

## MODELLING OF DYNAMIC MODES IN A DC MOTOR FOR ELECTRIC VEHICLE

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**Abstract.** The paper investigates mathematical models of a DC electric motor with independent excitation for electric vehicles and PWM as a speed control method. Differential equations of electromechanical state are written in normal Cauchy form. This representation simplifies the computational process, since integration is carried out by the explicit numerical method of Runge-Kutta of the fourth order, which is simpler than implicit and more accurate than single-step explicit methods. The symbolic programming language Force 2.0, which is a variant of the Fortran language, is used for modelling. Compared to mathematical packages of simulation modelling, it is convenient in terms of low time costs for compiling the program itself, since the program includes model equations together with initial conditions, a numerical method, and an integration procedure. The developed models take into account electromagnetic couplings of the motor's electrical circuits and make it possible to simulate dynamic operating modes. Such models can be used to analyse the operation of motors both autonomously and as an element of an electromechanical system, including valve converters. The operation and transient modes of a DC motor are simulated, the simulation results are given, and their analysis is presented. The results confirmed the correctness of the chosen approach to modelling and numerical methods, as well as compliance with the classical theory of electric machines.

**Keywords:** mathematical model, differential equations, numerical methods, DC motor, PWM controller

## MODELOWANIE TRYBÓW DYNAMICZNYCH W SILNIKU PRĄDU STAŁEGO DO POJAZDÓW ELEKTRYCZNYCH

**Streszczenie.** W artykule zbadano modele matematyczne silnika elektrycznego prądu stałego z niezależnym wzbudzeniem dla pojazdów elektrycznych oraz PWM jako metodę regulacji prędkości. Równania różniczkowe stanu elektromechanicznego zapisano w postaci normalnej Cauchy'ego. Takie przedstawienie upraszcza proces obliczeniowy, ponieważ całkowanie przeprowadza się za pomocą jawnej metody numerycznej Runge-Kutta czwartego rzędu, która jest prostsza od metod niejawnych i dokładniejsza od metod jawnych jednoetapowych. Do modelowania wykorzystano symboliczny język programowania Force 2.0, który jest odmianą języka Fortran. W porównaniu z pakietami matematycznymi do modelowania symulacyjnego jest on wygodny pod względem niskich kosztów czasowych kompilacji samego programu, ponieważ program zawiera równania modelu wraz z warunkami początkowymi, metodą numeryczną i procedurą integracji. Opracowane modele uwzględniają sprzężenia elektromagnetyczne obwodów elektrycznych silnika i umożliwiają symulację dynamicznych trybów pracy. Modele takie mogą być wykorzystywane do analizy pracy silników zarówno autonomicznie, jak i jako element układu elektromechanicznego, w tym przetworników zaworowych. Symulowano pracę i tryby przejściowe silnika prądu stałego, podano wyniki symulacji i przedstawiono ich analizę. Wyniki potwierdziły poprawność wybranego podejścia do modelowania i metod numerycznych, a także zgodność z klasyczną teorią maszyn elektrycznych.

**Słowa kluczowe:** model matematyczny, równania różniczkowe, metody numeryczne, silnik prądu stałego, sterownik PWM

### Introduction

DC motors are a key element in the transition from Industry 4.0 to Industry 5.0 and form the foundation of electromobility [3]. Control systems for these motors play a crucial role in optimizing their operating modes, particularly in terms of energy consumption and torque generation [16]. For the study of electro-mechanical systems, mathematical models are especially useful, as they help reduce CO<sub>2</sub> emissions and conserve energy resources [2]. The required accuracy and adequacy of a model, as well as the need to account for various electromagnetic effects, depend on the specific modelling task. Therefore, modelling serves as one of the methodological foundations for studying electrical systems within the framework of circular production and strategic management [17].

A thorough understanding of mathematical modelling techniques is essential for the effective analysis of electric motor systems. The ability to construct adequate models allows researchers to investigate dynamic behaviours while accurately reflecting both electrical and mechanical quantities [15]. Analysis of the resulting graphical dependencies enables a deeper understanding of motor performance under various operating conditions, including emergency scenarios, ultimately supporting optimal and reliable operation in applications such as electric transportation.

Mathematical models of electric machines for control purposes must satisfy two often conflicting requirements. First, they must capture the main physical phenomena occurring in electric machines as control objects. At the same time, the models of the power unit must remain relatively simple to facilitate the practical synthesis of control laws [19, 25].

During the design phase of electric motors, two types of models are typically employed. The first type consists of linear models, which are used for approximate analysis and the synthesis

of system properties. The second type comprises more accurate, nonlinear models. Simpler linear models are often derived from these nonlinear models through linearization techniques.

Modern hardware and software tools enable the study of complex transient processes in electric motors. Static modes are a special case of dynamic behaviour, in such analyses, all derivatives in the governing equations are set to zero. The formation of transient processes in highly controllable systems is achieved by regulating the armature voltage, modifying the magnetic flux, and introducing additional resistance in the armature circuit [18].

