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OPTIMAL CONTROL OF ELECTRIC ENERGY QUALITY, BASED ON LEXICOGRAPHIC APPROACH

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Abstract. *Important components of large and complex energy systems are electric energy supply systems, which perform the function of providing electric energy quality at the clamps of electric energy consumers. Development and realization of special methods and measures, enabling to synthesize the corresponding technical facilities, which will provide the possibility to obtain the basic indices of electric energy quality within the limits of DCTU (State standard of Ukraine) EN 50160:2014, is relevant technical problem. Taking into account the fact that quality indices correspond to different operation modes of electric energy supply systems consumers, the problem of searching methods, which would adequately represent the process of energy consumption, arises. In such operation conditions of modern systems of electric energy supply it is expedient to use the methods of searching optimal solutions which would enable to influence simultaneously the improvement (or non-worsening) of the quality indices of electric energy. The given paper considers theoretical aspects of using one of the most suitable methods for the solution of this problem, which relates to multicriterial ones, when it is not possible to express in simple terms the need to increase or decrease one efficiency criterion (quality index) and it is necessary to consider not one criterion but a set of partial criteria which form vector criterion. Corresponding mathematical models for one of the widely used problems of electric energy quality optimization have been formulated and considered. Practical example of their application has been presented, it will give the possibility to simplify the computations and provide adequate realization of optimization processes.*

Keywords: lexicographic optimization, electric energy quality, symmetry, voltage regulation, mathematical modeling

OPTYMALNE ZARZĄDZANIE JAKOŚCIĄ ENERGII ELEKTRYCZNEJ W OPARCIU O PODEJŚCIE LEKSYKOGRAFICZNE

Streszczenie*. Ważnymi elementami dużych i złożonych systemów energetycznych są systemy zasilania energią elektryczną, które pełnią funkcję zapewniania jakości energii elektrycznej na zaciskach odbiorników energii elektrycznej. Opracowanie i wdrożenie specjalnych metod i środków umożliwiających syntezę odpowiednich urządzeń technicznych, które zapewnią możliwość uzyskania podstawowych wskaźników jakości energii elektrycznej w granicach DCTU (norma państwowa Ukrainy) EN 50160:2014, jest istotnym problemem technicznym. Biorąc pod uwagę fakt, że wskaźniki jakości odpowiadają różnym trybom pracy odbiorców systemów zasilania energią elektryczną, pojawia się problem poszukiwania metod, które odpowiednio odzwierciedlałyby proces zużycia energii. W takich warunkach eksploatacji nowoczesnych systemów zasilania energią elektryczną wskazane jest stosowanie metod poszukiwania optymalnych rozwiązań, które umożliwiłyby jednoczesny wpływ na poprawę (lub niepogorszenie) wskaźników jakości energii elektrycznej. W niniejszym artykule rozważono teoretyczne aspekty zastosowania jednej z najbardziej odpowiednich metod rozwiązania tego problemu, która odnosi się do metod wielokryterialnych, gdy nie jest możliwe wyrażenie w prosty sposób potrzeby zwiększenia lub zmniejszenia jednego kryterium efektywności (wskaźnika jakości) i konieczne jest uwzględnienie nie jednego kryterium, ale zestawu kryteriów cząstkowych, które tworzą kryterium wektorowe. Sformułowano i rozważono odpowiednie modele matematyczne dla jednego z powszechnie stosowanych problemów optymalizacji jakości energii elektrycznej. Przedstawiono praktyczny przykład ich zastosowania, który umożliwi uproszczenie obliczeń i zapewni odpowiednią realizację procesów optymalizacji.*

Słowa kluczowe: optymalizacja leksykograficzna, jakość energii elektrycznej, symetria, regulacja napięcia, modelowanie matematyczne

Introduction

For the solution of multicriterial problems it is necessary to choose the strategy where each of the partial criteria is provided with the greatest possible value.

In such problems the ideal is considered the case, when there exists the strategy that transforms in maximum each of the partial criteria. In this case this strategy is considered to be optimal. However, it is known that such case can hardly be found in practice. In general case, when each of the partial criteria has its set of the maximizing strategies, it is rather difficult to determine the optimal strategy. For the solution of any multicriterial problem it is necessary to take into consideration the data, regarding relative importance of partial criteria. In the given paper some principles of the lexicographic approach for searching optimal strategies or lexicographically maximizing sequences of strategies will be considered.

