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MODELLING THE WORKING CYCLE OF A HEAT PUMP SCROLL COMPRESSOR

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Abstract. *A new approach to the identification and modelling of the working cycle of a heat pump compressor is proposed. A mathematical model of the heat exchange process in the compressor circuit is constructed, taking into account the change in temperature and pressure of the compression chambers. The presented mathematical model is based on the material and energy balance equations of the heat pump working fluid. Transient characteristics are determined and their investigation is conducted.*

Keywords: heat pump, compressor, transfer function, transient characteristic, compression chamber

MODELOWANIE CYKLU PRACY SPIRALNEJ SPRĘŻARKI POMPY CIEPŁA

Streszczenie. *Zaproponowano nowe rozwiązanie identyfikacji i modelowania cyklu pracy sprężarki pompy ciepła. Stworzono model matematyczny procesu wymiany ciepła w obiegu sprężarki uwzględniający zmiany temperatury i ciśnienia w komorach sprężania. Przedstawiony model matematyczny oparty jest na równaniach bilansów materiałowych i energetycznych płynu roboczego pompy ciepła. Wyznaczono i zbadano charakterystyki przejściowe.*

Słowa kluczowe: pompa ciepła, sprężarka, funkcja przenoszenia, charakterystyka przejściowa, komora sprężania

Introduction

Geothermal heat pumps utilizing low-potential heat are currently the most optimal solution for autonomous heating and heat supply systems. Such systems provide the most economical solutions compared to traditional heat supply systems.

One of the main components of heat pumps is the compressor circuit, where the working fluid, typically freon, is compressed to a certain pressure value at which it is heated. After this, due to the increased temperature and high pressure, heat transfer occurs through a heat exchanger to the heating circuit of the consumption line.

The working cycle in the compressor occurs with a variable mass of freon. Research on such processes is described in many scientific works. In particular, modelling of individual heat pump components is performed in works [3, 7]. Special attention is given to the issue of energy balance of compression chambers. In this case, a working fluid with variable mass is considered, which in turn is due to the change in volumes of compression chambers.

An important characteristic of heat pumps is their durability, which depends on the service life of the scroll compressor. This issue is given important attention, particularly in work [6], where an approach for determining the operating duration of an aluminium scroll compressor is proposed. This approach is based on conducting modelling of scroll compressor operation using the finite element method. Based on the results of such research, parameters of crack propagation in the scroll compressor are determined.

Research on heat transfer processes in scroll compressors is described in detail in works [1, 8]. Thus, in work [8], based on energy and material balances, transient processes in these types of compressors are investigated. They are based on dynamic changes in the geometric dimensions of scroll compressor compression chambers. In work [1], based on analytical description of compression chamber volume, a mathematical model of scroll compressors is constructed.

Special attention should be paid to work in which research on individual elements of geothermal heat pumps is conducted. Thus, in work [5], an approach to investigating heat exchange processes in the heat pump condenser circuit is proposed, and a mathematical model of the heat exchange process between the heat pump condenser circuit and the consumption line circuit is constructed. In work [4], it is proposed to divide heat exchangers of evaporator and condenser circuits into sections for supplying coolant with increased pressure to obtain higher heat transfer temperature. Also, in work [2], a semi-empirical model of a scroll compressor is presented, which is tested on a test bench, and the obtained data are used for calibration and adequacy verification.

1. Description of scroll compressor operation process and problem formulation

As is known, the main elements of heat pump internal circuits are (Fig. 1): evaporator (1), throttling device (2), condenser (3), and compressor (4) (Fig. 1). Liquid freon enters through the throttling device into the evaporator, where, due to low-potential heat from the ground circuit, a transition from liquid to vapor state occurs. Then entering the compressor, freon is compressed to the required value. In this case, the freon temperature increases. After this, under high pressure, in the condenser, the heated freon transfers the obtained heat through a heat exchanger to the heating circuit.

