ANTI-ALIASING METHOD FOR SECOND-ORDER CURVES ON A HEXAGONAL RASTER

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Abstract. This paper presents a method for anti-aliasing of second-order curves using method of additional lines. The proposed method refers to supersampling methods, which allow achieving a high level of smoothing even for complex contours. For each type of second-order curves, the method is implemented using an evaluation function. The algorithm by which the method is implemented consists of six main sides. Direct formation of the internal and external additional lines of pixels is implemented in the fourth block of the algorithm. The pixels of both the internal and external additional lines are painted with a colour intensity of 50% of the colour intensity of the pixels of the main contour of the curve. This creates the appearance of a smooth transition and smoothes the steps. This allows reducing the aliasing effect on the boundaries of graphic objects of high curvature. The method significantly increases the speed of anti-aliasing implementation, due to the possibility of using it when constructing the main contour of the curve on a hexagonal raster. The method of additional lines for anti-aliasing of second-order curves on a hexagonal raster is equally effective for all types of second-order curves.

Keywords: anti-aliasing, second-order curves, hexagonal raster

METODA ANTYALIASINGU DLA KRZYWYCH RZĘDU DRUGIEGO NA RASTRACH HEKSAGONALNYCH

Streszczenie. W artykule przedstawiono metodę wygładzania krawędzi krzywych drugiego rzędu z wykorzystaniem metody linii dodatkowych. Proponowana metoda nawiązuje do metod supersamplingu, które pozwalają osiągnąć wysoki poziom wygładzania nawet w przypadku złożonych konturów. Dla każdego typu krzywych drugiego rzędu metoda jest implementowana za pomocą funkcji ewaluacyjnej. Algorytm, za pomocą którego implementowana jest metoda, składa się z sześciu głównych boków. Bezpośrednie formowanie wewnętrznych i zewnętrznych dodatkowych linii pikseli jest implementowane w czwartym bloku algorytmu. Piksele zarówno wewnętrznych, jak i zewnętrznych dodatkowych linii są malowane z intensywnością koloru równą 50% intensywności koloru pikseli głównego konturu krzywej. Tworzy to wrażenie płynnego przejścia i wygładza przejścia. Pozwala to na zmniejszenie efektu aliasingu na granicach obiektów graficznych o dużej krzywiźnie. Metoda ta znacznie zwiększa szybkość implementacji antyaliasingu ze względu na możliwość jej wykorzystania podczas konstruowania głównego konturu krzywej na rastrze heksagonalnym. Metoda dodatkowych linii do antyaliasingu krzywych drugiego rzędu na rastrze heksagonalnym jest równie skuteczna dla wszystkich typów krzywych drugiego rzędu.

Słowa kluczowe: antyaliasing, krzywe drugiego rzędu, raster heksagonalny

Introduction

An important challenge for modern computer graphics systems in generating realistic images is the significant reduction or complete elimination of the aliasing effect, also known as the "staircase" effect, which appears along the edges of graphical images when displayed on screens [2, 4, 6].

To entirely remove aliasing, a monitor with a resolution of at least 4000×4000 pixels is required. However, the current level of technology is not yet capable of providing such a resolution [4, 5].

Aliasing is particularly noticeable in images with high contour curvature, especially when rendering second-order curves. Since second-order curves – such as circles, ellipses, hyperbolas, and parabolas – are fundamental graphical primitives for all contours, the problem of reducing edge "staircasing" in graphical images translates into the task of anti-aliasing second-order curves.

1. Review of related publications

All anti-aliasing methods can be classified into three main groups: supersampling, multisampling, and post-processing (Figs. 1, 2).
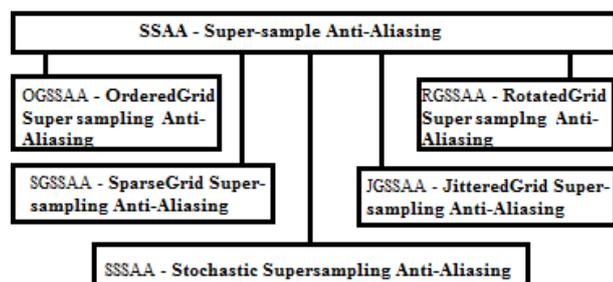

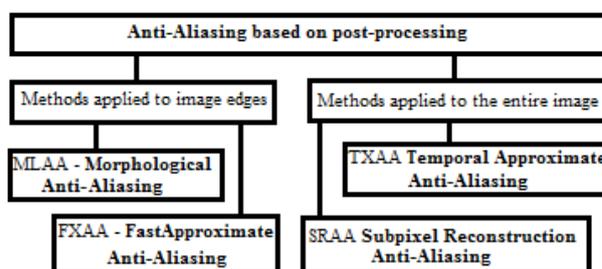*Fig. 1. Supersampling methods*

Fig. 2. Post-processing methods

Each of the methods presented in Fig. 1 has its own characteristics and advantages and is applied depending on quality and performance requirements [5].