1. Methods and materials of the research

In the determined case, i.e., in absence of random or uncertain factors, the selected strategy definitely determines the course and result of the operation [5, 9]. That is why, in the determined lexicographic optimization problem each strategy *"U"* is characterized by 5 numbers-values of partial criteria $K(u), K_1(u), \ldots, K_S(u)$. All the partial criteria, which form vector criterion are strictly arranged by the importance, so that, while comparing the pair of strategies, first of all the first criterion K_1 is used and the strategy for which the value of this criterion is greater is considered to be the best; if the value of the first criterion for both strategies is equal then the second criterion is taken and the preference is given to the strategy for which its value is greater; if the second criterion does not allow to identify the best strategy the third partial criterion is used, etc., up to K_S. Such problem is also called lexicographic problem of minimization [1, 4]:

$$K^* = lex\ \sup K(u), u \in U \tag{1}$$

sometimes it is briefly written as: find *lex* sup *K*(*u*), (1), *u* ϵ *U*, where *U* – is the set of all strategies.

If there exist lexicographic optimal strategies, then (1) will take the form: find *lex* max K(U), u ϵ U.

In some cases it is more convenient to formulate lexicographic problem so that all partial criteria become minimal. In such form the problem is lexicographic problem of minimization: find *lex* inf K(U), u ϵ U

The following methods are often used to find lexicographic optimal strategies:

1. Optimization of the processes with successive application of the criteria.
2. Optimization by the method of successive concessions.

Applying the methods of optimization with consistent application of criteria, any lexicographic problem can be reduced to the sequence of standard extremal problems:

$$\begin{aligned}
&1)\ \text{find}\ \max_{u\in U} K_1(u) = K_1^* \\
&2)\ \text{find}\ \max_{u\in U} K_2(u) = K_2^* \\
&\qquad K_1(u) = K_1^* \\
&\qquad \ldots\ldots\ldots\ldots \\
&s)\ \text{find}\ \max_{u\in U} K_s(u) = K_s \\
&K_r(u) = K_r^*\ r = 1,\ 2\ldots\ s-1
\end{aligned} \tag{2}$$

General process of such problem solution is reduced to the following procedure. First, the set of the control actions x_n+1 is searched, which would meet all the necessary conditions of the process optimality by the first partial criterion of the criterion, optimal value (which maximizes x_n+1) is found among these control actions and corresponding optimal value x_n+1, needed for further computations is calculated. Similarly problem 2) is solved at the known value x^*_n+1 and the value x^*_n+2 is determined, it is used at the next stages of the solution, etc. [6, 12].

Regarding lexicographic problem of electric energy quality optimization, the given procedure can be presented in the following manner. By means of the vector of the controlled variables, obtained as a result of the solution of the sub problem by the first partial criterion, new parameters of the mode, which must be represented in the following mathematical model to be solved by the second partial criterion, are determined. Further, the given algorithm is repeated [4, 8].

While solving separate partial problems mode parameters, not taken into account in the corresponding mathematical models, may be improve (side additive effect) or may worsen (side negative effect). But as the output data for the solution of each next task are the results, obtained in the process of the solution of the previous problem, lexicographically optimal solution will satisfy the most common goals of the problem.

While solving multicriterial problems, applying the successive concession method, first of all qualitative analysis of the relative importance of partial criteria is carried out [7].

First in importance criterion K_1 is maximized and its greatest value Q_1 is found. Then the value of the possible decrease (concession) $\Delta_1 > 0$ of K criterion is determined and the greatest value Q_2 of the second criterion K_2 is searched on condition that the value of the first criterion must not be less than $Q_1 < \Delta_1$. The value of concession $\Delta_2 > 0$ is assigned again but at another criterion, which together with the first concession is used for finding conditional maximum of the third criterion, etc.

Thus, any strategy, being the solution of the last problems of such sequence of the task is considered to be optimal:

$$\begin{aligned}
&\text{1) find } \sup_{u \in U} K_1(u) = Q_1 \\
&\text{2) find } \sup_{u \in U} K_2(u) = Q_2, \quad K_1(u) \geq Q_1 - \Delta_1 \qquad (3) \\
&\dots\dots\dots\dots\dots\dots\dots\dots \\
&\text{s) find } \sup_{u \in U} K_S(u) = Q_S, \quad K_r(u) \geq Q_r - \Delta_r, r = 1, 2, \dots, s-1
\end{aligned}$$

Comparisons (2) and (3) show that in case, when all concessions Δ_1 – are zero then successive concession method allocates only lexicographically optimal strategies. These strategies provide the greatest on U value to the first by importance K_1 criterion. In other extreme case, when the values of concession turn out to be very large, strategies, obtained by means of the given method, provide the greatest on U value to the last by importance partial criterion K_s.

