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Methods for Comparing Three-Dimensional Motion Trajectories

Metody porównujące trajektorie ruchu trójwymiarowego

Tomasz Waldemar Samorow*, Maria Skublewska-Paszkowska

Department of Computer Science, Lublin University of Technology, Nadbystrzycka 36B, 20-618 Lublin, Poland

Abstract

Analysis of three-dimensional motion trajectories plays an important role in medicine, sports, robotics, and the entertainment industry. This research aims to compare the performance of the following six trajectory analysis algorithms: Euclidean Distance, Mean Squared Error, Dynamic Time Warping, Fréchet Distance, Fuzzy C-Means, and Fuzzy Similarity in terms of scalability, accuracy, computational efficiency, and robustness to speed variations. The research was conducted on the 3DTennisDS dataset containing tennis stroke trajectories recorded with the Vicon motion capture system. Results showed that fuzzy methods offer the best combination of accuracy (Fuzzy Similarity: 0.92, FCM: 0.89) and computational efficiency while maintaining high resistance to dynamic movements. In conclusion, fuzzy algorithms provide the most balanced solution for trajectory comparison in practical applications.

Keywords: 3D motion; motion capture; tennis; trajectory-comparison algorithms

Streszczenie

Analiza trójwymiarowych trajektorii ruchu odgrywa kluczową rolę w medycynie, sporcie, robotyce oraz branży rozrywkowej. Celem niniejszego badania jest porównanie wydajności sześciu algorytmów analizy trajektorii: odległości euklidesowej, średniego błędu kwadratowego (MSE), dynamicznego dopasowania czasowego (DTW), odległości Frécheta, a także metod Fuzzy C-Means i Fuzzy Similarity pod względem skalowalności, dokładności, efektywności obliczeniowej oraz odporności na zmiany prędkości. Eksperyment przeprowadzono na zbiorze 3DTennisDS, obejmującym trajektorie uderzeń tenisowych zarejestrowane systemem motion capture firmy Vicon. Wyniki wykazały, że metody rozmyte zapewniają najlepsze połączenie wysokiej dokładności (Fuzzy Similarity: 0,92; FCM: 0,89) i niskich kosztów obliczeniowych, zachowując jednocześnie dużą odporność na dynamiczne ruchy. Podsumowując, algorytmy rozmyte stanowią najbardziej zrównoważone rozwiązanie do porównywania trajektorii w praktycznych zastosowaniach.

Słowa kluczowe: algorytmy porównywania trajektorii; przechwytywanie ruchu; ruch 3d; tenis

*Corresponding author

Email address: **s95553@pollub.edu.pl** (T. W. Samorow)

1. Introduction

The analysis of three-dimensional motion trajectories plays an important role in medicine, supporting the diagnosis of movement disorders, rehabilitation, and surgical techniques [1, 2]. In sports, it enables the improvement of athletes' techniques [3], and in robotics and engineering, it assists in designing control systems for robots and exoskeletons [4, 5]. Motion capture technology in the entertainment industry allows for acquiring data further utilized for creation of realistic characters and virtual worlds [6, 7]. Accurate comparison of motion trajectories enhances the efficiency and precision of systems, especially in the context of human-machine interaction, workplace safety, and crowd behavior analysis [8, 9]. The increasing importance of these applications necessitates the development and comparison of methods for effective analysis of three-dimensional motion trajectories.

Consequently, this topic was chosen out of curiosity about how different methods deal with data uncertainty, measurement noise or varying motion speeds. Comparing different comparison methods will help identify the most effective algorithms for specific applications, which can significantly impact practices in medicine, sports, robotics and occupational safety.

The aim of this study is to compare selected algorithmic methods used for the analysis and comparison of three-dimensional motion trajectories. The study is conducted utilizing trajectories obtained from recordings using an optical motion capture system.

We will focus on three groups of methods. The first group consists of methods based on distance and similarity measures, Euclidean distance method and the mean squared error method. The second group includes methods for finding similar elements in trajectories, the Dynamic Time Warping method and the Fréchet distance method. The third group comprises methods based on fuzzy measurement techniques, the C-means method, and fuzzy similarity measures. Against this methodological backdrop, the study addresses four research questions:

RQ1: Does the length of the trajectory affect the performance of algorithms?