### 1. Literature review

The rapid development of electric vehicles (EV) has significantly increased interest in electric drive systems, particularly in the modelling, control, and optimization of traction motors. The choice of motor type, its control strategy, and its dynamic behaviour directly affect vehicle efficiency, reliability, and driving performance, making accurate mathematical modelling a fundamental task in EV research [14]. Recent review studies [11, 21] provide comprehensive overviews of electric motor technologies, vehicle energy models, and drivetrain structures for electric vehicles. These works show that DC motors remain attractive for low- and medium-power EV applications due to their high starting torque, linear control characteristics, and ease of integration with power electronic converters. However, their dynamic performance is strongly influenced by electromagnetic coupling, armature dynamics, and load variations, which requires detailed mathematical representations [9]. The importance of accurate motor modelling at the system level is further emphasized in [23], where realistic electric motor models were shown to be critical for predicting power consumption under real operating conditions. These results confirm that simplified steady-



state motor representations are insufficient for analysing EV behaviour in transient driving cycles. Dynamic electromagnetic effects also strongly influence mechanical performance. In [6], noise and vibration in automotive DC motors were analysed using time-synchronous averaging, demonstrating that transient electromagnetic processes significantly affect mechanical stability and acoustic performance. This highlights the need for electromechanically coupled dynamic models. Advanced control strategies for EV motors have been actively developed in recent years. A robust DC motor control scheme based on extended state observers was presented in [10], ensuring prescribed performance under disturbances and parameter uncertainties. Structural, thermal, and operational aspects of EV drive motors were investigated in [5], showing that electrical, thermal, and mechanical dynamics must be considered simultaneously for reliable operation.

The accuracy of motor models is strongly dependent on parameter identification. The review in [8] demonstrated that errors in electrical and mechanical parameters lead to significant deviations in transient response and control performance, reinforcing the importance of physically consistent dynamic models. The increasing complexity of EV drive systems has led to the development of multi-motor and energy management strategies. In [24], a dual-motor energy management system based on optimization algorithms was proposed, requiring accurate motor dynamics to allocate power efficiently between drives.

Motor control in new-energy vehicles was further studied in [12], where speed and torque control under variable operating conditions were analysed. Practical limitations in DC motor speed control were discussed in [4], revealing that converter nonlinearities and unmodeled dynamics degrade control quality if the motor model is oversimplified.

High-fidelity dynamic modelling is also essential for controller design. In [13], a DC motor model combined with an optimized PID controller significantly improved position and speed tracking performance, confirming that precise dynamic motor representations are crucial for achieving fast and stable transient responses. Advanced torque control methods for new-energy vehicle motors presented in further confirm that accurate dynamic electromagnetic modelling is indispensable for high-precision EV drive systems. Despite the large body of research on motor control and EV energy management, the number of comprehensive models that simultaneously describe the dynamic modes of DC motors and their influence on the behaviour of the electric drive remains limited. There is also a strong need to integrate DC motor models with modern control strategies in order to improve both energy efficiency and dynamic performance of EV traction systems. In contemporary research, software-based simulation environments such as MATLAB/Simulink are widely used for modelling and analysis of electric drives. However, all application-oriented simulation packages and programming languages have inherent advantages and limitations, and the choice of a modelling environment depends on the objectives of the study and the required accuracy and computational efficiency. Algorithm-oriented modelling approaches based on high-performance programming languages provide an attractive alternative for dynamic simulation of electromechanical systems. In this context, the use of FORCE 2.0 (a FORTRAN-based language) enables high computational speed, efficient execution of long-term simulations, and multi-variant parametric studies. Such models allow both steady-state and transient operating modes to be analysed, including the dynamic behaviour of current, torque, and rotational speed, making it possible to predict drive response under different operating conditions and to effectively integrate control systems.

Therefore, there is a clear need for detailed DC motor models formulated in Cauchy normal form that incorporate electromagnetic coupling and allow accurate time-domain simulation of transient and dynamic operating modes. These models are essential for analysing DC motors both as standalone

units and as elements of EV electromechanical drive systems with power electronic converters. In this context, the present study aims to develop and investigate a mathematical model of a DC motor and to analyse the dynamic operating modes of the EV electric drive, including load variations, parameter sensitivity, and integration into the overall vehicle propulsion system.

## 2. Researches methodology

Differential equations describing the electromagnetic and mechanical states of rotating DC electric machines characterize their transient regimes. These equations are derived from calculated equivalent circuits, which correspond to systems with either lumped or distributed parameters [22]. Accordingly, it is useful to introduce the concept of an electromechanical circuit, which is defined by the electromechanical state equations of a rotating machine. Any change in such a circuit induces a corresponding change in its state, affecting electrical quantities (voltages and currents), magnetic fluxes, mechanical quantities (torques and rotational speeds), and, consequently, their associated energies. Transient processes necessarily evolve over time, otherwise, significant instantaneous power variations would occur, which are physically unrealistic [7]. The calculation and analysis of transient processes constitute a central task in the study of DC motors, as these processes describe the system's transition from one steady state to another. This analysis is based on the integration of differential state equations with specified initial conditions [1]. Electromechanical processes should be described using a minimal system of differential equations in standard Cauchy form, solved with respect to the first derivatives of the time-dependent functions. The dynamic behaviour of rotating electrical machines in autonomous operating modes is analysed through time discretization of these differential equations, following an explicit integration scheme.

Based on the review of existing mathematical models of DC motors for electric vehicles, it can be concluded that such models should be constructed using the theory of electromagnetic circuits (Fig. 1).