That is why, the value of concessions, assigned for multi-criterial problem may be considered as a measure of the deviation of partial criteria priority from rigid lexicographic. Values of Δ_r concessions are assigned successively as a result of studying of partial criteria interrelations. In spite of the labor intensity in computational terms nowadays successive concession method is more advanced from the analytical and applied points if view. The advantages of the given method are the following: it enables to find the best resultant of really extremal values of all criteria and clearly shows at the expense of what "concessions" this can be achieved [6, 8].

Term "random concession" does not mean arbitrary actions, violating strict analyticity of the given method of making compromise decisions [9, 14]. Similar "concessions" are assigned only in the area of coincidence of the solution with the extremum (minimum and maximum), where, as a rule, criterion values change very slowly. That is why "sacrificing small values of one criterion near its optimum, in the given case considerable improvement of another criterion can be achieved".

Problems of electric energy quality optimization, as it was stated above, are multicriterial problems. It is quite natural to consider in such problems the set of partial criteria for the selection of the actions from the set of alternatives. In this connection, in the process of mathematical modeling and solution of the problems, dealing with the improvement of electric energy quality the problem of ranking according to the degree of advantages of objectives from vector criterion of optimality is important [12, 13].

For this purpose it is necessary to take into account the data, regarding the relative importance of partial criteria. The problem of comparison of strategy "quality" if vector criterion is available will be considered in details.

In the determined problem it is believed that if the inequality is satisfied

$$K_r(u) \geq K_r(\upsilon), \ r = 1 \dots s \qquad (4)$$

then the strategy U is not worse than strategy υ in terms of vector criterion ($u \geq \upsilon$). In case, if

$$K_r(u) = K_r(\upsilon), \ r = 1 \dots s \qquad (5)$$

then strategy U and υ are considered to be equivalent. Equivalent strategies must be considered as reciprocal.

Strategy U is considered to be better than υ strategy ($u > \upsilon$), if (4) takes place and if at least one of these inequalities is strict. In this case while transition from υ to u anything will be lost by none of the partial criteria, but there will be a gain relatively one of them [7, 10].

Strategy υ is called efficient in case if there is no strategy u, $u > v$, i.e., for which inequalities (4) were satisfied and at least one of them – strictly. In other words, strategies which cannot be improved by vector criterion are efficient strategies. For this reason as optimal strategies the efficient strategies must be chosen.

Thereby, studying the problem of strategies comparison by vector criterion enables to decrease the initial set of all the strategies U to the set of the efficient strategies $U°$. However, it is necessary to take into consideration that in some cases prior to solution of the problem regarding the selection of the best strategies, only as a result of studying the properties and structure of the $U°$ set important and non-trivial conclusions could be obtained.

As a rule, the set of efficient strategies contains not one but many strategies, which usually are non equal by the "quality". An important conclusion follows that for the selection of the optimal strategy from the whole set of the efficient strategies in each multicriterial problem it is necessary to use the additional data regarding the objective of the operation, i.e., the information which "in the process of formation only of vector criterion, without indication of the relative importance of partial criteria remained unformalized and therefore unused".

Thus, the approach to the problem of the process optimization, taking into account some criteria, greatly depends on the possibility of the assessment of the nonequivalence degree of various partial criteria and methods of assigning this nonequivalence.

The simplest solution of multicriterial problem of optimization is when all the partial criteria can be ranked by importance i.e., when it is possible to allocate "more important" criterion, next "more important" criterion, etc. The information, regarding the relative importance of the partial criteria can be obtained, relying on the content side of the optimization problem [3, 11].

Problem of multicriterial optimization of special modes in electric energy supply systems will be considered. In general case the problem contains the components of currents and voltages of reverse and zero sequence of the basic frequency, higher harmonics of currents and voltages. It may happen that all these components will have various phases' values. Deviation of positive sequence voltages may take place. In such, most general optimization problem setting, partial criteria may represent minimized functions of the corresponding indices of electric energy quality. Certainly, they are not equivalent relatively reaching the aim of special model optimization.

It is not expedient to perform mode optimization on the basic frequency without filtration of higher harmonics because:

a) power filters of higher harmonics can considerably change mode on the basic frequency;
b) mode optimization on the basic frequency can worsen harmonic spectrum.

Thus, filtration problem, among other partial aims should be given the priority.

Similarly, without voltage balancing of the source, it is not expedient to perform balancing and symmetry of loads.

In the process of mode optimization on the basic frequency first it is better to perform mode balancing. This is caused by the fact that balancing device as a rule is asymmetrical star of the power reactive elements with the grounded neutral and they may change the components of the forward and reverse sequences in the system. Balancing devices realized in the form of asymmetrical triangle of power reactive elements, in general case do not influence on the changes in the system of zero sequence ccomponents. That is why, among the partial objectives, aimed at mode optimization, mode balancing on basic frequency should be given preference. After that, having performed mode balancing, we can move to mode optimization in direct sequence.