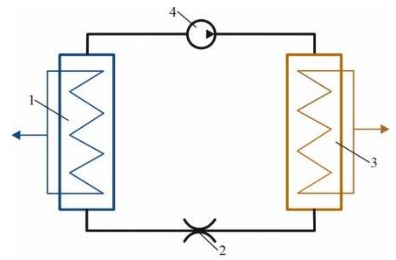

Fig. 1. Heat pump schematic diagram

Scroll compressors have gained the widest application in heat pumps currently. A scroll compressor has two scrolls: one stationary and the other moving (Fig. 2). The moving scroll rotates relative to the other at a given speed and has orbital rotation, in which the axes of this moving scroll always remain parallel to the same axes of the stationary scroll. The working cycle of a scroll compressor occurs in one turnover of the moving scroll along its orbit. In this case, crescent-shaped chambers are formed between the scrolls. Freon suction into the scroll compressor occurs from the heat pump evaporator. Then, passing through the formed cavities, freon is compressed. Discharge is carried out through a channel made in the central part of the stationary scroll.

To create a mathematical description and conduct research on a scroll compressor, it is necessary to know its geometric characteristics, particularly to determine the available compression chambers of controlled volume. Freon with volume V_{in} enters the internal cavity of the scroll compressor through the suction pipe into chambers 1 and 5 (Fig. 2). Such volume is limited by the compressor housing and scrolls. Chambers 3 are compression chambers where gas moves from chambers 1 and 5 during the rotational movement of the compressor scroll. The internal part is the discharge chamber 7.

When the compressor motor shaft rotates, the moving scroll performs orbital motion, which leads to the formation of suction volume V_{in}. At the moment of scroll closure, the maximum value V_{in} is formed. Subsequently, gas moves to chamber 3, where at the moment of scroll closure, the maximum compression volume $V_{st}<V_{in}$. The number of compression chambers depends on the number of scroll compressor turns. When moving toward the center, the volumes of compression chambers gradually decrease. At the end of the compression process, pairwise compression chambers combine into one discharge chamber 7 with discharge volume V_{out}. Subsequently, freon with volume V_{out} exits the compressor through discharge opening 8.

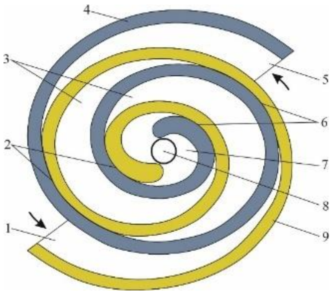

Fig. 2. Scroll compressor scheme: 1, 5 – suction chambers; 2, 6 – boundaries between scrolls; 3 – compression chambers; 4 – stationary scroll; 7 – discharge chamber; 8 – discharge opening; 9 – moving scroll

2. Mathematical model of the working cycle of heat pump scroll compressor circuit

The task of our research is to construct a mathematical model of suction, compression, and discharge processes of a scroll compressor, taking into account freon supply from the heat pump evaporator. Main assumptions for the stated problem: the refrigerant inside the chambers is homogeneous; the influence of gravity and kinetic energy is negligible; heat transfer between the compressor shell and freon is also negligible.

For a complete description of a scroll compressor, it is necessary to calculate the state parameters of each compression chamber. The working process in a scroll compressor occurs with variable freon mass. This is related to the continuous change in compression chamber volumes in the processes of suction, compression, and discharge. Simultaneously with the change in freon mass quantity, its temperature, volume, and pressure also change.

The energy balance equation with variable freon mass [3, 7] for each compression chamber can be written as:

$$c_v\frac{d(m_fT_f)}{\mathrm{d}t} = kF(T_s - T_f) + \frac{k_f-1}{k_f}(Q_{in}-Q_{out})h_{out} - \\ -Q_{out}(h_{in}-h_{out}) - p_f\frac{dV}{dt} \qquad (1)$$

where $m_f = m_f\,(t)$ – is the freon mass in the compression chamber, k – is the heat transfer coefficient from freon to the compression chamber walls, F – is the heat transfer surface between the compression chamber wall and freon, V – is the freon volume in the compressor cavity, c_v – is the specific heat capacity of freon, Q_{in}, Q_{out} – are the freon flow rates at the inlet and outlet of the compressor cavity respectively, h_{in}, h_{out} – are the specific enthalpy of freon at the inlet and outlet of the compressor cavity respectively, k_f – is the adiabatic index, $T_f = T_f\,(t)$ – is the freon temperature, T_s – is the compressor wall temperature, $p_f = p_f\,(t)$ – is the pressure in the compression chamber.