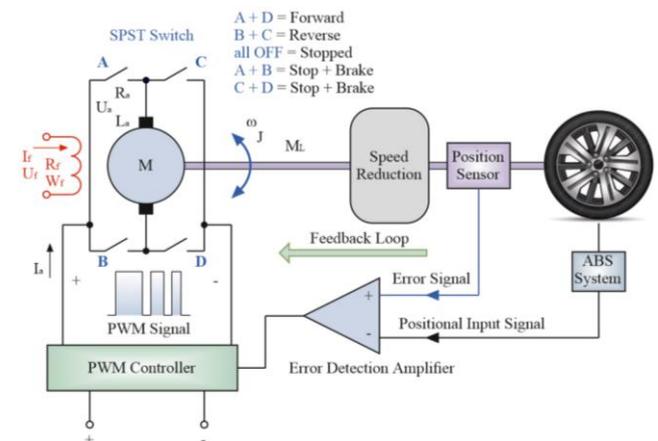


Fig. 1. Electrical diagram of a DC motor with independent excitation and its control system

The linear speed of the electric vehicle (km / h) is determined by the speed of the electric motor, the transmission ratio  $k = 9.67$  and the wheel radius  $R_{wh}$ :

$$V = 3.6 \frac{\omega_m}{k} R_{wh}$$

These models are relatively simple to implement, as the excitation and armature windings are electrically independent. The DC motor under consideration features a salient-pole design. For a comprehensive description of electromechanical processes, it is necessary to construct calculation schemes with distributed parameters, analogous to those used for synchronous machines.

When developing the mathematical model, the following assumptions were made:

- the armature reaction in the motor is fully compensated, the magnetic flux is constant, and the active resistance of the armature circuit remains unchanged during motor operation despite heating;
- saturation of the magnetic circuit of the excitation winding is neglected, both along the main magnetic flux path and along the leakage paths.

Under these assumptions, the motor's system of equations is linear, allowing for an analytical solution. In practice, motor parameters may vary during operation, which leads to a nonlinear system of equations. When the magnetic flux changes, taking into account the magnetization curve, the resulting variations in model parameters can be incorporated by adding the necessary differential or algebraic equations. In the proposed model, however, the magnetic flux is assumed to remain constant.

To formulate the dynamic equations of the complete system, it is necessary to include differential equations that describe the dynamics of a DC motor with independent excitation. These equations are written in standard (normal) form for both the armature and excitation windings, neglecting the voltage drop at the brush contacts. Given that  $E(t) = c\omega(t)$  and  $M(t) = ci(t)$ , we obtain the equations of the classical model:

$$u_a = R_a i_a + L_a \frac{di_a}{dt} + c\omega \quad (1)$$

$$u_f = R_f i_f + L_f \frac{di_f}{dt} \quad (2)$$

where  $u_a, u_e$  – voltages on the armature and on the excitation winding;  $i_a, i_e$  – armature and excitation currents;  $R_a, R_f$  – resistances of the armature and excitation windings;  $L_a, L_f$  – inductances of the armature and excitation windings;  $c$  – design factor (coefficient of electromotive force (EMF) and torque;  $\omega$  – angular velocity of the armature (shaft).

The equation of mechanical state has the form:

$$M_{em} - M_L = J \frac{d\omega}{dt} \quad (3)$$

where  $M_{em} = ci_a$  – electromagnetic moment;  $M_L$  – static load moment;  $J$  – moment of inertia of the armature.

For convenience, we write equations (1), (2) in Cauchy normal form:

$$\frac{di_a}{dt} = \frac{u_a - R_a i_a - c\omega}{L_a} \quad (4)$$

$$\frac{d\omega}{dt} = \frac{M_{em} - M_L}{J} = \frac{ci_a - M_L}{J} \quad (5)$$

The mechanical characteristic equation has the form:

$$\omega = \frac{U_{nom} - I_a \cdot R_a}{c} \quad (6)$$

The shaft rotation speed (rpm) is expressed in terms of angular velocity according to equation (6):

$$n = \frac{30\omega}{\pi} \quad (7)$$

In matrix form, equations (4), (5) take the form:

$$\frac{d}{dt} \begin{bmatrix} i(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{c}{L_a} \\ \frac{c}{J} & 0 \end{bmatrix} \begin{bmatrix} i(t) \\ \omega(t) \end{bmatrix} + \begin{bmatrix} \frac{u_a}{L_a} \\ -\frac{M_L}{J} \end{bmatrix} \quad (8)$$

or

$$y(t) = A \cdot x(t) + B \quad (9)$$

where  $A = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{c}{L_a} \\ \frac{c}{J} & 0 \end{bmatrix}$  – matrix of state variables;

$x(t) = \begin{bmatrix} i(t) \\ \omega(t) \end{bmatrix}$  – vector of state variables;  $B = \begin{bmatrix} \frac{u_a}{L_a} \\ -\frac{M_L}{J} \end{bmatrix}$  – vector

of free terms;  $y(t) = \frac{d}{dt} \begin{bmatrix} i(t) \\ \omega(t) \end{bmatrix}$  – vector of state variable

derivatives.

From this model, it follows that the state variables of the system are the shaft speed and the current in the armature circuit. These variables are directly related to the armature's moment of inertia, resistance, and inductance – that is, to the mechanical and electrical inertias of the motor – which significantly influence the performance of an electric vehicle. According to equation (6), the rotational speed can be controlled by varying the voltage applied to the power converter, the resistance of the armature circuit, or the magnetic flux (coefficient  $c$ ). The implementation of armature voltage regulation can be considered from the perspective of power electronics, specifically using an H-bridge, pulse-width modulation (PWM), and a controller. An H-bridge is an electronic circuit that allows the current direction in a given section of the circuit to be reversed. Such circuits are used in DC motors to reverse the rotation direction and regulate motor speed. The advantages of this control method include lower power consumption and the possibility of operating from an unregulated power source. However, the control process is more complex than in continuous regulation.