Rule of partial objectives preferences, suggested above in the problems of multi -objective optimization is rather schematic and can be used for the solution of partial optimization problems. On the other hand, this rule can be taken as the basic for the solution of any problems of electric energy quality optimization.

In the determined problem it is believed that if the inequality is satisfied (4) then the strategy U is no worse than the strategy υ in terms of vector criterion ($u > \upsilon$).

In case if (5) then it is considered that strategies u and υ are equivalent. Equivalent strategies must be considered as equally valid.

Strategy u is considered to be better than strategy υ ($u > \upsilon$), if (4) takes place and at least one of these inequalities is strict. In this case when moving from υ to u nothing will be lost by any of the partial criteria, but relatively one of the them there will be definitely a gain.

Strategy υ is called efficient in case if there is no strategy u that ($u > \upsilon$), i.e., the strategy for which inequalities (4) would be satisfied and at least one of them – strictly. In other words, efficient are the strategies which cannot be improved according to vector criterion. Proceeding from this assumption, efficient strategies must be selected as optimal strategies.

Thus, studying the problem regarding the compression of the strategies by vector criterion enables to reduce the initial set of all the strategies U to the set of efficient strategies $U°$.

However, it should be taken into account that in some cases, prior to the solution of the problem, regarding the selection of the best strategies, important and far from trivial conclusions could be obtained only as a result of studying the properties and structure of the set $u°$

As a rule, the set of the efficient strategies contains not one but many strategies, which usually are not similar by "quality". From this statement the important conclusion follows that for the selection of optimal strategy from the whole set of efficient strategies in each multicriterial problem it is necessary to use additional information, regarding the objective of the operation i.e., the information which "in the process of vector criterion formation, without indication of the relative importance of partial criteria, remained non-formalized and therefore unused".

Thus, the approach to the problem of process optimization, taking into account some criteria depends on the possibility of assessment the degree of inequality of various partial criteria and methods of assigning this inequality [2, 3].

In the simplest possible way multicriterial problem of the optimization can be solved when all the partial criteria can be ranked by importance i.e., when "more important" criterion, next "more important" criterion etc. can be allocated. Information, regarding the relative importance of partial criteria can be obtained, based on the content side of the optimization problem.

The problem of multi-objective optimization of special modes in electrical energy supply systems will be considered in general case it contains the components of currents and voltages, reverse and zero sequences of the basic frequency, higher harmonics of currents and voltages.

It may happen that all these components will have different phase values. Deviation of positive sequence voltage may happen; in such most general optimization problem setting partial criteria may represent minimized function of the corresponding indices of electric energy quality. Certainly they are not equivalent relatively to the obtaining of the general objective of special modes optimization. Without filtration of higher harmonics it is not expedient to perform the optimization of the mode on the basic frequency because:

a) power filters of higher harmonics can considerably change mode on the basic frequency;
b) mode optimization on the basic frequency can worsen spectrum of harmonics.

Thus, filtration problem should be given priority among other partial objectives.

Similarly, without balancing voltages of the source it is not expedient to perform equilibration and balancing of loads.

In the process of mode optimization on the basic frequency first it is better to perform mode equilibration. This is due to the fact that equilibration devices are as a rule, asymmetrical star of power reactive elements with grounded neutral and can change the components of the direct and reverse sequences in the system. Balancing devices, executed in the form of asymmetrical triangle of power reactive elements in general case do not influence the changes in the system of the components of zero sequence. That is why; equilibration of the mode should be given priority among the partial objectives of mode optimization on the basic frequency. After that, having performed mode balancing we can move to mode optimization on the direct sequence.

Rule of the partial objectives preferences in the multi-objective optimization problems, suggested above, is rather schematic and can be applied for the solution of the partial optimization problems. On the other hand, this rule can serve as a base for the solution of any problems of electric energy quality optimization.

2. Results of the research

The possibility of applying the suggested lexicographic approach to the creation of the mathematical models of optimization of electrical energy quality in electric energy supply systems will be considered.

Modern systems of Electric Energy Supply (EES) are characterized by the great extent, multistage voltages transformation, and specific asymmetric consumers of electric energy. In such systems much attention must be paid to the energy quality provision, this objective can be achieved if all the parameters, quality of energy depends on, are in acceptable limits. Characteristics problem for such EES is optimization of nonsymmetrical mode and voltage regulation in the connection nodes of electric loads.

This problem is interpreted as multi-objective optimization problems of non-linear programming. Its multi-criteria character follows from the fact that two criteria of optimality are explicitly present. First it is availability of the voltage of the reverse sequence of the mode U_2, value of which as a result of balancing must be minimal. Second, it is the value of the voltage of the direct sequence of the mode on the load buses U_1^b which must take the value, close or equal to nominal voltage.