RQ2: Which of the algorithms provides the most accurate trajectory alignment?

RQ3: Which algorithm operates the fastest in terms of computation?

RQ4: Which algorithm best aligns trajectories with variable motion speeds?

2. Related Work

2.1. Motion-Capture Hardware and Data Formats

Progressions in motion capture technology and sophisticated equipment have maximized the collection and processing of motion data. For example, Vicon systems support accurate documentation of movement with the use of markers and high-definition cameras [10, 11], storing data in the universal C3D format [12]. Analysis is facilitated by open-source tools and libraries like Kinetics Toolkit [13] and Biomechanical ToolKit [14], which support processing and visualization in Python and MATLAB.

Researchers often develop custom code to assist in studies, for example gait analysis [11] or motion segmentation during ergometer exercises [3]. Databases like the KIT Whole-Body Human Motion Database [4] offer standardized motion records for research purposes.

2.2. Distance- and Similarity-Based Metrics

Beginning with metrics based on distance and similarity: a fundamental way uses distance measures like the Euclidean distance, which compares corresponding points of two trajectories at each time frame, measuring positional differences [15, 16]. It is effective for synchronized trajectories but limited while dealing with variable speeds or temporal shifts.

Another method is the Mean Squared Error (MSE), which calculates the average squared difference between trajectory points. It allows for a global assessment of similarity [17, 18], although it is sensitive to outliers and noise, potentially affecting accuracy.

2.3. Temporal Alignment Techniques

While comparing sequences of different lengths or rates of motion, Dynamic Time Warping (DTW) with Euclidean distance as the underlying metric becomes handy. DTW flexibly synchronizes time sequences, minimizing differences even with temporal shifts. It has applications in gait recognition using data from marker less systems [19] and in analysing sign language trajectories, where an optimized version, GLR-DTW, increased accuracy [20].

The Fréchet distance, which considers the shape of the trajectory and the order of points, is useful in analysing complex movements [18, 21]. In studies of temporomandibular joint mobility, it enabled the assessment of differences between trajectories of normal condyles and those replaced with prostheses [18].

2.4. Fuzzy Measurement and Clustering Methods

Clustering methods such as Fuzzy C-Means (FCM) [22] place trajectories into clusters with membership degrees, a property which is useful for cases where data is uncertain or clusters are overlapping [23, 24]. For the analysis of crowd movement, this algorithm allowed the more precise modelling and prediction of the movement patterns of pedestrians [23].

Fuzzy similarity measures are applied to data affected by noise, flexibly aligning trajectories and reducing the impact of discontinuities. In analysing vehicle behaviour, they improved trajectory precision by modelling uncertainty [25], while in air signature verification systems, they enabled more accurate comparison of signature trajectories [26].

3. Methodology

3.1. Algorithms Under Comparison

To address the research questions, we will apply and compare six algorithms across various experimental conditions. The methods include:

Euclidean Distance (eq. 1) - Direct point-to-point comparison [27]

$$d(p,q) = \sqrt{\sum_{i=1}^{n} (q_i - p_i)^2} \tag{1}$$

Mean Squared Error (eq. 2) - Average squared difference between trajectory points [28]

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}\left(Y_i - \widehat{Y_i}\right)^2 \tag{2}$$

Dynamic Time Warping (eq. 3) - Flexible temporal alignment of sequences [19]

$$DTW(P,Q) = \min_{w \in W}\left\{\sum_{(i,j)\in w} d\left(p_i, q_j\right)\right\} \tag{3}$$

Fréchet Distance (eq. 4) - Shape-based comparison considering point order [21]

$$\delta_F(f,g) = \inf_{\alpha,\beta} \max_{t\in[0,1]} \left|f\left(\alpha(t)\right) - g\left(\beta(t)\right)\right| \tag{4}$$

Fuzzy C-Means (eq. 5) - Soft clustering with variable membership [29]

$$J_m = \sum_{i=1}^{N}\sum_{j=1}^{C} u_{ij}^m |x_i - c_j|^2 \tag{5}$$

Fuzzy Similarity (eq. 6) - Flexible alignment with noise tolerance [30]

$$S(A,B) = \frac{\sum_{x\in X} \min\left(\mu_A(x), \mu_B(x)\right)}{\sum_{x\in X} \max\left(\mu_A(x), \mu_B(x)\right)} \tag{6}$$

3.2. Benchmark Dataset

All algorithms will be tested on the 3DTennisDS [31] dataset, a motion capture collection of tennis strokes performed by tennis players at the Lublin University of Technology in Poland. This dataset contains recordings of various tennis strokes (forehand, backhand, volleys) performed with and without ball, captured using the Vicon motion capture system.