In the pulse-width modulation method, voltage pulses of constant amplitude are applied to the armature. As a result, motor operation consists of alternating brief periods of acceleration and braking, which occur so rapidly that the motor achieves an effective average rotational speed. Figure 2 illustrates the H-bridge circuit, whose output provides the regulated voltage applied to the motor armature.

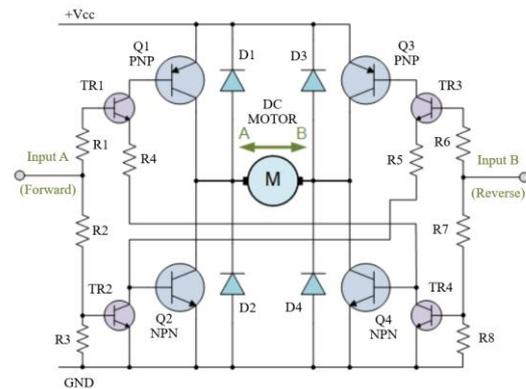


Fig. 2. H-bridge circuit for reversing and speed control of a DC motor

The operating principle of the H-bridge is based on the following: when the upper transistor in one arm conducts current and the lower transistor in the opposite arm conducts simultaneously (and vice versa), current flows through the motor winding. If both upper or both lower transistors in the same bridge arm are turned on simultaneously, no current flows through the winding. By properly controlling the transistors, it is possible to reverse the direction of the current in the winding, thereby changing its polarity [20].

The sequence of pulses applied to the transistors and their duty cycle are generated using pulse-width modulation (PWM), which is implemented in the microcontroller. This determines both the magnitude and the polarity of the voltage applied to the motor armature winding. Each transistor is controlled individually through a driver, which serves as an interface between the microcontroller and the H-bridge.

Current flows through the winding when one upper transistor and the diagonally opposite lower transistor are turned on, specifically  $Q_1$  and  $Q_4$ , while  $Q_2$  and  $Q_3$  are turned off. When  $Q_1$  and  $Q_4$  are turned off and  $Q_2$  and  $Q_3$  are turned on, the current direction in the winding reverses, and consequently, the rotation direction of the motor changes. The duty cycle diagrams for the transistor control signals and the corresponding voltage on the motor armature are shown in Fig. 3.

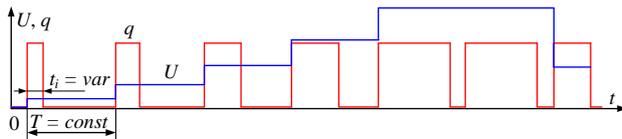


Fig. 3. The principle of voltage regulation by changing the pulse duration

The range of control signal voltages, over which the duty cycle varies from its minimum to maximum value, defines the dynamic operating range of the PWM controller. In an electric vehicle, the PWM controller generates a pulsed mode of operation for the H-bridge, applying a regulated voltage to the motor armature winding. The switching frequency of the transistors in the bridge arms is set to 10 kHz.

The voltage value is determined by the formula:

$$U_a = U_{nom} \cdot \gamma = U_{nom} \cdot \frac{t_i}{T} = U_{nom} \cdot t_i \cdot f_c \quad (10)$$

where  $\gamma$  is the pulse duty cycle, and the inverse value  $q = 1 / \gamma$  is the pulse duty cycle. The value of  $\gamma$  lies in the range from 0 to 1. By changing the turn-on time of the transistors, the voltage at the output of the bridge can be changed.

Let us consider a mathematical model that takes into account the electromagnetic coupling between the excitation and armature windings in the form of mutual inductances  $L_{fa}$ ,  $L_{af}$ :

$$\frac{d\Psi_a}{dt} = u_a + c\omega\Phi - R_a i_a \quad (11)$$

$$\frac{d\Psi_f}{dt} = u_f - R_f i_f \quad (12)$$

where  $\Psi_a$ ,  $\Psi_f$ ,  $i_a$ ,  $i_f$ ,  $R_a$ ,  $R_f$  are the total flux linkages, currents and resistances of the armature and excitation windings, respectively;  $\Phi$  is the magnetic flux.

Therefore:

$$\frac{di_a}{dt} = S_a u_a + T_a u_f + E_a \quad (13)$$

$$\frac{di_f}{dt} = T_f u_a + S_f u_f + E_f \quad (14)$$

where the coefficients  $S_a$ ,  $T_a$ ,  $T_f$ ,  $E_a$ ,  $S_f$ ,  $E_f$  are determined from equations (15):

$$S_a = \frac{1}{L_a + L_{af}L_{fa}/L_f}, \quad T_a = -\frac{S_a L_{af}}{L_f}, \quad T_f = -\frac{S_a L_{fa}}{L_f},$$

$$E_a = S_a \left( \frac{L_{af} r_f i_f}{L_f} - c\omega\Phi - r_a i_a \right), \quad S_f = \frac{1 - L_{fa} T_a}{L_f}, \quad (15)$$

$$E_f = -\frac{L_{fa} E_a + r_f i_f}{L_f}$$

In compensated motors, the saturation of the magnetic core can be taken into account approximately according to the magnetization curve:

$$\Phi = \Phi(i_f), \quad L_f = L_f(i_f) \quad (16)$$

In the unsaturated state in a DC motor:

$$\Phi = L_f i_f / w_f, \quad L_f = const \quad (17)$$

where  $w_f$  is the number of turns of the excitation winding.

