad usage in highly productive systems of three-dimensional image formation. Their essential disadvantage is the computational complexity of raising the cosine of the angle to a high power, which decreases the speed of the object's surface shading.

Therefore, BRDFs, which lie in the usage of low-degree approximate expressions, are used for the highly productive formation of graphic scenes.

Cosine-quadratic BRDF [21] provides the approximate reproduction of glare features at the object's surface, using the second-degree expression

$$(\xi n(\cos(\psi) - 1) + 1)^2 \tag{3}$$

The standard formula variant lies in the usage of a multiplier $\xi = \frac{1}{2}$.

The essential disadvantage of the cosine-quadratic BRDF is that the accuracy of glare reproduction in the epicentre zone is not sufficiently high.

Another approximate expression is Schlick BRDF [20], which is calculated according to the formula

$$\cos(\psi) / (n - n \cdot \cos(\psi) + \cos(\psi)) \tag{4}$$

This BRDF is characterized with an unacceptable accuracy level of glare's attenuation zone reproduction.

There are also BRDFs based on the calculation of the angle between vectors. They allow controlling the form of glare but usually need extra calculations for the determination of angle arccosine.

For example, Lyon BRDF [14] involves the approximate calculation of the a angle between vectors as the length of difference vector between $\vec{L}$, $\vec{R}$, it also involves the control of glare form by using the k parameter

$$\left(1 - \frac{\frac{n}{2}a^2}{2^k}\right)^{2^k} \tag{5}$$

Physically accurate BRDFs [7, 20] are used in the photorealistic systems of three-dimensional rendering, where it is necessary to highly accurately represent the key aspects of light reflection from surface material.

The main peculiarity of physically accurate BRDFs is taking into account the surface microscopic irregularities. For this, the material surface is represented as the set of microfacets – tiny mirror areas. In addition, the dependency of the amount of reflected light on the angle of illumination (Fresnel coefficient calculation [5, 15, 18]), the masking of microfacet illumination by other microfacets (geometry attenuation coefficient calculation [4, 5, 15, 18]), light wavelength, physical effects, such as polarization or retroreflection, can be taken into account.

Cook-Torrance BRDF is a basic model for the most physically accurate light reflectance models [7, 8, 12, 16, 25, 26, 28, 30], it is calculated using the formula

$$\frac{DFG}{4\cos(\theta_i)\cos(\theta_o)} \tag{6}$$

where D – microfacet distribution function [15, 18], F – Fresnel coefficient [15, 18], G – light attenuation coefficient [15, 18].

D the function of the Cook-Torrance model can be calculated using the Beckmann distribution [1, 3, 22, 28]

$$\frac{e^{-\tan^2\theta_h/\alpha^2}}{\pi\alpha^2\cos^4\theta_h} \tag{7}$$

where α – surface roughness.

Walter BRDF [7, 8] is characterized by the usage of GGX distribution [1, 22, 30], which provides a more realistic glare attenuation zone representation. For the calculation of the distribution, the formula is used

$$\frac{\alpha^2{}_g\chi^+(\vec{N} \cdot \vec{H})}{\pi\cos^4\theta_h(\alpha^2{}_g + \tan^2\theta_h)^2} \tag{8}$$

where $\chi^+(x) = 1$ if $x > 0$, otherwise $\chi^+(x) = 0$, $\alpha^2{}_g$ – parameter of distribution width.

Physically accurate BRDFs are used when a highly realistic visualization has a higher priority than a fast formation of a three-dimensional image. Respectively, physically accurate BRFDs do not always comply with the requirements of highly productive graphics systems.

On the other side, the usage of existing empirical BRDFs in the highly productive systems of three-dimensional rendering is limited due to the insufficient accuracy of glare reproduction at the surfaces of objects.

Therefore, the development of the new low-degree BRDFs is actual.

The paper aims to develop and optimize a modified cosine-quadratic light reflection function (BRDF) to enhance the accuracy of glare rendering on surfaces in three-dimensional scenes.

The proposed approach focuses on improving the precision of glare reproduction while maintaining high computational efficiency**.** This enables the model to be applied in real-time for video games, VR, visualization, and other graphics systems.

2. Development of modified cosine-quadratic function

Let us find the optimal value ξ for the cosine-quadratic function.