The primary data files in the dataset are C3D format files named tpX_stroketype_sY.c3d, where X represents the participant identification and Y indicates the stroke number. The dataset includes data from 10 participants for each stroke type, with 10 recording sessions per participant. Each participant was prepared according to the Plug-in Gait Model with 39 body markers, while tennis rackets were fitted with 7 additional markers.

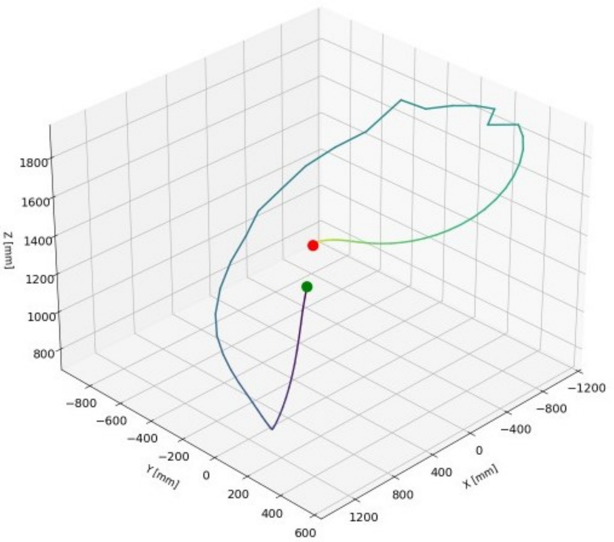

Figure 1: Three-dimensional path of the racket-hand (RH2) marker during Participant 1's forehand stroke.

3.3. Research stand

The software tools and hardware utilized in this research are summarized in Table 1.

Table 1: Summary of software libraries and hardware used in the study

Category	Tools / Hardware
Language	Python 3.11; R (+ ggplot2)
Tools / Libraries	NumPy; SciPy; fastdtw; scikit-learn; scikit-fuzzy; EZC3d; BTK; psutil; pandas
Workstation	AMD Ryzen 5 3600 · 16 GB DDR4-3200 · NVIDIA GTX 1060 6 GB
OS	Windows 10 Pro 2009

For each algorithm execution, the following data will be systematically recorded: raw performance metrics (execution time, memory consumption, CPU usage), alignment quality metrics (RMSE, correlation, feature matching).

3.4. Conducting research

To ensure the reliability and reproducibility of the results, all tests were conducted under controlled system conditions. The workstation, as specified in Table 1, had no other resource-intensive applications running in the background during the measurement sessions. Each test scenario was automated using a Python script to ensure consistent execution and data collection.

3.4.1. Sensitivity to trajectory length

We begin by asking how an algorithm's runtime, memory consumption and alignment quality evolve as the analyzed motion grows longer. From participant 1's forehand-with-ball recordings we pair consecutive sessions (1 vs 2, 3 vs 4, …, 9 vs 10) and cut every trajectory into short (19-frame), medium (38-frame) and long (76-frame) snippets centered on the impact phase.

Each pair is fed to every metric; execution time and peak RAM are measured with time, while the metric-specific score (RMSE, DTW cost, Fréchet distance, fuzzy similarity, etc.) is stored by comparing short-, medium- and long-segment results we can tell whether a method scales linearly, quadratically, or otherwise with sequence length.

3.4.2. Alignment accuracy

To isolate geometric accuracy, five medium-length strokes (38 frames, identical duration and tempo; see §3.2) were compared pairwise. Each algorithm supplied its native distance 'd' (RMSE, DTW cost, Fréchet distance, 1−1-1− similarity, etc.). Every set of five distances was rescaled to a common quality index q ∈ [0,1] such that 1 denotes perfect overlap and 0 the poorest match:

$$q = 1 - \frac{d - d_{min}}{d_{max} - d_{min}} \tag{7}$$

where d_{min} and d_{max} are the minimum and maximum values returned by that metric for the five pairs. Repeating the procedure with z-scores and with a global min-max confirmed that the ranking of algorithms is unchanged, so (7) does not bias the comparison.