Important remark should be made, regarding the selection of the facilities to provide mode optimization both on the direct and reverse sequences. For the optimization of the asymmetrical modes in the loading node, as a rule, it is proposed to use static correcting devices, executed on the reactive elements, assembled according to the asymmetric triangle circuit.

Such devices are very efficient and can be used for local regulation of voltage [15].

The important point of the analysis and synthesis of multicriterial optimization problem is selection and creation of mathematical model. The authors recommend the following mathematical models for mode balancing and voltage regulation in loading node, these models will be written in the symbolic form:

$$\begin{cases} \cos ts_{estim}(x) \to \min \\ U_2(x) \le U_{2perm} \\ U_{\min} \le U_{\mathrm{I}}(x) \le U_{\max} \\ tg\varphi_i(x) \le tg\varphi_{iperm} \\ \boldsymbol{x} \ge 0 \end{cases} \quad (6)$$

$$\begin{cases} U_2(x) \to \min \\ U_2(x) \le U_{2perm} \\ U_{\min} \le U_{\mathrm{I}}(x) \le U_{\max} \\ \boldsymbol{x} \ge 0 \end{cases} \quad (7)$$

$$\begin{cases} Q_{inst}(x) \to \min \\ U_2(x) \le U_{2perm} \\ U_{\min} \le U_{\mathrm{I}}(x) \le U_{\max} \\ \boldsymbol{x} \ge 0 \end{cases} \quad (8)$$

$$\begin{cases} \cos ts_{estim}(x) \to \min \\ U_2(x) \le U_{2perm} \\ \boldsymbol{x} \ge 0 \end{cases} \quad \begin{cases} \cos ts_{estim}(x) \to \min \\ U_{\mathrm{I}}(x) \ge U_{\min} \\ \boldsymbol{x} \ge 0 \end{cases} \quad (9)$$

$$\begin{cases} \cos ts_{estim}(x) \to \min \\ U_2(x) \le U_{2perm} \\ \boldsymbol{x} \ge 0 \end{cases} \quad \begin{cases} U_{\mathrm{I}}(x) \to \max \\ \cos ts(x) \le \cos ts_{estim}(x) + \Delta \cos ts \\ \boldsymbol{x} \ge 0 \end{cases} \quad (10)$$

$$\begin{cases} Q_{inst}(x) \to \min \\ U_2(x) \le U_{2perm} \\ \boldsymbol{x} \ge 0 \end{cases} \quad \begin{cases} U_{\mathrm{I}}(x) \to \max \\ Q(x) \le Q_{inst}(x) + \Delta Q \\ \boldsymbol{x} \ge 0 \end{cases} \quad (11)$$

$$\begin{cases} U_2(x) \to \min \\ U_2(x) \le U_{2perm} \\ \boldsymbol{x} \ge 0 \end{cases} \quad \begin{cases} Q_{inst}(x) \to \min \\ U_{\min} \le U_{\mathrm{I}}(x) \le U_{\max} \\ \boldsymbol{x} \ge 0 \end{cases} \quad (12)$$

Notation in mathematical models:

$\cos ts_{estim}$ – calculation of the installation costs of the correcting device hrs/yr;

$U_2(x)$ – voltage of the reverse sequence on the loading buses, kV;

$U_{\mathrm{I}}(x)$ – voltage of the direct sequence on the loading buses, kV;

$tg\varphi_i(x)$ – reactive power factor of the compensated load;

$Q_{inst}(x)$ – installed power of the correcting device, kvar;

$\Delta \cos ts$, ΔQ – values of the concessions on costs for the installation of the correcting devices and reactive power consumption, kvar;

U_{2perm} – admissible range of values of the reverse sequence voltages in the trading node, kV;

$U_{\max}, U_{\min}$ – admissible range of values of the direct sequence voltages in the trading node, kV;

x – vector of the controlled variables which reflects currents by phases of the correcting device.

Models (6)…(8) from the point of view of mathematical programming are interpreted as non-linear and for their analysis require complex algorithms of non-linear programming. That is why; any simplification of these models on the condition of saving their adequacy to reality is relevant.

If the given multicriterial problem is considered from the position of lexicographic optimization (it is the most efficient method of general non-linear models simplification) then the sets of mathematical models (9) and (12) can find application.

In this case models (9) and (12) can be solved using the method of the sequential application of the criteria and models (10) and (11) – by the method of sequential concessions.

Performance of the suggested mathematical models was tested for the solution of multicriterial problem of mode balancing and voltage regulation in the loading node for the circuit, presented in Fig. 1.