The heat transfer coefficient can be described by a formula obtained for convective heat exchange in scroll compressors [7]:

$$k = 0.023\frac{\lambda}{d_e}Re^{0.8}\,Pr^{0.4}\left(1+1.77\frac{d_e}{R_k}\right) \qquad (2)$$

where λ – is the gas thermal conductivity coefficient, d_e – is the equivalent diameter, Re – is the Reynolds number, Pr – is the Prandtl number, R_k – is the scroll curvature radius in the middle of the compression chamber.

The equation for determining pressure in the compressor cavity can be written as [7]:

$$c_v\frac{d(m_fp_f)}{dt} = \frac{1}{RV}\left(kF(T_s-T_f) + Q_{in}h_{int} - Q_{out}\,h_{out} - \frac{k_f}{k_f-1}p_f\frac{dV}{dt}\right) \qquad (3)$$

where R – is the universal gas constant.

The material balance equation for freon in the compressor cavity has the form:

$$\frac{dm_f}{dt} = Q_{in} - Q_{out} \qquad (4)$$

Thus, as a result of certain transformations, we will have the following system of equations:

$$\begin{cases} c_vm_f\frac{dT_f}{dt} = kF(T_s-T_f) + (Q_{in}-Q_{out})\left(h_{out}\frac{k_f-1}{k_f} - c_vT_f\right) - Q_{out}(h_{in}-h_{out}) - p_f\frac{dV}{dt} \\ c_vm_f\frac{dp_f}{dt} = \frac{1}{RV}\left(kF(T_s-T_f) + Q_{in}h_{in} - Q_{out}h_{out} - \frac{k_f}{k_f-1}p_f\frac{dV}{dt}\right) - c_vm_f(Q_{in}-Q_{out}) \end{cases} \qquad (5)$$

To solve the stated problem, it is necessary to solve the obtained system of equations (5) simultaneously for each compression chamber, therefore we transform system of equations (5) to the following form:

$$\begin{cases} c_vm_{f,j}\frac{dT_{f,j}}{dt} = kF_j(T_s-T_{fj}) + (Q_{in,j}-Q_{out,j})\left(h_{out,j}\frac{k_f-1}{k_f} - c_vT_f\right) - Q_{out}(h_{in,j}-h_{out,j}) - p_{f,j}\frac{dV_j}{dt}, j=\overline{1-3} \\ c_vm_{f,j}\frac{dp_{f,j}}{dt} = \frac{1}{RV_j}\left(kF(T_s-T_{f,j}) + Q_{in,j}h_{in,j} - Q_{out,j}h_{out,j} - \frac{k_f}{k_f-1}p_{f,j}\frac{dV_j}{dt}\right) - c_vm_{f,j}(Q_{in,j}-Q_{out,j}), j=\overline{1-3} \end{cases} \qquad (6)$$

where $j = 1–3$ – are process parameters for suction, discharge, and compression chambers respectively.

To solve system of equations (6), we express the volume change with time dV/dt through the rate of change of orbital angle θ with time:

$$\frac{dV_j}{dt} = \frac{dV_j}{d\theta}\cdot\frac{d\theta}{dt} = \frac{dV_j}{d\theta}\omega, j=\overline{1-3}$$

where ω – is the angular velocity of the compressor motor shaft $\omega = 2\pi f$, f – is the compressor frequency.

As can be seen, this approach does not involve hydrodynamic analysis but represents a simplified thermodynamic model with lumped parameters that takes into account the geometric dimensions of refrigerant chambers.

During compressor operation, the moving scroll rotates counterclockwise in a circular orbit with a radius equal to the eccentricity of the scrolls, i.e., the distance by which the centers of the moving and stationary scrolls are separated from each other. The dependencies of volumes V of suction, compression, and discharge chambers on the orbital angle for different compression chambers are investigated in a number of works [1, 3, 7, 8]. To construct a mathematical description of compression chamber volume change, we will use the graphical dependencies presented in work [3], which are shown in Fig. 3.

Such graphical representations of volume change can be described by functional dependencies. As can be seen, the suction volume increases until reaching a maximum, then decreases when the suction chamber is closed. The change in compression chamber volume from the scroll rotation angle has a linear character. The discharge area volume sharply increases at the end of a complete scroll revolution, since at this moment the compression chambers open and become part of the discharge area.