The equation of the armature rotation takes the form (5).

Taking into account saturation in uncompensated motors is associated with the construction of the calculation scheme of the magnetic circuit as a circuit with distributed parameters, since here the magnetic flux is a function of the armature and excitation currents  $\Phi = f(i_f, i_a)$ . The calculation of eddy currents in structural elements can be performed only by an approximate method.

To solve the problem, taking into account the spatial distribution of physical state variables, it is necessary to use the equation of a quasi-stationary electromagnetic field in partial derivatives. The resulting differential equations must be combined into a single system, taking into account mutual influences.

Algebraic differential equations (4), (5), (6) and (5), (13), (14), (17) constitute, respectively, the models of a DC motor with independent excitation. Almost all existing numerical methods require multiple solutions of the Cauchy problem for different values of the initial conditions or parameters of the system. The initial conditions in our case are zero  $i_a(t) = 0$ ,  $\omega(t) = 0$ . Solving nonlinear or linear differential equations consists of finding the dependencies  $y_i(t)$  under known initial conditions  $t = t_0$  and  $y_{k0} = y_{k0}$  (initial values of the system variables).

To solve the problem, the multi-step explicit Runge-Kutta method of the fourth order was used, which is more complex, but more efficient and accurate than the single-step Euler method. Its essence is to replace the integral curve at each integration step with a straight line and calculate four coefficients, which are the right-hand side of the differential equations. According to this method, a certain value of the function (current or rotation speed) is calculated from several previous values of  $y(t)$ . The explicit scheme has the form:

$$k_1 = h \cdot f(t_i, y_i), \quad k_2 = h \cdot f\left(t_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right),$$

$$k_3 = h \cdot f\left(t_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right), \quad k_4 = h \cdot f(t_i + h, y_i + k_3), \quad (18)$$

$$y_{i+1} = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$$

The error according to this method is  $h^5$ , and considering that the integration step is much smaller than unity, it is more accurate, since for the Euler method, the error is  $h^2$ .

Figure 4 shows the algorithm of the program for solving differential equations.

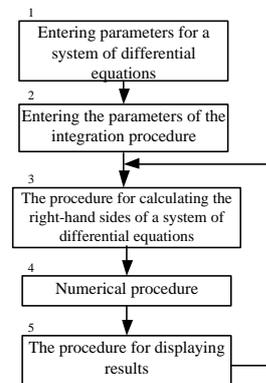
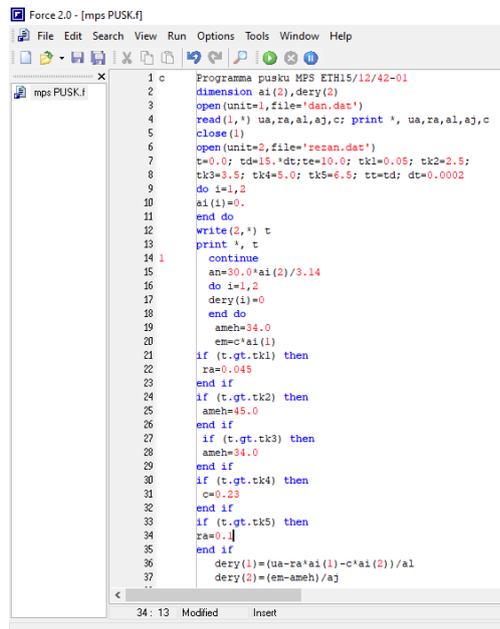


Fig. 4. Integration algorithm flowchart

### 3. Results

To test the developed algorithm for forming transient processes of starting and operation of DC electric motors with independent excitation, a program was developed in the FORCE 2.0 environment (FORTRAN prototype). Transient processes were studied for the first model. A fragment of the program is shown in Fig. 5.



```

1 c
2 Programma pusku MFS ETH15/12/42-01
3 dimension ai(2),dery(2)
4 open(unit=1,file='dan.dat')
5 read(1,*) ua,ra,ai,aj,c; print *, ua,ra,ai,aj,c
6 close(1)
7 open(unit=2,file='rezan.dat')
8 t=0.0; td=15.*dt;te=10.0; tk1=0.05; tk2=2.5;
9 tk3=3.5; tk4=5.0; tk5=6.5; tt=td; dt=0.0002
10 do i=1,2
11 ai(i)=0.
12 end do
13 write(2,*) t
14 print *, t
15 continue
16 an=30.0*ai(2)/3.14
17 do i=1,2
18 dery(i)=0
19 end do
20 ameh=34.0
21 em=c*ai(1)
22 if (t.gt.tk1) then
23 ra=0.045
24 end if
25 if (t.gt.tk2) then
26 ameh=45.0
27 end if
28 if (t.gt.tk3) then
29 ameh=34.0
30 end if
31 if (t.gt.tk4) then
32 c=0.23
33 end if
34 if (t.gt.tk5) then
35 ra=0.1
36 end if
37 dery(1)=(ua-ra*ai(1)-c*ai(2))/ai
38 dery(2)=(em-ameh)/aj

```

Fig. 5. Program window in the FORCE 2.0 environment

To solve the tasks set, other application packages can be used, such as Mathcad, Mathematica, Maple, MATLAB/Simulink, and others. All of them have their advantages and disadvantages. The use of the MATLAB/Simulink simulation modelling package is limited by a set of unified blocks and Simulink capabilities and the accuracy of existing models of the SimPowerSystems library.