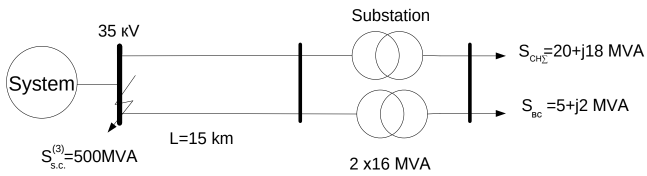

Fig. 1. Calculation scheme of the electric grid

Let asymmetric load (for instance, electric furnace) be connected to phases B, C of low voltage step-down substation. Initial data:

- wire AC-120:
  r_0 = 0.27 Ohm/km; x_0 = 0.405 Ohm/km
- transformer TW-16/35/10 (two-winding):
  ΔP_{sc} = 122 kW; $\Delta P_{no\text{-}loading}$ = 39 kW; U_{sc} = 8%; $I_{no\text{-}loading}$ = 3%

For the solution of the problem mathematical model (9) was used. Energy essence of this problem lies in balancing of the mode on the first sub-model with voltage limitations of the reverse sequence, established by DSTU (State Standard of Ukraine) EN50160:2014 [5]. The second sub model from the expression (9) requires minimalization of the calculation costs function on the condition of maintaining minimally allowable voltage level $U_{min.adm}$ on the load buses.

In this case the usage of the mathematical model, where the limitations would be set not on three values of the direct sequence voltage but only one (for any phase) is possible. It should be noted that the output data for this sub-model must be mode parameters, obtained after the solution of the sub problem of mode balancing on the base of the first sub-model.

One more important remark concerning the controlled variables should be made. In the given case the variables are currents, flowing on the phases of the correcting devices, performed in the form of the triangle of the capacitive elements:

$$x_1 = I_{C_{AB}}, \; x_2 = I_{C_{BC}}, \; x_3 = I_{C_{CA}}$$

First sub-model from (9) functionally has the form

$$\begin{cases} \cos ts_{estim} = \cos ts_0 + \begin{vmatrix} \cos ts_1 \\ \cos ts_2 \\ \cos ts_3 \end{vmatrix}^t \cdot X + \frac{1}{2} \cdot \gamma \cdot R \cdot 10^{-3} \cdot X^t \cdot \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} \cdot X \to \min \\ U_2 = Z_2 \left\{ u_0 + \begin{vmatrix} u_1 \\ u_2 \\ u_3 \end{vmatrix}^t \cdot X + \frac{1}{2} \cdot X^t \cdot \begin{vmatrix} 2/3 & -1/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{vmatrix} \cdot X \right\}^{1/2} = 0.02U \\ x \geq 0 \end{cases}$$

where

$$\cos ts_0 = \cdot \gamma \cdot R \cdot 10^{-3} \cdot (I_A^2 + I_B^2 + I_C^2)$$

$$\cos ts_1 = U \cdot (K_{spec} \cdot p + \gamma \Delta p) + \gamma \cdot R \cdot 10^{-3} \left[-2 \cdot I_A \cdot \sin(\varphi_A + 30^0) + 2 \cdot I_B \cdot \sin(30^0 - \varphi_B) \right]$$

$$\cos ts_2 = U \cdot (K_{spec} \cdot p + \gamma \Delta p) + \gamma \cdot R \cdot 10^{-3} \left[-2 \cdot I_B \cdot \sin(\varphi_B + 30^0) + 2 \cdot I_C \cdot \sin(30^0 - \varphi_C) \right]$$

$$\cos ts_3 = U \cdot (K_{spec} \cdot p + \gamma \Delta p) + \gamma \cdot R \cdot 10^{-3} \left[-2 \cdot I_C \cdot \sin(\varphi_C + 30^0) + 2 \cdot I_A \cdot \sin(30^0 - \varphi_A) \right]$$

I_A, I_B, I_C – are the currents in the corresponding phases A, B, C; φ_A, φ_B, φ_C – are angular shifts between phase voltages and currents; γ – is cost of 1 kW of active losses of electric energy, UAH/kW×yr; C_{ce} – is averaged cost of electric energy; β – is additional rate of double rate tariff UAH/kW×yr; α – is basic rate of double-rate tariff with maximum load using; T_m – is annual amount of hours of maximum active load, hrs/yr; T_a – annual time of electric unit switching, hrs/yr; K_{spec} – is unit cost of 1 kvar of reactive power of the capacitors or reactors, hrs/kvar; p – is total depreciation rate for fixed assets, operation, current repair, 1/yr; ΔP – is specific costs of the active power per 1 kvar of reactive power of the capacitors or reactors, kW/kvar; U – is linear voltage, kV; R – is active resistance of the supply line, Ohm;