To describe the volume change V and its current state for each compression chamber, we will approximate the empirical dependencies of volume change versus time (orbital angle).

We will approximate the empirical curve $V_1(t)$ for the suction chamber using a 3rd degree polynomial:

$$V_1(t) = a_{12} + b_{12}t + c_{12}t^2 + d_{12}t^3 \tag{7}$$

where $0 \leq t \leq t_k$, t_k – is the time of complete scroll rotation, and a_{12}, b_{12}, c_{12}, d_{12} – are polynomial coefficients to be found by the least squares method.

The empirical curves $V(t)$ for the compression and discharge chambers are approximated using cubic splines. The interval partition is performed at the moment t_l when the compression and discharge chambers merge.

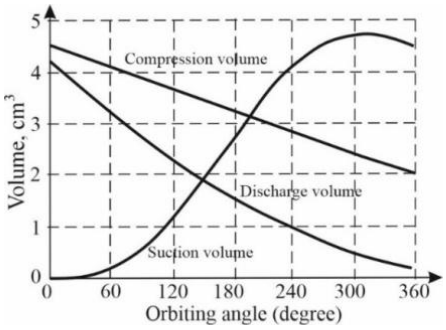

Fig. 3. Volume change in compression chambers [3]

For compression and discharge chambers, we write the approximation polynomials as:

$$V_j(t) = \begin{cases} a_{j1} + b_{j1}t + c_{j1}t^2 + d_{j1}t^3, t_0 \leq t \leq t_l, j = 2{,}3 \\ a_{j2} + b_{j2}(t - t_l) + c_{j2}(t - t_l)^2 + d_{j2}(t - t_l)^3, t_l \leq t \leq t_k, j = 2{,}3 \end{cases} \tag{8}$$

where $a_{j1} = V_j(t_0) = V_j(0)$, $a_{j2} = V_j(t_l)$.

We will select coefficients a_{j1}, b_{j1}, c_{j1}, d_{j1}, a_{j2}, b_{j2}, c_{j2}, d_{j2}, $j = 2, 3$, such that at the interval boundaries (t_0, t_l) continuity is ensured for both function $V_j(t)$, and its first $V'_j(t)$ and second $V''_j(t)$ derivatives at the boundaries of the intervals and (t_l, t_k) derivatives. Thus, the following conditions must be satisfied:

$$\begin{cases} V_j(t_0) + b_{j1}(t_l - t_0) + c_{j1}(t_l - t_0)^2 + d_{j1}(t_l - t_0)^3 = V_j(t_l), j = 2{,}3 \\ V_j(t_l) + b_{j2}(t_k - t_l) + c_{j2}(t_k - t_l)^2 + d_{j2}(t_k - t_l)^3 = V_j(t_k), j = 2{,}3 \\ b_{j1} + 2c_{j1}(t_l - t_0)^2 + 3d_{j1}(t_l - t_0)^3 = b_{j2}, j = 2{,}3 \\ 2c_{j1} + 6d_{j1}(t_l - t_0)^2 = 2c_{j2}, j = 2{,}3 \end{cases} \tag{9}$$

To determine all unknown coefficients $a_{j1}, b_{j1}, c_{j1}, d_{j1}, a_{j2}, b_{j2}, c_{j2}, d_{j2}, j = 2, 3$, it is necessary to add to the system of equations (9) with two more equations. For this, we set boundary conditions:

$$V''_j(t_0) = 2c_{j1} = 0,\ V''_j(t_k) = 2c_{j2} + 6d_{j2}(t_k - t_l) = 0 \tag{10}$$

Taking into account (7–10), we write the approximation relations for all compression chambers as:

$$V_j(t) = \begin{cases} V_1(t) = a_{12} + b_{12}t + c_{12}t^2 + d_{12}t^3 \\ \begin{cases} V_j(t_0) + b_{j1}t + d_{j1}t^3, t_0 \leq t \leq t_l, j = 2{,}3 \\ V_j(t_l) + b_{j2}(t - t_l) + c_{j2}(t - t_l)^2 + d_{j2}(t - t_l)^3 \\ \qquad t_l \leq t \leq t_k, j = 2{,}3 \end{cases} \end{cases} \tag{11}$$