The parameters of the ETH 15/12/42-01 motor for electric vehicles manufactured by Balkancar with the following nominal data were used for the model: power  $P_{nom} = 15$  kW, armature supply voltage  $U_{nom} = 120$  V, angular velocity  $n_{nom} = 4200$  rpm, armature current  $I_{nom} = 150$  A, moment of inertia of the mechanical part of the electric motor  $J = 0.3$  kg·m<sup>2</sup>, armature winding resistance  $R_a = 0.05$  Ohm, inductance of the motor armature circuit  $L_a = 45$  mH, design factor taking into account the magnetic flux of the excitation winding  $c = 0.26$ .

The integration of the equations is carried out by the explicit numerical Runge-Kutta method with a time discretization step of  $h = 0.0002$  s. The number of points is  $n = t_{end}/h = 10/0.0002 = 50000$ . We assume that the electric motor is powered by a high-power electrical network, i.e.,  $U_{nom} = 120$  V = const. Figures 6–8 show the dependences  $n = f(t)$ ,  $M_E = f(t)$ ,  $n = f(M_E)$ . Let us show how the rotation speed of such an electric motor can be regulated.

The start of an electric motor with a nominal load torque  $M_{nom} = 9550 P_{nom}/n_{nom} = 9550 \cdot 15/4200 = 34$  Nm to a steady speed is simulated. The angular velocity of ideal idling is  $\omega = U_{nom}/c = 522$  s<sup>-1</sup>. To reduce the starting current of the armature and the torque at the moment of starting, a starting resistance  $R_s = 0.1$  Ω is introduced into the armature circuit. The total resistance of the armature circuit  $R_{a\Sigma} = R_s + R_a = 0.15$  Ω. At the time point 0.5 s, it is removed and the engine accelerates to the nominal speed of rotation, which is 4182 rpm. At the time point 2.5 s, an increase in the load on the engine shaft to 45 Nm is simulated, and the speed is 4113 rpm.

At time  $t = 3.5$  s the torque decreases to the nominal, while the speed again becomes close to the nominal, namely 4190 rpm. In the time interval  $5 < t < 6.5$  s the speed is regulated by reducing the coefficient  $c$  by 10% ( $c = 0.23$ ). In practice, this is a change in the magnetic flux of the excitation winding, which is realized by a proportional change in the excitation current by increasing the resistance of the excitation circuit by 10%. As can be seen from Fig. 6, the speed according to equation (5) increases by 10% to a value of 4720 rpm.

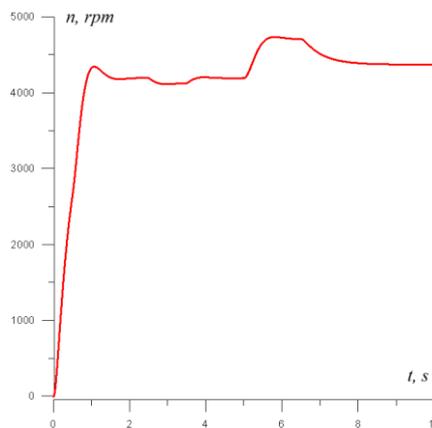


Fig. 6. Anchor rotation speed graph

In the time interval  $6.5 < t < 10$  s, the speed is regulated by introducing additional resistance into the armature circuit to the value of  $R_a = 0.1$  Ω with a flux reduced by 10%. According to equation (5), the speed decreases to 4371 rpm. It should be noted that at the moment of switching, there is a slight overregulation of the speed of rotation within 5%, which is possible in practice. As can be seen from Fig. 6, the electric motor, taking into account the starting resistance, reaches a stable nominal speed of rotation  $n = 4184$  rpm in about 1 s.

The amplitude value of the starting torque in Fig. 7 is approximately 260 Nm, respectively, the multiplicity of the starting torque to the nominal is  $260/34 \approx 7$ , which corresponds to the theory of engine starting. The shape of the armature current completely reproduces the shape of the electromagnetic torque through the coefficient  $c$ . At steady speeds ( $\omega = \text{const}$ ), the electromagnetic and mechanical torques are balanced.

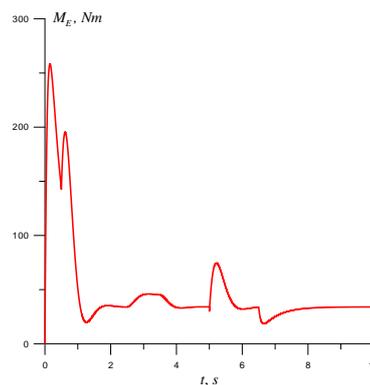


Fig. 7. Dependence of the electromagnetic moment in the transient process

The indicators of the main transient characteristics (Fig. 6 and Fig. 7) are summarized in Table 1.

Table 1. Parameters of the transient characteristics

Time to reach steady state (Fig. 6)	Rotational-speed overshoot (Fig. 6) at $t_s = 1 \dots 1.5$ s	Error in the steady state (Fig. 6)	Starting torque ratio (Fig. 7)
$\approx 1.6$ s	3.5%	$(4200-4182)/4200 = 0.4\%$	$260/34 \approx 7.6$

The static characteristic in Fig. 8 is a reflection of the characteristics  $n = f(t)$  and  $M_{em} = f(t)$  and corresponds to the mechanical characteristic of a DC motor.