One of these methods is specifically designed for anti-aliasing images with high edge curvature – this is the FXAA (Fast Approximate Anti-Aliasing) post-processing method. In this approach, the rendered image is analysed to detect all pixels located along the image boundaries. This is achieved by examining whether a pixel has neighbouring pixels that do not belong to the graphical primitive.

The next step involves per-pixel edge smoothing, as there are no clearly defined edges, only the pixels positioned along the image boundaries. Finally, colour blending is performed specifically for the detected boundary pixels and their immediate surroundings [5].

In addition to these traditional approaches, specialized methods have been developed for the hexagonal raster, adapted to its specific structure. These methods reduce the visibility of the "staircase" effect and improve the smoothing of image contours.

Most existing anti-aliasing methods designed for square rasters prove to be ineffective when applied to a hexagonal grid, as they do not account for its structural characteristics. Therefore, there is a need to develop new anti-aliasing methods specifically adapted to the hexagonal raster [5, 9, 10].

For a hexagonal raster, supersampling is applied with adapted approaches that take into account the geometry of the hexagon, allowing for more precise contour smoothing. Adapted supersampling achieves a high level of anti-aliasing by considering the pixel arrangement within the hexagonal grid.

This method ensures high-quality smoothing even for complex contours, although it requires significant computational resources.

The application of subpixelization (Fig. 3) for anti-aliasing allows for the adaptation of subpixel placement in areas of high curvature.

For a hexagonal raster, this provides more accurate contour representation through interpolation based on the pixel placement, which is especially important for second-order curves [1, 5, 7].
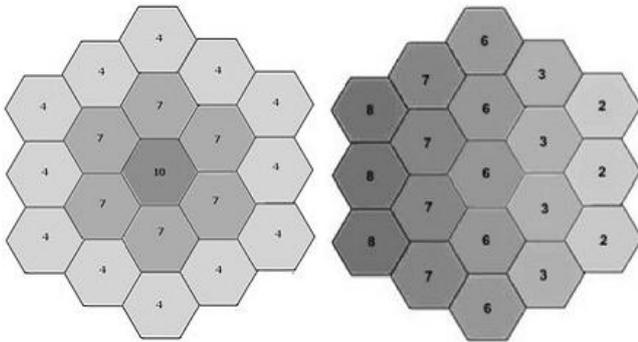

Fig. 3. Variants of colour intensity distribution for subpixelization: a) from the centre of the pixel b) from the edge of the pixel

Overall, anti-aliasing methods for the hexagonal raster include adaptive approaches that take into account the specific characteristics of the hexagonal pixel (Fig. 4).
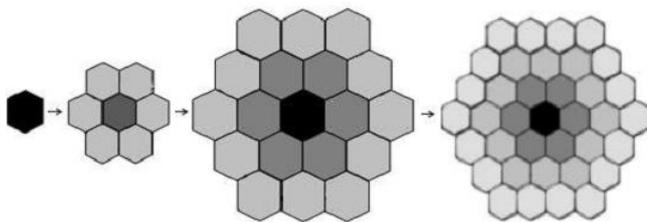

Fig. 4. Possible subpixelization

When a hexagonal pixel is divided into subpixels, the colour intensity of each subsequent subpixel segment decreases in proportion to its distance from the center. This means that the colour intensity can be determined proportionally to the number of subpixels that fall within the area bounded by the polygon [1, 5].

Colour representation in the hexagonal raster format has its unique characteristics. Dividing a hexagonal pixel into subpixels provides the opportunity for more detailed colour representation (Fig. 5). This, in turn, allows for the placement of elements for rendering primary colours in a way that significantly improves image quality due to an expanded colour gamut [3, 8].
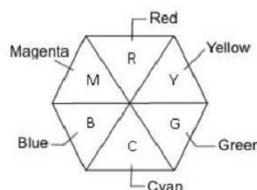

Fig. 5. Colour representation in a hexagonal pixel

Supersampling and multisampling provide high quality but have high resource requirements. Post-processing methods allow for faster performance, although they require further adaptation for the hexagonal raster.

The goal is to propose a method for anti-aliasing second-order curves on a hexagonal raster that would improve the quality of contour representation, reducing the aliasing effect.