Where coefficient $a_{j1}, b_{j1}, c_{j1}, d_{j1}, a_{j2}, b_{j2}, c_{j2}, d_{j2}, j = 2, 3$ are found from the system of equations:

$$\begin{cases} V_j(t_0) + b_{j1}(t_l - t_0) + d_{j1}(t_l - t_0)^3 = V_j(t_l), j = 2{,}3 \\ V_j(t_l) + b_{j2}(t_k - t_l) + c_{j2}(t_k - t_l)^2 + d_{j2}(t_k - t_l)^3 = V_j(t_k), j = 2{,}3 \\ b_{j1} + 3d_{j1}(t_l - t_0)^3 = b_{j2}, j = 2{,}3 \\ 3d_{j1}(t_l - t_0)^2 = c_{j2}, j = 2{,}3 \\ 2c_{j2} + 6d_{j2}(t_k - t_l) = 0, j = 2{,}3 \end{cases} \tag{12}$$

Next, we find transfer functions for temperature and pressure for each compression chamber. For this, we will assume that volume and its change in the above equations are written in increments relative to the basic steady-state regime. Since the task is to determine the transfer function, initial conditions are zero.

Applying the Laplace operator, we transform equation (6) to the form:

$$\begin{cases} \left(c_v m_{f,j} p_{f,j} + kF_j + c_v\left(Q_{in,j} - Q_{out,j}\right)\right) T_{f,j} = h_{out,j} \frac{k_f - 1}{k_f} Q_{in,j} + f_{1,j}, j = \overline{1-3} \\ \left(c_v m_{f,j} p_{f,j} + \frac{k_f}{(k_f - 1)RV_j} \frac{dV_j}{dt}\right) p_{f,j} = \left(\frac{1}{RV_j} h_{int_j} - c_v m_{f,j}\right) Q_{in,j} + f_{2,j}, j = \overline{1-3} \end{cases} \tag{13}$$

where:

$f_{1,j} = kF_j T_s - h_{out,j} \frac{k_f - 1}{k_f} Q_{out,j} - \left(h_{in,j} - h_{out,j}\right) Q_{out,j} - p_{f,j} \frac{dV_j}{dt}$, $\mathrm{j} = \overline{1\text{–}3}$, $f_2 = \frac{1}{RV_j}\left(kF_j\left(T_s - T_{f,j}\right) - Q_{out,j} h_{out,j}\right) + c_v m_{f,j} Q_{out,j}$,

p – is the Laplace operator.

We determine transfer functions for each equation of system (13), setting the control action as $Q_{in,j}$:

$$\begin{cases} W_{T_{f,j}}(p) = \frac{h_{out,j}\left(k_f - 1\right)}{k_f\left(c_v m_{f,j} p_{f,j} + kF_j + c_v\left(Q_{in,j} - Q_{out,j}\right)\right)}, j = \overline{1-3} \\ W_{p_{f,j}}(p) = \frac{\left(k_f - 1\right)\left(h_{int_j} - c_v m_{f,j}\right)}{c_v m_{f,j} p_{f,j} R V_j \left(k_f - 1\right) + k_f \frac{dV_j}{dt}}, j = \overline{1-3} \end{cases} \tag{14}$$

In the first transfer function of system of equations (14), the output quantity is $T_{f,j}$, which is determined caused by the control action $Q_{in,j}$ under the disturbance influence $f_{1,j} = 0$. In the second transfer function, the output quantity is $p_{f,j}$, caused by the control action $Q_{in,j}$ under the disturbance influence $f_{2,j} = 0$. The obtained transfer functions are not explicitly expressed since they include volume magnitude and its derivative, which must be calculated at each time moment (orbital angle) for each compression chamber. Therefore, to obtain transient characteristics, we will use equation (6), (11) and construct them using Simulink application of MatLab program simulation model using the derivative order reduction method.