For this, we use a definite integral over the angle variable ψ of the expression, which corresponds to the amount of hemispherical reflected light [2, 6, 13] under the condition of maximizing the reflected energy

$$2\pi \int_0^{\lim(n)} f_r(\psi, n) \cdot \cos(\psi) \cdot \sin(\psi) \cdot d\psi \tag{9}$$

where $f_r(\psi, n)$ – BRDF, $\lim(n)$ – boundary coordinate of BRDF calculation.

Additionally, we integrate this expression over the surface shininess coefficient $n \in [16, 256]$.

$$\int_{16}^{256} 2\pi \int_0^{\lim(n)} f_r(\psi, n) \cdot \cos(\psi) \cdot \sin(\psi) \cdot d\psi dn \tag{10}$$

To maximize the accuracy of glare's epicentre reproduction, we take $\lim(n)$ as $a\cos(\exp(-0.511/n))$, it approximately corresponds to the inflexion point, which separates the epicentre and attenuation zones.

We equate the expression forms at $f_r(\psi,n)=\cos(\psi)^n$ and $f_r(\psi,n)=(\xi\cdot n\cdot(\cos(\psi)-1)+1)^2$, eliminating 2π

$$\int_{16}^{256}\int_{0}^{\lim(n)}\cos(\psi)^n\cdot\cos(\psi)\cdot\sin(\psi)\cdot d\psi dn= \\ =\int_{16}^{256}\int_{0}^{\lim(n)}(\xi\cdot n\cdot(\cos(\psi)-1)+1)^2\cdot\cos(\psi)\cdot\sin(\psi)\cdot d\psi dn \tag{11}$$

The left part of the expression is calculated, and the right part is simplified. We get the equation

$$1.098=-0.71068\cdot\xi+0.12026\cdot\xi^2+1.40119 \tag{12}$$

Next, we find from the equation that $\xi=0.463$. Then

$$f_r(\psi,n)=(0.463\cdot n\cdot(\cos(\psi)-1)+1)^2 \tag{13}$$

To simplify the calculations, it's possible to use the approximate value $\frac{29}{64}$ instead of $\xi=0.463$.

The obtained function is denoted as F_{VOL}. The expression with $\xi=1/2$ is denoted as F_2. Fig. 2 shows the plots of F_B (Blinn-Phong BRDF), F_S (Schlick BRDF), F_{VOL}, F_2 for $\psi\in[0;\pi/2]$ when $n=16$.

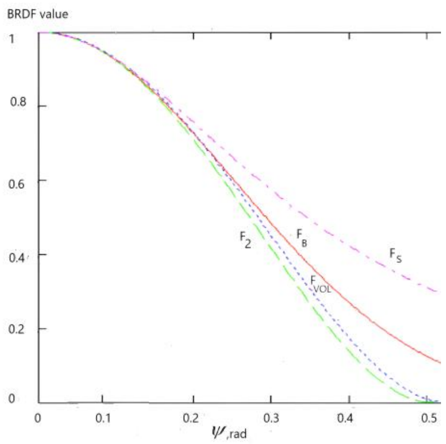

Fig. 2. Plots of F_B, F_S, F_{VOL}, F_2

As shown in Fig. 2, F_{VOL} provides a more accurate F_B approximation in comparison with F_2, F_S.

Fig. 3 shows the plots of maximum relative errors δ of F_{VOL}, F_2, F_S from F_B in the glare's epicentre zone with respect to $n\in[16;1000]$.

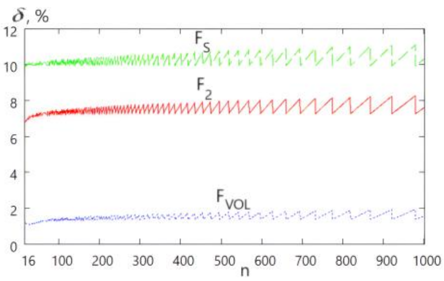

Fig. 3. Plots of maximum relative errors of F_{VOL}, F_S, F_2 from F_B in glare's epicentre zone

As shown in Fig. 3, the maximum δ of F_{VOL} from F_B in the glare's epicentre zone is significantly lower than the maximum δ between F_2, F_B and F_S, F_B, it has the value 1.9%.

Fig. 4 shows the three-dimensional plot of relative errors δ between F_{VOL} and F_B in the glare's epicentre zone.