3.4.3. Computational efficiency

Accuracy is irrelevant if a method is too slow for real-time feedback, therefore the third experiment focuses exclusively on computational cost. To ensure measurement stability, tests were run on an idle system.

Re-using all fifteen trajectory pairs (short, medium, long) from the first experiment, we execute fast metrics 100 times and slower ones 5 times, taking medians to smooth out performance fluctuations.

From the raw numbers we derive the time-per-frame, a length-normalised figure of merit, and compute scaling factors such as "long-runtime divided by short-runtime."

Plotting time against memory on log-axes lets us spot outliers that are, for instance, twice as slow and five times heavier than their peers.

3.4.4. Robustness to speed variation

Finally, to mimic athletes who execute a stroke faster or slower than the reference, we synthetically warp each of the five medium-length segments from participant 1's sessions 1-5. The warping factors applied were 0.5×, 0.75×, 0.8×, 0.9×, 1.1×, 1.2×, 1.5×, and 2.0×. Every warped trajectory is compared back to its original (1.0x speed); thus, each metric must align motions whose geometry is identical but whose tempo differs by up to ±100 %.

For every algorithm we chart the alignment error as a function of speed factor, calculate the average percentage error increase per 10 % speed change, locate the point where error exceeds an RMSE of 10 mm (failure threshold) and compute the variance of errors across all factors (consistency). (consistency).

4. Experimental Evaluation

4.1. Results

The experiments evaluated six trajectory-comparison algorithms on four metrics: scalability with trajectory length, alignment accuracy, computational efficiency, and robustness to speed variation.

Execution time for Euclidean Distance and MSE grew only 1.04 × when trajectory length quadrupled (Fig. 2). Dynamic Time Warping increased 4.75 × and Fréchet Distance 14.77 ×, whereas Fuzzy Similarity remained length-invariant (1.00 ×) and Fuzzy C-Means rose 1.74 ×.

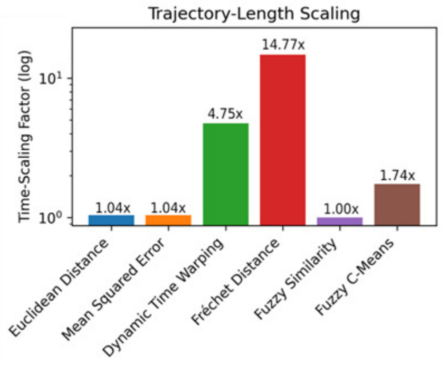

Figure 2: Trajectory-length scaling factor when the sample count is quadrupled (log scale; lower is better).

Regarding alignment accuracy (Fig. 3), Fuzzy Similarity obtained the highest score at 0.92. It was followed by FCM with a score of 0.89 (using the Fuzzy Partition Coefficient) and DTW at 0.87. The remaining methods scored lower: Fréchet distance at 0.76, Euclidean distance at 0.68, and MSE at 0.65.

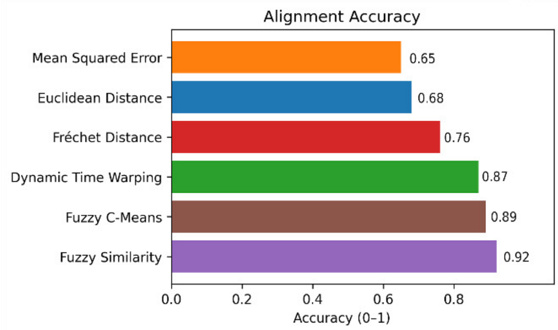

Figure 3: Alignment accuracy of the six algorithms (all values normalized to the 0 – 1 range).

In terms of computational efficiency (Fig. 4), the distance-based methods had the lowest resource consumption, with execution times under 0.15 ms and memory usage between 3–6 KB. Fuzzy Similarity's execution time was approximately 0.30 ms with 15–16 KB of memory. FCM's requirements were higher, at 5–8 ms and 55–161 KB. DTW and Fréchet distance were the most demanding; DTW took 10–45 ms and ≈226 KB of memory, while Fréchet's execution time exceeded 45 ms.