2. The method of second-order curve antialiasing using additional pixel lines

The antialiasing method using additional pixel lines around the primary curve with reduced colour intensity is based on an algorithm that utilizes the evaluation function method but with modified parameters for the auxiliary curve. This approach allows for the formation of an additional pixel line that follows the contour of the primary curve but with lower colour intensity. Such a technique significantly reduces the "stair-step" effect and ensures smoother colour intensity transitions in the image.

For each type of second-order curve (circle, ellipse, parabola, and hyperbola), the antialiasing method is implemented using an evaluation function (EF). The evaluation function method is characterized by its computational simplicity and high accuracy in generating a stepwise trajectory. The obtained value of the evaluation function determines which nodes of the hexagonal grid should be selected to construct the primary contour.

Similarly, by modifying parameters (e.g., the radius for a circle or the semi-axis length for an ellipse), it is possible to generate a contour shifted by one pixel inward or outward.

For a circle, the primary contour is defined by the radius R. As is known, the evaluation function EF for circular interpolation at a point with coordinates x_i, y_i has the following form:

$$EF_i = (x_i^2 + y_i^2) - R^2 \tag{1}$$

To generate the inner auxiliary line, radius $(R-1)$ is used, while for the outer auxiliary line, radius $(R+1)$ is applied. Since the evaluation function method for a circle is based on minimizing the value of

$$(x_i^2 + y_i^2) - R^2 \tag{2}$$

the function:

$$EF_i = (x_i^2 + y_i^2) - (R^2 - 1)^2 \tag{3}$$

is evaluated for the inner line, whereas:

$$EF_i = (x_i^2 + y_i^2) - (R^2 + 1)^2 \tag{4}$$

is used for the outer line. Thus, the auxiliary lines replicate the shape of the primary contour but are scaled down or enlarged by one pixel (Fig. 6).
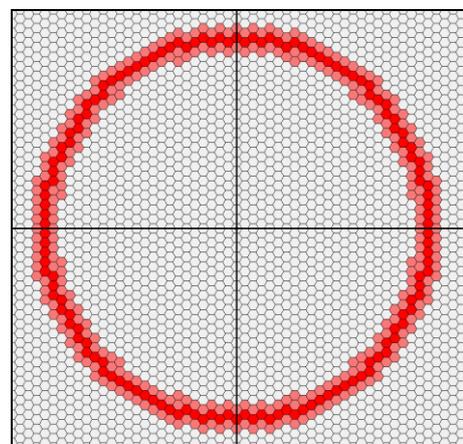

Fig. 6. The auxiliary lines surrounding the primary circular contour

For an ellipse (Fig. 7), the primary contour is defined by the semi-axis lengths a and b. Using the evaluation function:

$$EF_i = (\frac{x^2}{a^2} + \frac{y^2}{b^2}) - 1 \tag{5}$$

The inner auxiliary line can be generated using formula:

$$\left(\frac{x^2}{(a_{int})^2} + \frac{y^2}{(b_{int})^2}\right) - 1 = 0 \qquad (6)$$

While the outer line can be generated using formula:

$$\left(\frac{x^2}{(a_{ext})^2} + \frac{y^2}{(b_{ext})^2}\right) - 1 = 0 \qquad (7)$$

$$a_{ext} = a + \partial, \quad b_{ext} = b + \partial \qquad (8)$$

where ∂ is chosen so that the displacement corresponds to one pixel on the hexagonal grid.
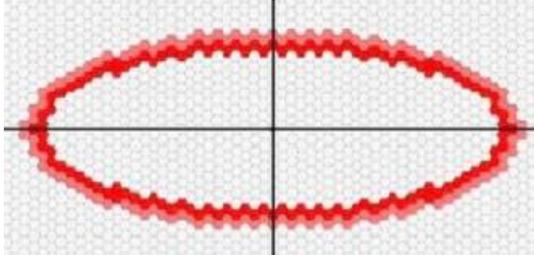