$$U_0 = \frac{1}{9} \cdot \Big\{ I_A^2 + I_B^2 + I_C^2 - 2 \cdot I_A \cdot I_B \cdot \cos \cdot \left[60^0 - (\varphi_A - \varphi_B) \right] - 2 \cdot I_B \cdot I_C \cdot \cos \cdot \left[60^0 - (\varphi_B - \varphi_C) \right] - 2 \cdot I_A \cdot I_C \cdot \cos \cdot \left[60^0 - (\varphi_C - \varphi_A) \right] \Big\}$$

$$U_1 = \frac{1}{9} \cdot (3 \cdot I_A \cdot \cos\varphi_A + \sqrt{3} \cdot I_A \cdot \sin\varphi_A - 3 \cdot I_B \cdot \cos\varphi_B + \sqrt{3} \cdot I_B \cdot \sin\varphi_B - 2 \cdot \sqrt{3} \cdot I_C \cdot \sin\varphi_C)$$

$$U_2 = \frac{1}{9} \cdot (3 \cdot I_B \cdot \cos\varphi_B + \sqrt{3} \cdot I_B \cdot \sin\varphi_B - 3 \cdot I_C \cdot \cos\varphi_C + \sqrt{3} \cdot I_C \cdot \sin\varphi_C - 2 \cdot \sqrt{3} \cdot I_A \cdot \sin\varphi_A)$$

$$U_3 = \frac{1}{9} \cdot (3 \cdot I_C \cdot \cos\varphi_C + \sqrt{3} \cdot I_C \cdot \sin\varphi_C - 3 \cdot I_A \cdot \cos\varphi_A + \sqrt{3} \cdot I_A \cdot \sin\varphi_A - 2 \cdot \sqrt{3} \cdot I_B \cdot \sin\varphi_B)$$

Formalization of this model, which meets the requirements of the example, leads to such problem of quadratic programming:

Its solution gives the following control vector:

$$\begin{cases} 12604000 - \begin{vmatrix} 3793 \\ 4600 \\ 3328 \end{vmatrix}^t \cdot X + \frac{1}{2} \cdot X^t \cdot \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} \cdot X \to \min \\ \begin{vmatrix} -0.361 & 0.431 & -0.071 \\ -0.29 & -0.167 & 0.548 \end{vmatrix} \cdot X = \begin{vmatrix} 35.31 \\ 37.4 \end{vmatrix} \\ x \geq 0 \end{cases}$$

$$x_1^t = (578.2;\ 680.7;\ 694.9)$$

As a result of such partial optimization balancing of the mode (K_{U2} = 2%) is performed and voltage level at load buses is increased to 9.52 kV. Characteristic of such mode is shown in vector diagram (Fig. 2)

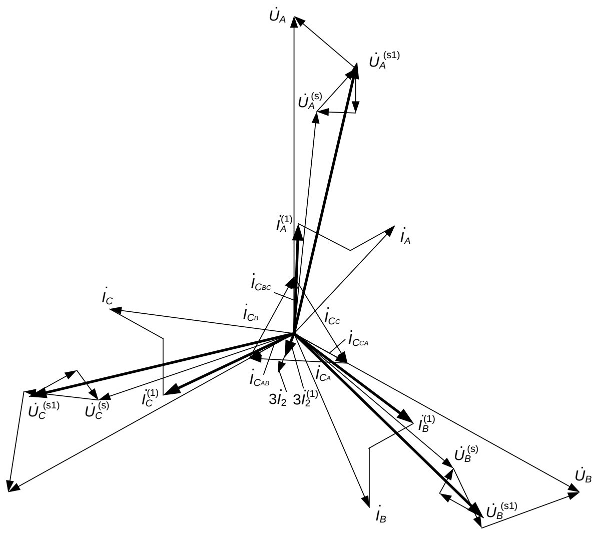

Fig. 2. Vector diagram of mode parameters of the system after partial optimization by the first sub-model (9)

For reaching minimal admissible voltage level it is necessary to form the second sub-model (9). Its formalization by the parameters of new mode, taking into account the switched on correcting device (control vector x_1t), gives such problem of quadratic programming:

$$\begin{cases} 7745000-\begin{vmatrix}107.04\\611.04\\688.04\end{vmatrix}^t\cdot X+\dfrac{1}{2}\cdot X^t\cdot\begin{vmatrix}2&1&1\\1&2&1\\1&1&2\end{vmatrix}\cdot X\rightarrow\min \\ \begin{vmatrix}0.0644&0.0644&0.0644&-1&0&0&0&0\\0.0473&0.0473&0.0473&0&-1&0&0&0\\-0.0171&-0.0171&-0.0171&0&0&-1&0&0\\0.0644&0.0644&0.0644&0&0&0&-1&0\\0.047&0.047&0.047&0&0&0&0&-1\end{vmatrix}\cdot X=\begin{vmatrix}35\\11\\11\\33\\28\end{vmatrix} \\ x\geq 0 \end{cases}$$

Its solution gives such control vector (A):

$$x_2^{\,t}=(198.5;198.5;168.5;7.35;17.17;0.817;0;5.35)$$

Connection of the correcting device (control vector x_2t) increases the voltage level on the load buses to the admissible value of 9.975 kV. Vector diagram, presented in Fig. 3 shows the characteristics of the mode after its complete optimization.