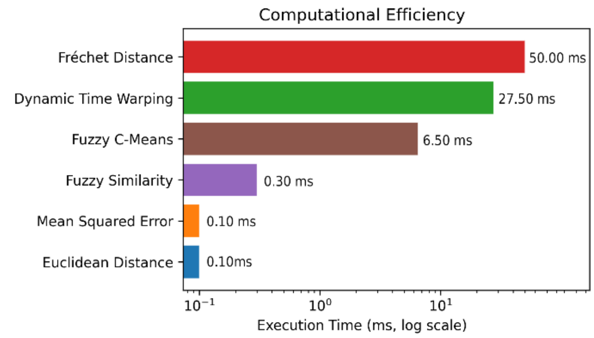

Figure 4: Computational efficiency on short segments (log-scale execution time in ms).

The assessment involved varying motion speeds from 0.5× to 2.0× (Fig. 5). The fuzzy methods showed the least change in error. For Fuzzy Similarity, the error changed by +0.22% per 10% speed shift, while for FCM, it decreased by –0.02%. Other algorithms were more sensitive: Fréchet distance had the largest error increase (+5.16%), followed by DTW (+2.72%). The Euclidean/MSE metrics were unstable, with their error decreasing by 1.37%.

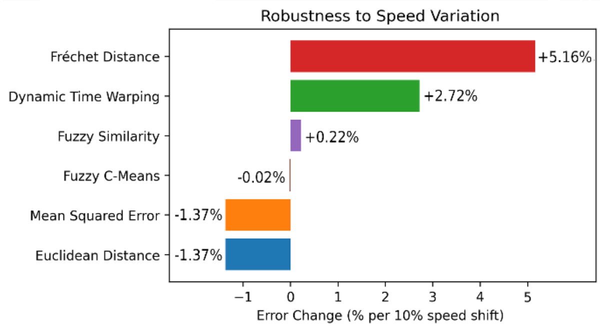

Figure 5: Change in alignment error per 10% speed shift (lower bars indicate higher robustness).

4.2. Discussion

The modest scaling of Fuzzy Similarity (1.00 ×) and FCM (1.74 ×) contrasts with DTW's quadratic growth (4.75 ×) and Fréchet's even steeper rise (Fig. 6), consistent with earlier reports of DTW's computational burden [32, 33, 34].

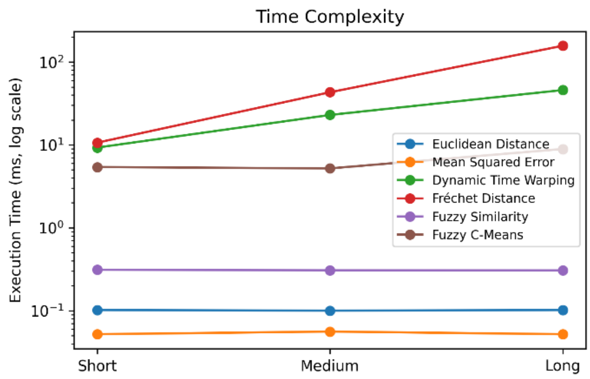

Figure 6: Time–complexity growth for each algorithm when trajectory length increases from short to long segments (log-ms).

Accuracy of Fuzzy Similarity led with 0.92, closely followed by FCM at 0.89, surpassing DTW (0.87). Izakian et al. document similar superiority of fuzzy hybrids over DTW on multiple datasets [29], while DTW's vulnerability to local noise has been noted by Gong et al. [32] and Zhao et al. [35]. These findings confirm that fuzzy metrics capture subtle shape variation without over-fitting.

Efficiency on distance measures remain the fastest, but Fuzzy Similarity's sub-millisecond runtime and FCM's single-digit-millisecond cost demonstrate that fuzzy methods can be lightweight when coupled with weighted DTW-based distances [29]. DTW's 10–45 ms runtime and large memory footprint mirror earlier quadratic analyses [33, 34].