3. Experimental studies of refrigerant passage process through heat pump compressor circuit

Model parameters were calculated based on data specified by heat pump and compressor equipment manufacturers. In particular, results were obtained for the XPV Copeland Scroll spiral compressor – this is a variable speed scroll compressor for R410A freon with inverter drive. The model of freon compression process constructed above was used to calculate the properties of refrigerant passing through suction, compression, and discharge chambers of the compressor. Based on transfer functions, as a result of simulation modelling in MatLab Simulink environment, transient characteristics for temperature h_{Tf} (Fig. 4) and pressure h_{pf} (Fig. 5) were obtained.

As can be seen from Fig. 4 and 5, with increasing freon mass flow rate, the quality of the transient process improves, namely the time constant decreases, which accordingly leads to a reduction in process inertia.

Amplitude-frequency (AFC) and phase-frequency (PFC) characteristics for temperature (Fig. 6) and compressor pressure (Fig. 7) were obtained.

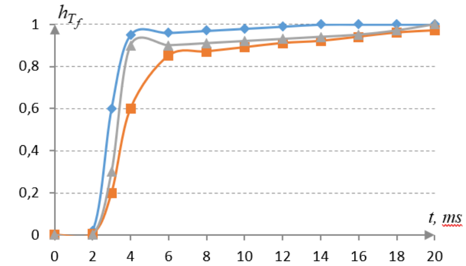

Fig. 4. Temperature transient characteristics of freon at different mass flow rates through the compressor: – 3.4 m^3/h; – 5.1 m^3/h ; – 8.9 m^3/h

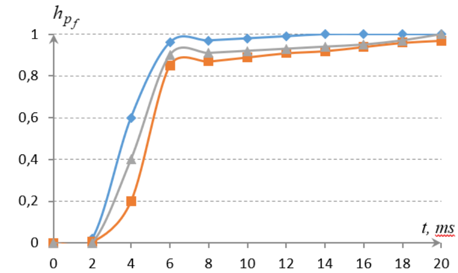

Fig. 5. Pressure transient characteristics of freon at different mass flow rates through the compressor: – 3.4 m^3/h; – 5.1 m^3/h ; – 8.9 m^3/h

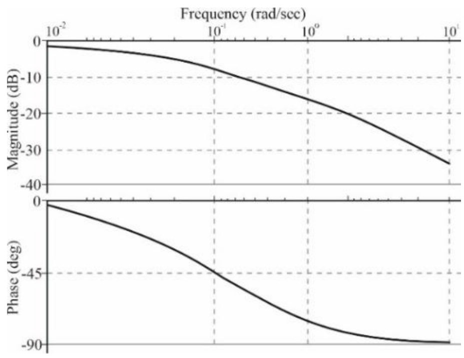

Fig. 6. AFC and PFC graphs of compressor by temperature

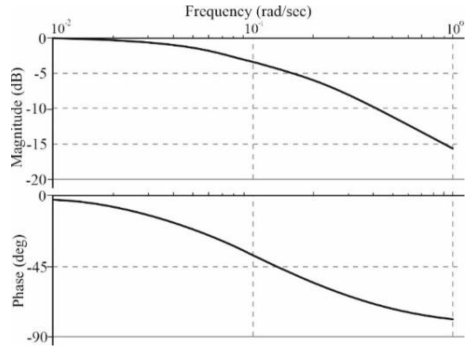

Fig. 7. AFC and PFC graphs of compressor by pressure

As can be seen from Fig. 6 and 7, in the AFC with decreasing frequency, the output signal amplitude increases and vice versa. Therefore, to obtain a quality transient process, it is necessary to maintain minor temperature fluctuations. From PFC analysis, we ee that output fluctuations in pressure and temperature lag behind input ones, and such lag varies within the range from 0 to 90°C. For a quality transient process, compressor productivity regulation is necessary, particularly using inversion.

4. Conclusion

An approach to modelling the working cycle in a compressor with variable freon mass, taking into account the change in compression chamber volume, is proposed. Our proposed model is based on material and energy balance equations of the heat pump working fluid. It is proposed to approximate empirical curves of volume change in suction, compression, and discharge compression chambers using cubic splines.

Transfer functions and transient characteristics of the compressor for temperature and pressure are obtained. From the transfer functions, it is established that with increasing freon mass flow rate, the quality of the transient process improves, namely the time constant decreases, which accordingly leads to a reduction in process inertia. Analysis of frequency characteristics showed that for a quality transient process, compressor productivity regulation is necessary, particularly using inverter technology.

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