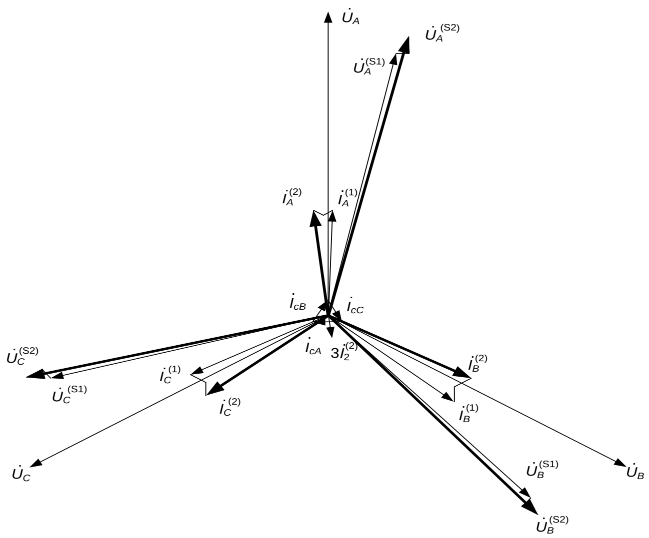

Fig. 3. Vector diagram of the complete optimization by the model (9)

Table 1 contains the comparative characteristics of the basic parameters of system operation made before and after optimization.

From the last vector diagram it is seen that as a result of the performed optimization calculations reactive power overcompensation of separate lines occurs. This should have been expected as the usage of the capacitor banks for the voltage regulation may lead to overcompensation. This provision seems like disadvantages at first glance but it is necessary due to the lack of other methods of the regulating impact. But the fact should not be forgotten, that due over -saturation at the system with the capacitive devices may lead to the violation of static durability of the load nodes. To take into account this circumstance it is expedient to introduce additionally in the sub-model (9) stability limitations of the load node

$$K_{stab}\,(x)\geq K_{st}$$

Table 1. Comparative characteristics of basic parameters of system operation mode

Basic parameters of system operation mode	Before optimization	After optimization by the criterion	
		U_2	U_1
Current value in phase A I^*_A, kA	$1.71e^{-j42^\circ}$	$1.33e^{-j1,76^\circ}$	$1.33e^{j5^\circ,38'}$
Current value in phase B I^*_B, kA	$2.2e^{-j37^\circ,24'}$	$1.715e^{j8^\circ,12'}$	$1.715e^{-j3^\circ,50'}$
Current value in phase C I^*_B, kA	$2.2e^{-j37^\circ,24'}$	$1.742e^{-j4^\circ,45'}$	$1.742e^{-j6^\circ,30'}$
Current of reverse sequence I^*_2, kA	$0.17e^{j157^\circ,36'}$	$0.11e^{j157^\circ,36'}$	$0.11e^{j187^\circ,38'}$
Voltages asymmetry coefficient ε_{U2}, %	2.3	2.0	2.0
Direct sequence voltage on the TR buses U_1^{III}, kV	9.100	9.520	9.975
Installed power of the capacitors Q_{inst}, kvar	-	17713	5940
Active power losses ΔP, kW	25.52	15.68	15.68

3. Conclusions

Thus, the task of the set of strictly ranked by the importance partial criteria "enables not only to allocate certain strategies as optimal but arrange all the strategies by the degree of their advantages similar to how the words are arranged while compiling a dictionary. That is why, the above-mentioned benefit ratio is often called lexicographic and multicriterial problems with strictly arranged by the importance criteria – are called lexicographic optimization problems".

It is shown that as the correcting device for the simultaneous mode balancing and voltage regulation asymmetric triangle of reactive elements can be used.

Models and methods of multicriterial optimization, enabling to realize intentionally mode balancing and voltage regulation on the load bused on the principles of lexicographic optimization are suggested.

Numerical example, given above, showed the efficiency of using mathematical models, suggested in the paper; these models are aimed at improvement of the indices of the electric energy quality.

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