Robustness of Fuzzy Similarity's minimal error drift (+0.22 %) and FCM's slight improvement (–0.02 %) under speed change affirm the hypothesis of temporal invariance; Fréchet's +5.16 % rise underscores its geometric rigidity. Prior studies on continuous DTW variants and learned warp distances corroborate fuzzy and learned-warp resilience to timing noise [34, 36].

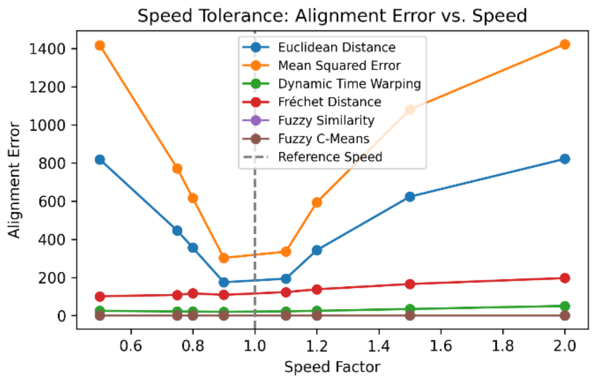

Figure 7: Speed–tolerance curves: alignment error versus speed factor (0.5x–2.0x). The dashed line marks the reference speed.

5. Conclusions

The purpose of this study was to evaluate six prominent trajectory-comparison algorithms—Euclidean Distance, Mean-Squared Error, Dynamic Time Warping, Fréchet Distance, Fuzzy C-Means, and Fuzzy Similarity. This evaluation was conducted against four key criteria: scalability with respect to trajectory length, alignment accuracy, computational efficiency, and robustness to speed variations.

The research successfully achieved this by conducting experiments on the 3DTennisDS motion-capture dataset. Key accomplishments include demonstrating fuzzy methods, particularly Fuzzy Similarity (0.92 accuracy, almost length-invariant) and Fuzzy C-Means (0.89 accuracy, modest runtime increase), offer the most balanced overall performance. In contrast, distance-based measures, while fastest, were sensitive to temporal distortions; Dynamic Time Warping provided good accuracy but with quadratic time complexity, and Fréchet Distance was the slowest and most length-dependent.

The research addressed four specific questions. Findings indicated that: (RQ1) algorithm performance, particularly runtime, is affected by trajectory length, with fuzzy methods showing more favorable scaling than DTW; (RQ2) fuzzy algorithms generally provide more accurate trajectory alignment compared to classical distance measures; (RQ3) while distance-based metrics are computationally fastest, fuzzy methods offer a strong balance of accuracy and efficiency; and (RQ4) elastic methods like DTW are more robust to speed variations than rigid metrics, with fuzzy methods also showing high robustness. These experimental outcomes provided comprehensive answers to the initial research questions

This study has two main limitations that also suggest directions for future work. First, the evaluation used a single dataset (3DTennisDS) specific to tennis, which limits the generalizability of the findings. Future work should test these algorithms on more diverse movements, such as gait or industrial tasks, to confirm their effectiveness.

Second, the selection of algorithms was based on the author's discretion and was not exhaustive. The performance ranking is therefore relative to the chosen set. Future studies could expand this comparison to include other methods, such as those based on deep learning.

References

[1] B. Błaszczyk, Biomechanika układu ruchu człowieka, PZWL, Warsaw, 2012.

[2] T.-W. Lu, C.-F. Chang, Biomechanics of human movement and its clinical applications, Kaohsiung Journal of Medical Sciences 28(2) (2012) S13–S25, https://doi.org/10.1016/j.kjms.2011.08.004.

[3] K. Kowalczyk, M. Skublewska-Paszkowska, Metoda docinania faz ruchu podczas wiosłowania na ergometrze na podstawie danych trójwymiarowych, Journal of Computer Sciences Institute 5 (2017) 155–158, https://doi.org/10.35784/jcsi.614.

[4] C. Mandery, Ö. Terlemez, M. Do, N. Vahrenkamp, T. Asfour, The KIT whole-body human motion database, In Proceedings of the 2015 International Conference on Advanced Robotics (ICAR) (2015) 329–336, https://doi.org/10.1109/ICAR.2015.7251476.