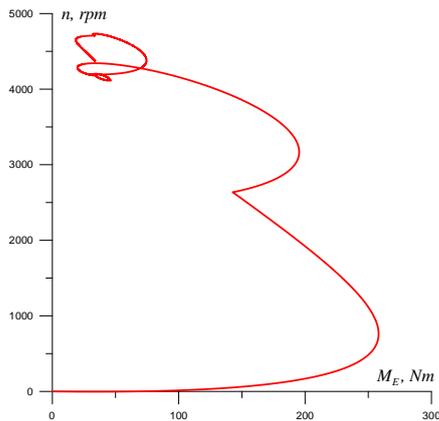


Fig. 8. Mechanical characteristics of a DC motor

Figure 9 shows a fragment of the results file.

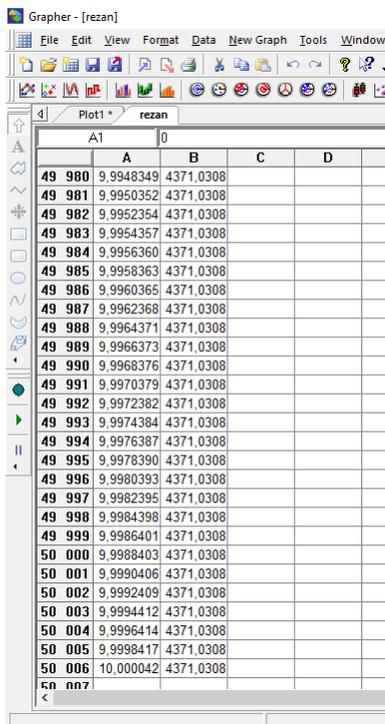


Fig. 9. Mechanical characteristics of a DC motor

Figures 10, 11, and 12 show the time dependences of the shaft rotation speed, armature current, and static characteristic with the pulse-width method of voltage regulation. Start-up occurs at a reduced torque of  $M_L = 5$  Nm, since the motor will not start at reduced voltages.

At times 0.0001, 0.25, 0.75, 1.25, 1.75, 2.25, 2.75, 3.25, 3.75, 4.25 s by changing the duty cycle from 0 to 1 in increments of 0.1 (transistor turn-on time) there is a gradual increase in the voltage on the motor armature. The armature voltages for these times are 12 V, 24 V, 36 V, 48 V, 60 V, 72 V, 84 V, 96 V, 108 V, 120 V, respectively. After the engine accelerates at a torque less than the nominal speed is slightly higher than the nominal and is 4360 rpm, and when the nominal load is applied it decreases to 4172 rpm, which is 99% of the nominal. From Fig. 11 it can be seen that in a steady-state process, at  $t \approx 6$  s with nominal parameters, the motor armature current is 135 A, which is 92% compared to the nominal. Noting the above, it can be stated that the model is adequate. This is confirmed by the calculation of electromechanical quantities with passport data. That is, the computer experiment corresponds to the theory of DC electric machines.

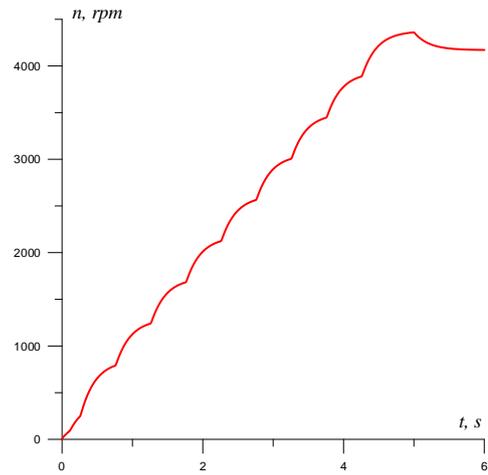


Fig. 10. Armature voltage graph with pulse-width modulation method

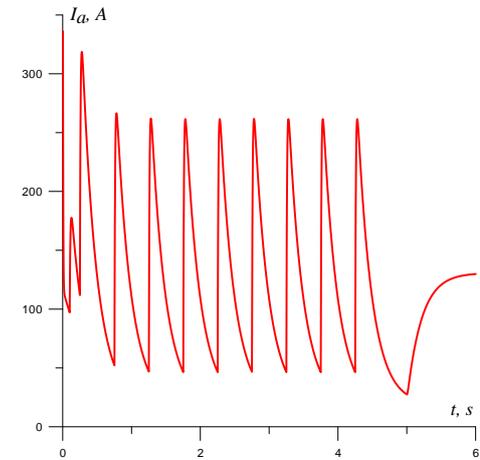


Fig. 11. Armature current graph with pulse-width method

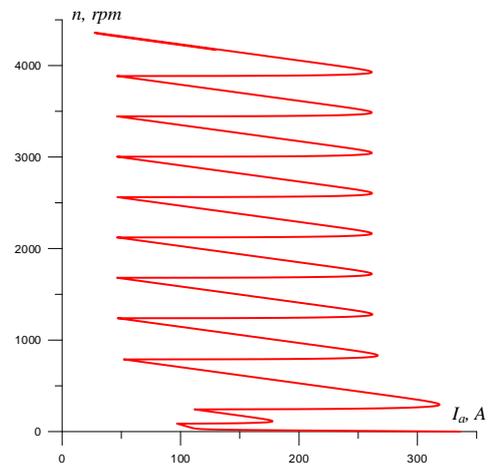


Fig. 12. Electromechanical characteristics of the motor with pulse-width modulation

#### 4. Discussion

The simulation results obtained for the DC motor model were compared with the nominal operating parameters of the ETH 15/12/42-01 traction motor. According to the nameplate data, the nominal angular velocity is 4200 rpm at the rated power of 15 kW and rated armature current of 150 A. Under rated load, the developed model demonstrates a steady-state angular velocity of approximately 4182...4190 rpm, which corresponds closely to the nominal value and confirms the correctness of both the mathematical model and the numerical solution algorithm. In addition, during the transient process the starting torque reaches approximately 260 Nm, which gives a starting torque ratio of about seven relative to the nominal torque of 34 Nm.