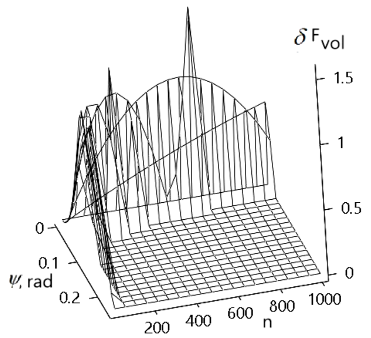

Fig. 4. Plot of relative errors between F_{vol} and F_B in the glare's epicentre zone

Fig. 5 shows the plots of maximum absolute errors Δ of F_{VOL}, F_S, F_2 from F_B with respect to $n\in[16;1000]$.

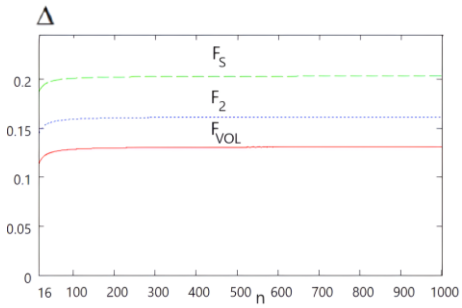

Fig. 5. Plots of maximum absolute errors of F_{VOL}, F_S, F_2 from F_B

Fig. 6 shows the three-dimensional plot of absolute errors Δ between F_{VOL}, F_B.

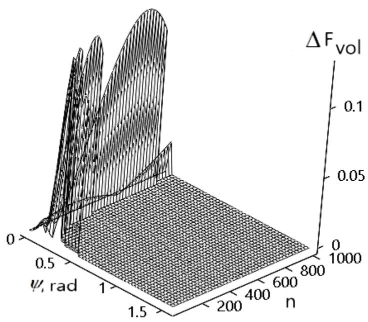

Fig. 6. Three-dimensional plot of absolute errors between F_{VOL} and F_B

Maximum Δ between F_{VOL}, F_S, F_2 and F_B corresponds to the glare's attenuation zone. Therefore, a more realistic reproduction of glare's attenuation zone in comparison with F_S, F_2 is additionally achieved.

The disadvantage of the proposed function is the increase of its values after reaching a zero level. Therefore, after the point, where $F_{VOL}=0$, the zeroing of its values is necessary. This point can be calculated using the formula

$$a\cos\left(\frac{29n-64}{29n}\right) \tag{14}$$

In the software tool Idx3d, it was visualized the test figures "Dinosaur" (Fig. 7) and "Teapot" (Fig. 8), using F_{VOL}.

Therefore, the F_{VOL} usage provides a highly productive and sufficiently realistic glare formation at the surfaces of three-dimensional objects.

Fig. 7. The results of the three-dimensional figure "Dinosaur" visualization

Fig. 8. The results of the three-dimensional figure "Teapot" visualization

3. Conclusions

This paper presents a modified cosine-quadratic reflectance model that significantly enhances the accuracy of glare reproduction on material surfaces. Unlike traditional BRDF functions, the proposed model achieves a 4.5-fold improvement in glare epicentre rendering while maintaining computational efficiency.

For the first time, the study introduces an optimized parameter selection method based on double integral calculations of hemispherical reflectivity. The results demonstrate that the new function provides a more realistic light distribution, reducing errors in both glare epicentre and attenuation zones.

The proposed approach is particularly relevant for high-performance 3D rendering systems, including video games, virtual reality, and real-time visualization applications. Future work may focus on further refining the model for broader material types and integrating it into advanced shading techniques.

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Prof. Oleksandr N. Romanyuk
e-mail: rom8591@gmail.com

Doctor of Technical Sciences, professor, Head of Department of Software Engineering of Vinnytsia National Technical University.
Research field: formation and processing of graphic images.

https://orcid.org/0000-0002-2245-3364

Prof. Volodymyr I. Lytvynenko
e-mail: immun56@gmail.com

Doctor of Science, Head of the Department of Informatics and Computer Sciences, Professor, Kherson National Technical University.
Science interests: Modeling of complex systems in economics, biology, medicine, machine learning methods, neural networks, Bayesian networks, evolutionary algorithms.

https://orcid.org/0000-0002-1536-5542

M.Sc. Yevhen K. Zavalniuk
e-mail: qq9272627@gmail.com

Ph.D. student, Department of Software Engineering of Vinnytsia National Technical University.
Research interests include the formation and processing of graphic images.

https://orcid.org/0009-0005-1202-4653