[5] B. Ren, Z. Zhang, C. Zhang, S. Chen, Motion trajectories prediction of lower limb exoskeleton based on long short-term memory (LSTM) networks, Actuators 11(3) (2022) 73, https://doi.org/10.3390/act11030073.

[6] A. Menache, Understanding Motion Capture for Computer Animation and Video Games, 2nd ed., Morgan Kaufmann, Burlington, 2011, https://doi.org/10.1016/C2009-0-62989-5.

[7] J. Tang, K. Kim, K. Wang, From screen to reality: exploring the evolution and integration of motion capture technology for virtual digital humans, Asia-Pacific Journal of Convergent Research Interchange 10(4) (2024) 301–316, https://doi.org/10.47116/apjcri.2024.04.12.

[8] M. H. Rahman, M. R. Hasan, N. I. Chowdhury, M. A. B. Syed, M. U. Farah, Predictive health analysis in Industry 5.0: a scientometric and systematic review of motion capture in construction, Digital Engineering 1 (2024) 100002, https://doi.org/10.1016/j.dte.2024.100002.

[9] X. Song, X. Chen, X. Li, J. Sun, X. Hou, S. Cui, Z. Zhang, H. Xiong, X. Wang, Pedestrian trajectory prediction based on deep convolutional LSTM network, IEEE Transactions on Intelligent Transportation Systems 24(2) (2023) 1667–1679, https://doi.org/10.1109/TITS.2020.2981118.

[10] N. Goldfarb, A. Lewis, A. Tacescu, G. S. Fischer, Open Source Vicon Toolkit for motion capture and gait analysis, Computer Methods and Programs in Biomedicine 212 (2021) 106414, https://doi.org/10.1016/j.cmpb.2021.106414.

[11] N. Darras, C. Joseph, A. Murphy, Clinical Gait Analysis Manager: freeware application to store, process and present gait analysis data, Gait & Posture 106 (2023) S233, https://doi.org/10.1016/j.gaitpost.2023.08.019.

[12] S. Bagheri, C3Dtools: Free access biomechanical toolkit, https://www.c3dtools.com, [12.01.2025].

[13] F. Chénier, Kinetics Toolkit: an open-source Python package to facilitate research in biomechanics, Journal of Open Source Software 6(66) (2021) 3714, https://doi.org/10.21105/joss.03714.

[14] A. Barrea, S. Armand, Biomechanical ToolKit: open-source framework to visualize and process biomechanical data, Computer Methods and Programs in Biomedicine 114(1) (2014) 80–87, https://doi.org/10.1016/j.cmpb.2014.01.012.

[15] J. Valčík, J. Sedmidubský, P. Zezula, Assessing similarity models for human-motion retrieval applications, Computer Animation and Virtual Worlds 27(5) (2016) 484–500, https://doi.org/10.1002/cav.1674.

[16] D. Schueller, C. Beecks, M. Hassani, J. Hinnell, B. Brenger, T. Seidl, I. Mittelberg, Automated pattern analysis in gesture research: similarity measuring in 3D motion capture models of communicative action, Digital Humanities Quarterly 11(2) (2017) 1–14, http://www.digitalhumanities.org/dhq/vol/11/2/000309/000309.html, [22.01.2025].

[17] J. Beernaerts, E. Debever, M. Lenoir, B. De Baets, N. Van de Weghe, A method based on the Levenshtein distance metric for the comparison of multiple movement patterns described by matrix sequences of different length, Expert Systems with Applications 115 (2019) 373–385, https://doi.org/10.1016/j.eswa.2018.07.076.

[18] J. Wang, J. Hua, R. Ding, L. Zou, H. Li, L. Zhang, Q. Sun, D. He, Comparative study of normal condyle and temporomandibular joint prosthesis movement during mouth opening by dynamic MRI and CT, Quantitative Imaging in Medicine and Surgery 13(7) (2023) 4606–4620, https://doi.org/10.21037/qims-22-1239.

[19] A. Świtoński, T. Krzeszowski, H. Josiński, B. Kwolek, K. Wojciechowski, Gait recognition on the basis of markerless motion tracking and DTW transform, IET Biometrics 7(5) (2018) 415–422, https://doi.org/10.1049/iet-bmt.2017.0134.