Fig. 7. The auxiliary lines surrounding the primary elliptical contour

The contours of hyperbolas and parabolas have an asymmetric shape; however, individual segments of parabolas and hyperbolas can be approximated using elliptical segments with varying focal radii. Therefore, to generate an auxiliary pixel line for a parabola or hyperbola, a similar approach to that of an ellipse is applied – by modifying the parameters of the approximating ellipse and shifting its semi-axes by one pixel outward or inward. This allows for the creation of an inner or outer auxiliary line around the primary parabolic or hyperbolic contour (Fig. 8).
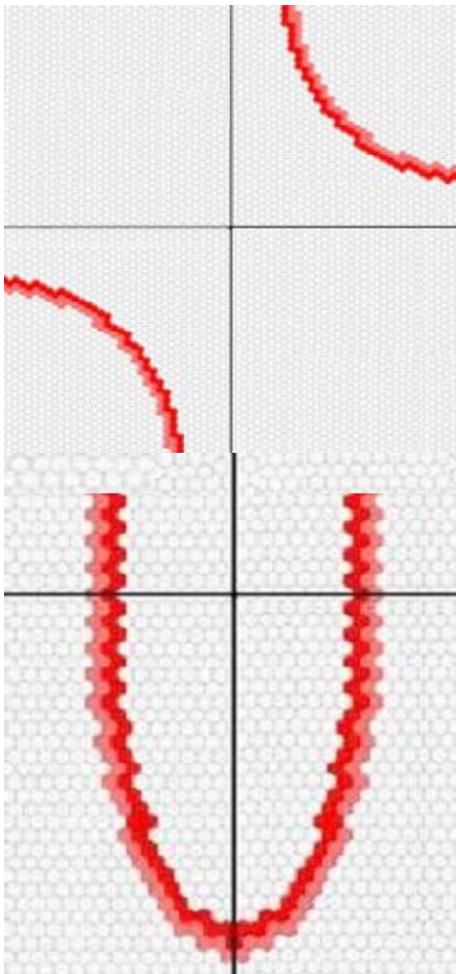

Fig. 8. Approximation of hyperbolas and parabolas using elliptical segments

After identifying the pixels that form the inner and/or outer auxiliary line, all these pixels are filled with a colour whose intensity is 50% of the primary contour's intensity. This creates a smooth transition from the contour to the background.

Step-by-Step Algorithm of the Proposed Antialiasing Method for Second-Order Curves Using Auxiliary Pixel Lines:

1) Selection of the curve type (circle, ellipse, parabola, or hyperbola) to which the proposed antialiasing method will be applied. The key parameters include the radius for a circle, semi-axes for an ellipse, focal distance for a parabola, and specific parameters for a hyperbola.
2) Construction of the primary contour of the curve based on the selected curve type.
3) Approximation of parabolas and hyperbolas using ellipses: If the curve type is a parabola or a hyperbola, local approximation by elliptical segments is applied to generate auxiliary lines.
4) Generation of the inner and/or outer auxiliary pixel line. For a circle, the additional lines are defined using radii $(R-1)$ and $(R+1)$. For an ellipse, $(a-\partial),(b-\partial)$ and $(a+\partial),(b+\partial)$ are used for the inner and outer lines. For a parabola or hyperbola, the parameters of the approximating ellipses are adjusted to form the inner or outer auxiliary pixel line.
5) Calculation of the auxiliary pixel line using the evaluation function method with modified parameters. The auxiliary pixels are rendered with a colour intensity of 50% of the primary contour's intensity.
6) Rendering of the primary contour with full intensity and the auxiliary pixel line with 50% intensity, ensuring a smooth transition and reducing the "stair-step" effect.

Figure 9 presents a flowchart of the described algorithm, illustrating the antialiasing method using an auxiliary pixel line.

This algorithm ensures high performance and accuracy in generating auxiliary pixel lines. The use of an evaluation function allows for consideration of the hexagonal grid's characteristics and different types of second-order curves.

As a result, a smooth transition between the contour of any curvature and the background is achieved, significantly enhancing the visual perception of images.
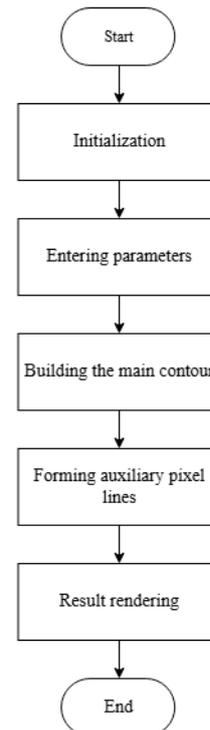

Fig. 9. Flowchart of the antialiasing algorithm using auxiliary pixel lines

3. Conclusion

Antialiasing is a set of technical and algorithmic tools used to eliminate the "stair-step" effect that occurs when creating graphical images and displaying them on rendering devices. The main goal of antialiasing is to smooth out sharp boundaries between colours to achieve a visual effect of a smooth transition.

A method of antialiasing second-order curves on a hexagonal grid has been developed, with the main idea being the construction of additional pixel lines around the primary curve with half the colour intensity.

This helps reduce the aliasing effect on the boundaries of highly curved graphical objects. Additionally, the evaluation function method is used to construct the auxiliary pixel lines. This method significantly improves the performance of the antialiasing implementation due to its applicability during the construction of the primary contour of the curve.

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