The transient processes reproduced in the model also demonstrate features characteristic of real electromechanical systems. After switching events, such as removal of the starting resistance or change in the load torque, the system exhibits slight overshoot of the rotational speed of up to about 5%, which subsequently decays during the stabilization period. The time required to reach the steady-state operating mode is approximately one second when the starting resistance is applied, which corresponds to the expected dynamics of DC motors used in traction applications. An important factor influencing the accuracy of the transient processes is the choice of the integration step in the numerical Runge-Kutta algorithm. In the presented studies, the time discretization step was selected as  $h = 0.0002$  s, which provides a sufficiently high temporal resolution of the electromechanical processes and ensures numerical stability of the solution.

Given that the truncation error of the fourth-order Runge-Kutta method is proportional to  $h^5$ , further decreasing the step would only marginally improve accuracy while significantly increasing computational time. Conversely, increasing  $h$  may lead to smoothing and phase distortion in the transient responses, especially during fast switching intervals and PWM operation, and therefore should be avoided. Additional studies were conducted using PWM of the armature voltage. Under PWM control, the armature current waveform follows the switching cycles, and the effective voltage applied to the motor determines the average developed torque and shaft speed. At reduced PWM duty cycles the motor is unable to overcome the static load torque, which is why start-up was simulated at a lower load torque of 5 Nm. This behaviour reflects the physical constraint that the electromagnetic torque must exceed the resisting torque to initiate rotation. At the same time, PWM introduces additional ripple in the armature current and electromagnetic torque, which is captured in the simulation results.

The presence of this ripple may slightly influence the dynamic characteristics, especially when the PWM frequency is comparable to the electrical time constant of the armature circuit. In this case, the choice of time step  $h$  in the numerical model becomes even more critical, as insufficient temporal resolution may lead to loss of information regarding the PWM-induced current pulsations. The developed model adequately reproduces the starting torque ratio, steady-state rotational speed, overshoot level, and stabilization time. This confirms that the mathematical description of the DC motor and the selected numerical method are suitable for studying electromechanical transients and evaluating the effect of control parameters such as PWM duty cycle and magnetic flux weakening. The obtained results also show that the combination of a sufficiently small integration step and properly selected PWM parameters ensures reliable representation of both steady-state and transient operating modes of the electric motor.

## 5. Conclusions

The developed mathematical models of a separately excited DC motor formulated in the normal Cauchy form provide a consistent and physically grounded framework for analysing the dynamic behaviour of electric drives in electric vehicle applications. This state-space representation allows the direct application of explicit fourth-order Runge-Kutta integration, which offers a favourable compromise between numerical accuracy and computational efficiency when simulating fast electromechanical transients. As a result, the proposed approach enables reliable time-domain analysis of key dynamic phenomena, including current buildup, torque development, speed regulation, and overshoot during start-up and load variations. The simulation results demonstrate that the developed models accurately reproduce the essential dynamic characteristics of the DC traction motor and remain in close agreement with nominal motor parameters, with deviations not exceeding  $\approx 10\%$ . This level of accuracy confirms the adequacy of the selected mathematical

formulation and numerical methods, as well as their consistency with the classical theory of electric machines.

Moreover, the inclusion of electromagnetic coupling between the electrical circuits and the mechanical subsystem allows the model to capture interactions that are often neglected in simplified steady-state or quasi-static approaches. An important advantage of the proposed modelling framework is its flexibility for integration into more complex electromechanical systems. The developed DC motor model can be used not only for standalone motor analysis, but also as a component of an electric vehicle traction drive, including power electronic converters and control systems. This makes it suitable for studying transient regimes such as acceleration, regenerative braking, and rapid load changes, which dominate the real operating conditions of electric vehicles. The obtained results indicate that such dynamic modelling is a valuable tool for the development of energy-efficient control strategies and power management algorithms.

Accurate prediction of current, torque, and speed dynamics enables better tuning of PWM-based controllers and helps to reduce energy losses, improve dynamic response, and enhance overall drive efficiency. In the context of low-emission electric transport systems, this contributes to improved vehicle performance, extended driving range, and reduced stress on electrical and mechanical components. Furthermore, the use of an algorithm-oriented simulation approach based on explicit numerical integration offers transparency and computational efficiency, which are essential for multi-variant parametric studies and long-term simulations. This creates favourable conditions for analysing the influence of motor parameters, load profiles, and control settings on the dynamic performance of the electric drive.

Future work will focus on extending the present model to include nonlinear effects such as magnetic saturation, eddy current losses, and temperature-dependent parameter variations, which become significant under high-load and high-speed operating conditions. In addition, the integration of the model with real-time digital control systems and hardware-in-the-loop platforms will further increase its practical relevance and allow validation under conditions close to real vehicle operation. These developments will support the design of more reliable, efficient, and adaptive electric drive systems for next-generation electric vehicles.

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