[20] W. Li, Z. Luo, X. Xi, Movement trajectory recognition of sign language based on optimized dynamic time warping, Electronics 9(9) (2020) 1400, https://doi.org/10.3390/electronics9091400.

[21] K. Buchin, T. Ophelders, B. Speckmann, Computing the similarity between moving curves, Computational Geometry 73 (2018) 2–14, https://doi.org/10.1016/j.comgeo.2017.01.002.

[22] M. Skublewska-Paszkowska, P. Powroźnik, P. Karczmarek, E. Łukasik, J. Smołka, Fuzzy C-Means Clustering for Motion Capture Tennis Time-Series Data, IEEE Access 12 (2024) 150975–150996, https://doi.org/10.1109/access.2024.3463201.

[23] W. Lu, X. Wei, W. Xing, W. Liu, Trajectory-based motion pattern analysis of crowds, Neurocomputing 247 (2017) 213–223, https://doi.org/10.1016/j.neucom.2017.03.074.

[24] Z. Izakian, S. M. Mesgari, A. Abraham, Automated clustering of trajectory data using particle swarm optimization, Computers, Environment and Urban Systems 55 (2016) 55–65, https://doi.org/10.1016/j.compenvurbsys.2015.10.009.

[25] X. Wang, J. Xu, Y. Song, Q. Zheng, J. Lv, W. Yan, Q. Cai, Z. Dai, Vehicle behavior analysis using reconstructed 3D parameters for road safety, Safety Science 144 (2021) 105419, https://doi.org/10.1016/j.ssci.2021.105419.

[26] A. R. Alobaidi, T. Dhieb, T. M. Hamdani, A. Wali, K. Ouahada, A. M. Alimi, In-air signature verification system based on beta-elliptical approach and fuzzy perceptual detector, IEEE Access 11 (2023) 134058–134073, https://doi.org/10.1109/ACCESS.2023.3336860.

[27] M. M. Deza, E. Deza, Encyclopedia of Distances, 2nd ed., Springer, Berlin, 2013, 5–6, https://doi.org/10.1007/978-3-642-30958-8.

[28] J. Fürnkranz, P. K. Chan, Mean squared error, In C. Sammut, G. I. Webb (Eds.), Encyclopedia of Machine Learning, Springer, Boston, 2010, 666–667, https://doi.org/10.1007/978-0-387-30164-8_528.

[29] H. Izakian, W. Pedrycz, I. Jamal, Fuzzy clustering of time-series data using dynamic time-warping distance, Engineering Applications of Artificial Intelligence 39 (2015) 235–244, https://doi.org/10.1016/j.engappai.2014.12.015.

[30] C. P. Pappis, N. I. Karacapilidis, A comparative assessment of measures of similarity of fuzzy values, Fuzzy Sets and Systems 56 (1993) 171–174, https://doi.org/10.1016/0165-0114(93)90141-4.

[31] M. Skublewska-Paszkowska, P. Powroźnik, E. Łukasik, J. Smołka, Tennis patterns recognition based on a novel tennis dataset – 3DTennisDS, Advances in Science and Technology Research Journal 18(6) (2024) 159–176, https://doi.org/10.12913/22998624/191264.

[32] Z. Gong, Y. Chen, Dynamic State Warping, arXiv preprint arXiv:1703.01141v1 (2017), https://arxiv.org/abs/1703.01141.

[33] S. Soheily-Khah, P.-F. Marteau, Sparsification of the alignment path in DTW, arXiv preprint arXiv:1711.04453v1 (2017), https://arxiv.org/abs/1711.04453.

[34] K. Buchin, A. Nusser, S. Wong, Computing Continuous Dynamic Time Warping of Time Series in Polynomial Time, In Proceedings of the 38th Int. Symp. on Computational Geometry (SoCG 2022), LIPIcs 224 (2022) 22:1–22:16, https://doi.org/10.4230/LIPIcs.SoCG.2022.22.

[35] Q. Zhao, L. Itti, Shape Dynamic Time Warping, arXiv preprint arXiv:1606.01601v1 (2016), https://arxiv.org/abs/1606.01601.

[36] J. Rhodes, A. Lee, TimewarpVAE, arXiv preprint arXiv:2310.16027v2 (2023), https://arxiv.org/abs/2310.16027.